CH 2 – MULTIPLYING WHOLE NUMBERS

❑ **Multiplying Whole Numbers – A New Approach**

Recall from Chapter 1 one of the ways we could expand 3,857:

$$3,857 \ = \ 3,000 + 800 + 50 + 7$$

This is the kind of expansion we need for this section on multiplying whole numbers. Here are some more examples:

$$796 \ = \ 700 + 90 + 6$$

$$23,908 \ = \ 20,000 + 3,000 + 900 + 8$$

I know that you know how to multiply whole numbers, but I wonder if you understand what the process truly entails.

EXAMPLE 1: **By expanding the 79, multiply 6 by 79.**

Solution: By expanding the 79, we can write the problem as

$$6 \times (70 + 9)$$

Now comes the interesting part. To carry out this multiplication of 6 times the sum in the parentheses, we first multiply the 6 by the 70 <u>and</u> then we multiply the 6 by the 9, and keep the addition sign between these two products:

$$(6 \times 70) + (6 \times 9)$$

This gives:

$$420 + 54$$

Adding gives us our final answer:

$$\boxed{474}$$

> The key concept here is that the 6 is to be multiplied by <u>both</u> the 70 and the 9. We can say that the 6 has been *distributed* to both the 70 and the 9. Just to make sure that this strange multiplication method works, just multiply 6 by 79 the regular way to confirm our answer.

EXAMPLE 2:   **By expanding the 428, multiply 5 by 428.**

Solution: When 428 is expanded, the problem becomes

$5 \times (400 + 20 + 8)$

Notice that there are three terms inside the parentheses, so we *distribute* the 5 to all three terms:

$(5 \times 400) + (5 \times 20) + (5 \times 8)$

Performing all the multiplications first gives

$2,000 + 100 + 40$ [Always multiply before adding.]

and we get the final answer of

2,140   Multiply the old way to confirm our answer.

EXAMPLE 3:   **By expanding both numbers and distributing, calculate 37 × 85.**

Solution: Expand each number, but this time, instead of using the × for multiplication, we'll use parentheses next to each other:

$(37)(85) = (30 + 7)(80 + 5)$

This time we have two numbers in each set of parentheses, so to distribute we will multiply both numbers in the first set of parentheses by both numbers in the second set. That is,

1.   multiply 30 by 80
2.   multiply 30 by 5
3.   multiply 7 by 80
4.   multiply 7 by 5

Writing out these four multiplications gives

$(30)(80) + (30)(5) + (7)(80) + (7)(5)$

which becomes:

$$2400 \quad + \quad 150 \quad + \quad 560 \quad + \; 35 \; = \quad \boxed{3,145}$$

The key concept is that each of the numbers 30 and 7 must be multiplied by each of the numbers 80 and 5. And again, check your answer by multiplying the regular way.

EXAMPLE 4:   **By expanding both numbers and distributing, calculate 29 × 438.**

Solution: Expand the numbers:

$(20 + 9)(400 + 30 + 8)$

Multiply each number in the first set of parentheses by each number in the second set of parentheses:

$20(400) + 20(30) + 20(8) + 9(400) + 9(30) + 9(8)$

$= \quad 8,000 + 600 + 160 + 3,600 + 270 + 72$

$= \quad \boxed{12,702}$

❑ **The Distributive Property**

The **Distributive Property** is the official title we give to the idea that, for example,

$3(90 + 5) \; = \; 3(90) + 3(5)$

We say that the 3 has been *distributed* to both the 90 and the 5. Is this a long, obscure way to do simple multiplication? It sure is! But practicing the Distributive Property with simple numbers is the best way to get ready for the Distributive Property in algebra when letters come into play, and where you'll use it constantly.

Homework

1. Use the Distributive Property (that is, by expanding the numbers) to find each **product** (check by multiplying the old way):

    a. 7×32

    c. $9 \cdot 234$

    e. $9(83)$

    g. $(7)(1{,}056)$

    i. 123×456

    b. $84(53)$

    d. $(53)(439)$

    f. 50×72

    h. $38 \cdot 129$

    j. 203×999

    > Algebra Preview:
    >
    > The *Distributive Property* can be written abstractly as
    >
    > $$a(b + c) = ab + ac$$

2. [Optional] Distribute to simplify each expression:

    Hint: Use the Distributive Property in the box above.

    a. $x(y + z)$ b. $a(b - c)$ c. $n(t + w)$

    d. $7(x + y)$ e. $3(c + d - e)$ f. $w(x - y + z)$

❑ **Algebra Preview – Equations**

Here's what you're going to see (possibly later in this course – certainly in Algebra). Suppose you're asked to solve the equation

$$3(n + 7) + 2n = 3$$

The <u>first</u> thing you are going to do is *distribute*:

$$3n + 21 + 2n = 3$$

and then proceed from there.

Solutions

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1. Note: Only two solutions are given here – you can check the rest by the standard method for multiplying.

c. $9 \cdot 234 \;=\; 9(200 + 30 + 4)$

Now multiply the 9 by the 200, then by the 30, and then by the 4:

$$= \; 9(200) + 9(30) + 9(4)$$

$$= \; 1{,}800 + 270 + 36$$

$$= \; \mathbf{2{,}106}$$

d. $(53)(439) \;=\; (50 + 3)(400 + 30 + 9)$

Now multiply both numbers in the first parentheses by all three numbers in the second parentheses:

$$= \; 50(400) + 50(30) + 50(9) + 3(400) + 3(30) + 3(9)$$

$$= \; 20{,}000 + 1{,}500 + 450 + 1{,}200 + 90 + 27$$

$$= \; \mathbf{23{,}267}$$

2. a. $xy + xz$ b. $ab - ac$ c. $nt + nw$

d. $7x + 7y$ e. $3c + 3d - 3e$ f. $wx - wy + wz$

❑ **To ∞ and Beyond**

Use the distributive property to simplify (expand):

$$(a + b)(x + y + z)$$

"Opportunity is missed by most people because it is dressed in overalls and looks like work."

– Thomas Edison