Ch 3 – Exponents

❑ Introduction

We learned about exponents in Chapter 1. For example, we learned that $10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$. The ***base*** doesn't have to be 10. Here are some more examples of exponents:

$2^3 = 2 \times 2 \times 2 = \mathbf{8}$

$5^4 = 5 \cdot 5 \cdot 5 \cdot 5 = \mathbf{625}$

$12^2 = (12)(12) = \mathbf{144}$

$19^1 = \mathbf{19}$

$10^0 = \mathbf{1}$

> Raising a number to the second power is called **squaring**. Thus, the *square* of 12 is 144.

Homework

1. Work each problem:

a. 7×10^0 b. $10^0 + 10^0$ c. $(6 + 4)^0$

d. $10^0 + 10^1 + 10^2$ e. $10^3 - 10^0$ f. $10^2 - 10^0$

In this chapter we'll see lots of exponents, but more importantly, we're going to start learning how to think about the logic of algebra. A common failing of algebra students is that they seem to think that if they don't "know" something, they're stuck. This is really sad, because most of math can be "figured out" at the moment with just a little common sense and the willingness to experiment.

For example, next year in Algebra you'll be required to memorize the "Five Laws of Exponents." But, you know what? We can figure out these laws of exponents right now even though we've barely begun Pre-Algebra. Check out the following four examples.

❑ Analyzing Exponent Expressions

<u>EXAMPLE 1:</u> True or False: $\mathbf{2^2 \times 2^3 = 2^5}$

<u>Solution:</u> We work each side of the equality separately. If the sides match, then the statement is true; otherwise it's false.

$2^2 \times 2^3$	2^5
4×8	$\underline{32}$ ✓
$\underline{32}$	

$[2^5 = 2 \times 2 \times 2 \times 2 \times 2]$

The two sides match, and therefore it's a **true** statement.

<u>EXAMPLE 2:</u> True or False: $\mathbf{(3 + 5)^2 = 3^2 + 5^2}$

<u>Solution:</u> Again, we work out the answer on each side:

$(3 + 5)^2$	$3^2 + 5^2$
8^2	$9 + 25$
$\underline{64}$	$\underline{34}$

The two sides <u>don't</u> match, and hence it's a **false** statement.

EXAMPLE 3: True or False: $\left(\frac{10}{2}\right)^3 = \frac{10^3}{2^3}$

Solution: To work out the left side of the statement, the most logical approach would be to do the inside of the parentheses first. Taking into account that a fraction means division:

$\left(\frac{10}{2}\right)^3$	$\frac{10^3}{2^3}$	
5^3	$\frac{1000}{8}$	We conclude that the
$\underline{125}$	$\underline{125}$ ✓	statement is **true**.

EXAMPLE 4: True or False: $(2^2)^3 = 2^5$

Solution: Work out each side of this potential equality:

$(2^2)^3$	2^5
4^3	$\underline{32}$
$\underline{64}$	

The quantities do not match. The statement is **false**.

Homework

2. By working each side of the given expression, determine whether the statement is true or false:

a. $10^3 \cdot 10^2 = 10^5$

b. $2^2 \times 2^5 = 2^{10}$

c. $3^1 \cdot 3^4 = 3^4$

d. $10^3 \times 10^3 = 10^6$

3. True or False:

a. $10^2 + 10 = 10^3$ b. $2^4 + 2^3 = 2^7$

c. $3^2 + 3^3 = 36$ d. $4^3 + 4^3 = 2^7$

4. True or False:

a. $10^0 - 10^0 = 0$ b. $10^0 + 10^3 = 10^3$

c. $10^4 - 10^0 = 9{,}999$ d. $10^6 - 10^6 = 10^0$

5. True or False:

a. $\dfrac{2^5}{2^3} = 2^2$ b. $\dfrac{10^6}{10^2} = 10^3$

c. $\dfrac{2^{10}}{2^{10}} = 0$ d. $\dfrac{10^9}{10^9} = 1$

Another example of the Order of Operations is that exponents always come before multiplication. Thus,

3×5^2
$= 3 \times 25$
$= 75$

6. True or False:

a. $\left(\dfrac{10}{2}\right)^3 = \dfrac{10^3}{2^3}$ b. $\left(\dfrac{6}{3}\right)^2 = \dfrac{6^2}{3}$

c. $\left(\dfrac{12}{3}\right)^2 = \dfrac{12}{3^2}$ d. $\left(\dfrac{8}{2}\right)^3 = \dfrac{8^3}{2^3}$

7. True or False:

a. $(2^3)^2 = 2^5$ b. $(10^3)^2 = 10^6$

c. $(1^4)^5 = 1$ d. $(2^3)^4 = 2^7$

8. True or False:

a. $(10 + 2)^2 = 10^2 + 2^2$

b. $(3 + 4)^2 = 3^2 + 4^2$

c. $(5 + 4)^2 = 5^2 + 40 + 4^2$

d. $(2 + 6)^2 = 2^2 + 24 + 6^2$

9. True or False:

a. $(2 \times 3)^2 = 2^2 \times 3^2$

b. $(3 \cdot 4)^3 = 12^3$

c. $(2 \times 5)^3 = 8 \times 125$

d. $(4 \cdot 6)^2 = 10^2$

Solutions

1. a. 7 b. 2 c. 1 d. 111 e. 999 f. 99

2. a. T b. F c. F d. T

3. a. F b. F c. T d. T

4. a. T b. F c. T d. F

5. a. T b. F c. F d. T

6. a. T b. F c. F d. T

7. a. F b. T c. T d. F

8. a. F b. F c. T d. T

9. a. T b. T c. T d. F

❑ To ∞ and Beyond

Give an explanation why $2^0 = 1$.

Simplify: $(x - a)(x - b)(x - c)(x - d) \cdots (x - y)(x - z)$

Note: There are 26 factors being multiplied.

"Use what talents you possess. The woods would be very silent if no birds sang there except those that sang the best."

– *Henry Van Dyke*