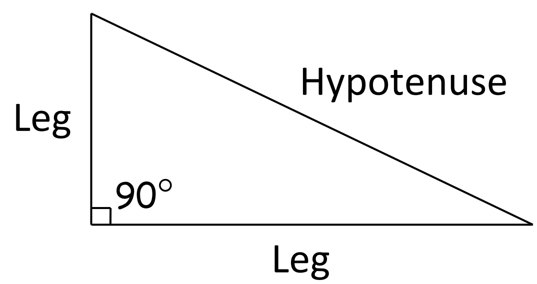
CH 8 – THE PYTHAGOREAN THEOREM

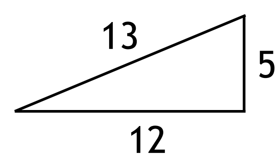
❑ *THE RIGHT TRIANGLE*

An angle of 90° is called a ***right angle***, and when two things meet at a right angle, we say they are ***perpendicular***. For example, the angle between a flagpole and the ground is 90°, and so the flagpole is perpendicular to the ground.

If we have a *triangle* with a 90° angle in it, we call the triangle a ***right triangle***. The two sides which form the right angle (90°) are called the ***legs*** of the right triangle, and the side opposite the right angle is called the ***hypotenuse***. Is it clear that the hypotenuse is always the longest side of the right triangle?



❑ *THE PYTHAGOREAN THEOREM*



A Classic Right Triangle

Ancient civilizations discovered that a triangle with sides 5, 12, and 13 (whether the unit of measurement was inches, cubits, or anything) would always be a right triangle -- that is, a triangle with a 90° angle in it. [Note that the hypotenuse must be the side of length 13.] But what if just the two legs are known? Is there a way to calculate the length of the hypotenuse? The answer is yes, and the formula dates back to 500 BC, the time of Pythagoras.

To discover this formula, let's rewrite the three sides of the above right triangle:

leg = 5 leg = 12 hypotenuse = 13

Since $5 + 12 \neq 13$, it's clear that the hypotenuse is not simply the sum of the two legs.

It should be clear that it can't possibly be the case that adding the two legs gives the hypotenuse. After all, the hypotenuse is the shortest distance between the two corners (vertices) of the triangle, so it must be less than "going the long way" along the legs.

Here's the secret: Use the idea of squaring. If we square the 5, the 12, and the 13, we get

$5^2 = \underline{25}$ $12^2 = \underline{144}$ $13^2 = \underline{169}$

and we notice (if we're as clever as Pythagoras was!) that the sum of 25 and 144 is 169:

$$25 + 144 = 169$$

In other words, a triangle with sides 5, 12, and 13 forms a right triangle precisely because

$$5^2 + 12^2 = 13^2$$

Now let's try to express this relationship in words -- it appears that

When you square the legs of a right triangle and add them together, you get the square of the hypotenuse.

As a formula, we can state it this way:

If a and b are the legs of a right triangle and c is the hypotenuse, then

$$a^2 + b^2 = c^2$$

❑ *SOLVING RIGHT TRIANGLES*

EXAMPLE 1: **The legs of a right triangle are 6 m and 8 m. Find the hypotenuse.**

Solution: We begin by writing the Pythagorean Theorem. Then we plug in the known values, and finally determine the hypotenuse of the triangle.

$a^2 + b^2 = c^2$ (the Pythagorean Theorem)

$\mathbf{6}^2 + \mathbf{8}^2 = c^2$ (substitute the known values)

$36 + 64 = c^2$ (square each leg)

$100 = c^2$ (simplify)

This last statement is called an **equation**, and states that the square of some number is 100. Well, what number, when squared, results in 100? A little experimentation yields the solution 10 (since $10^2 = 100$). Our conclusion is that $c = 10$, and thus

The hypotenuse is 10 m

Homework

1. In each problem, the two legs of a right triangle are given. Find the hypotenuse. Ask your teacher whether you may use a calculator.

a. 3, 4	b. 5, 12	c. 10, 24	d. 30, 16
e. 7, 24	f. 12, 16	g. 30, 40	h. 9, 40
i. 12, 35	j. 20, 21	k. 48, 55	l. 13, 84

2. In each problem, the length and width in inches of an LCD TV screen are given. Find the diagonal measure of the screen.

   a. $l = 8$, $w = 6$

   b. $l = 48$, $w = 14$

   c. $l = 48$, $w = 20$

3. In each problem, two traveling distances in miles are given. How many miles is the person from his starting point?

   a. first 9 miles North; then 40 miles West
   b. first 7 miles South; then 24 miles East
   c. first 10 miles East; then 24 miles South
   d. first 40 miles West; then 42 miles North

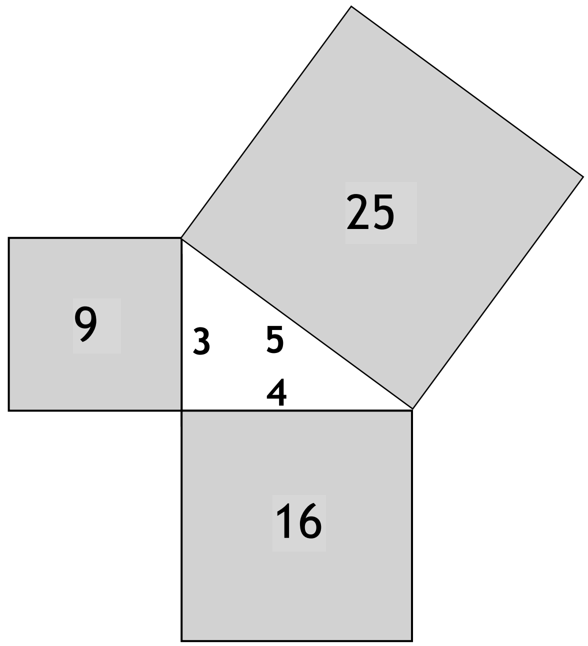
❑ *FINAL NOTES*

A Greek Perspective on the Right Triangle

As stated in Book I, Proposition 47, of Euclid's *Elements*:

> *"In right-angled triangles, the square on the side subtending the right angle is equal to the squares on the sides containing the right angle."*

Notice that the Pythagorean Theorem is stated geometrically -- it's not the algebraic formula we wrote. For example, consider the following right triangle with squares attached to each of the three sides:



The Greeks would not have claimed that $3^2 + 4^2 = 5^2$, but instead that the **areas** of the squares attached to the legs (the 9 and the 16) add up to the **area** of the square attached to the hypotenuse (the 25).

Using a Right Triangle in Construction

To see if a wall is truly vertical (straight up) relative to the floor, a contractor will do something like the following: Take a board (like a two-by-four) and lean it (diagonally) against the wall from some spot on the floor. Assume that the length of the board is 9 ft, that the distance from the bottom of the board to the wall (along the floor) is 6 ft., and that the board touches the wall 7 ft. above the floor. If the floor and the wall are truly perpendicular, then the numbers 6, 7, and 9 had better satisfy the Pythagorean Theorem (thus guaranteeing a 90° angle, indicating a vertical wall):

$$6^2 + 7^2 = 36 + 49 = 85, \text{ whereas } 9^2 = 81.$$

We do NOT have a right triangle, and so the angle between the floor and the wall is NOT 90° -- time for some reconstruction.

QUIZ

4. The legs of a right triangle are 15 and 36. Find the hypotenuse.

5. The legs of a right triangle are 44 and 483. Find the hypotenuse. [You may use a calculator on this one.]

6. The length and width of an LCD TV set are 42 in and 40 in. Find the diagonal measure of the screen.

7. Maria walks 7 km north and then 24 km west. How far is Maria from her starting point?

8. Why should you never argue with a 90° angle?

Solutions

1. a. 5 b. 13 c. 26 d. 34 e. 25 f. 20
g. 50 h. 41 i. 37 j. 29 k. 73 l. 85

2. a. 10 in b. 50 in c. 52 in

3. a. 41 mi b. 25 mi c. 26 mi d. 58 mi

4. 39 **5.** 485 **6.** 58 in **7.** 25 km

8. Because it's always right!