CH 8 – STATISTICS: WHOLE NUMBERS

❑ The Mean

Monica and Jarod are arguing over who is doing better in their Pre-Algebra class. Monica boasts the highest test score, so she claims she is doing better. But Jarod says his scores are more consistent, so he's doing better. How can we help them figure out who is doing better?

One way (of many) to settle this dispute is to calculate the ***mean*** of each of their test scores, and then simply compare the means to see who has the higher grade. The mean of a set of numbers is just a fancy word for what most people refer to as the *average*. The ***mean*** is calculated by adding all the numbers together and then dividing that sum by how many numbers were added.

For example, assume Monica's test scores in the class were 78, 92, 99, 63, and 93. To find the **mean** test score, we divide the sum of the five tests by 5:

$$\text{Monica's mean} = \frac{78+92+99+63+93}{5} = \frac{425}{5} = \mathbf{85}$$

For Jarod's scores of 94, 83, 88, 91, and 89, we get his mean:

$$\text{Jarod's mean} = \frac{94+83+88+91+89}{5} = \frac{445}{5} = \mathbf{89}$$

So instead of arguing about all 10 test scores, and trying to decide if Monica's score of 99 makes her scores better than Jarod's, or trying to estimate the damage done by Monica's score of 63, or argue about whose grades are more consistent, we simply compute each student's mean score, and let the math decide: Jarod wins.

Many times in business and science, the mean is not a fair summary of data, so we now study another measure of center, the *median*.

❑ The Median

The highway median is the "center divider," the portion of the highway that is in the middle of the lanes. Assuming a normal 6-lane highway, there are three lanes on each side of the median. In the same way, the ***median*** of a set of numbers is the middle number in the list of numbers.

There are two important issues here. One is that for the median to make any sense at all and truly be the middle number, we must first put the numbers in order. It doesn't matter whether we sort them in ascending (small to big) or descending (big to small) order. The second issue will be covered in homework problem 14.

For instance, let's calculate the **median** of Monica's test scores in the previous example. Notice that if we take the five scores in the order presented in the problem, the middle score would be 99:

78 92 99 63 93 the wrong way ☹

But that's absurd -- who could possibly claim that 99 (which is the biggest score!) is the middle score, just because it's in the middle of the list? So let's not forget to put the five numbers in order first; for no particular reason, we'll sort them in ascending order:

63 78 92 93 99 the right way ☺

Now which is the middle score? Do you see that the 92 has two scores below it and two scores above it? 92 is the middle score, and therefore, **the median is 92**.

To help you keep track of the meanings of *mean* and *median*, notice that the words "median" and "middle" each have "d" as their third letter.

❑ Contrasting the Mean and the Median

Consider the following set of numbers:

18 47 89 103 143

Add the five numbers together and divide by 5 to get a mean of 80. Also, since the numbers are already in ascending order, the median is 89.

Now watch what happens when we change the 143 at the end of the data listed above to 8,573, and leave the rest of the numbers alone:

18 47 89 103 **8,573**

Calculate the new mean and you should get 1,766, way bigger than the original mean of 80. But now check this out: The median is still 89! So, even though the mean jumped up considerably when we increased the last number, the median stayed exactly the same.

Conclusion: Extremely small or extremely large numbers may wreak havoc on the mean, but may have no effect on the median.

Homework

1. Find the median of 37, 77, 58, 41, 67
2. Find the median of 53, 29, 3, 55, 98
3. Find the mean of 74, 68, 12, 54
4. Find the mean of 5, 27
5. Find the mean of 16, 41, 40, 70, 78
6. Find the median of 44, 30, 99
7. Find the median of 63, 72, 99
8. Find the mean of 17, 10, 32, 65

If you take Statistics, you'll find two notations for the ***mean***. The *population mean* is denoted by μ, the Greek letter mu (pronounced myoo); the *sample mean* is written $\bar{x}$ (read as *x*-bar).

9. Find the median of 29, 75, 65

10. Find the mean of 52, 32, 93

11. Find the mean of 8, 18

12. Find the median of 48, 73, 71, 2, 9, 15, 99

13. Your friend is calculating the mean of 50 numbers, where the smallest number in the bunch is 17 and the largest is 99.

    a. What would you say if your friend calculated a mean of 123?

    b. What about a mean of 88?

    c. What about a mean of 15?

14. You may have noticed something about the concept of median and all the homework problems on median: Every problem contained an <u>odd</u> quantity of numbers, namely either 3, 5, or 7 numbers. What if there's an <u>even</u> number of numbers? For example, how do we find the median of the four numbers

    23 30 38 97

    where there's no middle number? We'll calculate the number that is <u>halfway</u> between the two numbers closest to the middle. In other words, we'll find the *mean* of 30 and 38, which is

    $$\frac{30+38}{2} = \frac{68}{2} = 34$$

    Thus, our conclusion is that **the median is 34**, even though 34 was not one of the original numbers. This is alright, because we remember that the main feature of the median is that there are as many numbers <u>below</u> it as there are <u>above</u> it. And sure enough, two numbers (23 & 30) are below the median 34 and two numbers (38 & 97) are above it.

    And don't forget to put the numbers in order before you calculate the median.

Calculate the median of each set of numbers:

a. 27 25 100 18

b. 6 22 21 37 13 2

c. 50 15 12 9 12 8

d. 17 45 100 0

e. 34 113 287 77 89 50

f. 6 8 4 7 7 3 4 2

Solutions

1. 58 **2**. 53 **3**. 52 **4**. 16 **5**. 49 **6**. 44

7. 72 **8**. 31 **9**. 65 **10**. 59 **11**. 13 **12**. 48

13. Only the 88 makes sense. Ask if you don't understand.

14. a. 26 b. 17 c. 12 d. 31 e. 83 f. 5

The new "zipper" **MEDIAN** on the Golden Gate Bridge

"Life does not need to be changed.

Only your intent and actions do."

– Swami Rama