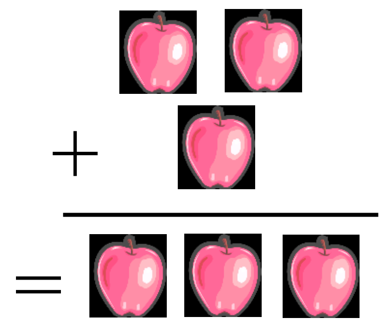
CH 15 – COMBINING LIKE TERMS

❑ Introduction



We've solved lots of equations so far in this course, but those equations had the variable in just one place. What shall we do if we come across an equation like $7x - 2x - 9 = 12x + 1$, with that darned variable all over the place? I suppose you can guess that this chapter will give us the tool we need to solve that kind of equation.

❑ The Common-Sense Approach to Combining Like Terms

If we add 5 inches and 6 inches, we get a sum of 11 inches:

5 inches + 6 inches = 11 inches

If we subtract \$14 from \$50, we get a difference of \$36:

\$50 – \$14 = \$36

If 7 widgets were sold last week and 3 widgets were sold this week, then the two-week total is 10 widgets. We can write

7 widgets + 3 widgets = 10 widgets

And if we let w represent a widget, we can write this fact as

$7w + 3w = 10w$,

and we say that $7w$ and $3w$ are ***like terms***, because each term represents some number of the exact same item.

Similarly, if 500 widgets were stolen from a warehouse that originally stored 2,000 widgets, we can write the amount remaining as 2,000 widgets – 500 widgets = 1,500 widgets, or

$$2{,}000w - 500w = 1{,}500w$$

We can also do this simplifying with more than two terms. For example, if we add 7 apples and 9 apples, then subtract 6 apples, and lastly add 10 additional apples, we can find the final number of apples like this:

$$7A + 9A - 6A + 10A = 20A$$

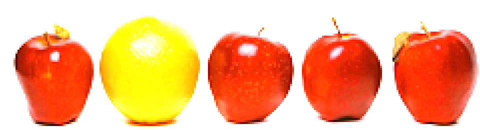
Homework

1. Work each problem, using the idea of *combining like terms*:

a.	9 feet + 12 feet	b.	12 lbs – 3 lbs
c.	14 apples + 14 apples	d.	\$123 + \$234
e.	100 meters – 98 meters	f.	17 widgets + 3 widgets
g.	93 cm – 90 cm	h.	3 iPads + 7 iPads
i.	€17 – €7 (€ = euro)	j.	$34\ \text{m}^2 + 16\ \text{m}^2$ (m^2 = square meter)

❑ Some Terms Cannot be Combined

We cannot do this adding (or subtracting) unless the items we're adding or subtracting are exactly the same. This is the old "*you can't add apples and oranges*" idea. Consider trying to analyze the sum "4 apples + 1 orange." Although you may "add" these together and say that the total is 5 pieces of fruit, you must agree that the sum is not 5 apples and it's not 5 oranges.



We say that "4 apples" and "1 orange" are ***unlike terms***; they cannot be added together. The terms $3x$ and $2w$ are also unlike terms and so the sum $3x + 2w$ cannot be simplified. Similarly, $7n$ and 5 are unlike terms and cannot be added together to make a single term.

This "simplification" process (assuming we have like terms) is called ***combining like terms*** and is used constantly in algebra. It basically says that regardless of what a variable might mean, we can combine two or more terms with the same variable into a single term by merely doing the arithmetic on the numbers connected to the variables; these numbers are called ***coefficients***. For example, when we write

$$5u + u = 6u$$

we say that the first term has a *coefficient* of 5, the second term has a *coefficient* of 1 (understood), and the sum (the answer) has a *coefficient* of 6.

❑ Examples

A. $10c + 20c = 30c$

B. $9x + x = 9x + 1x = 10x$

C. $-7y + 4y = -3y$

D. $\frac{2}{3}t - \frac{2}{3}t = 0t = 0$

E. $18m - 20m = -2m$

F. $-10h - 3h = -13h$

G. $25a - 13a - a = 11a$

H. $7x + 9y$ cannot be simplified (unlike terms)

I. $13N - 2$ cannot be simplified (unlike terms)

J. $7x - 7 - 9x + 12 = 7x - 9x - 7 + 12 = -2x + 5$

Homework

2. Your study partner is resisting the idea of like terms, and insists that $3x + 7$ ought to equal $10x$. By letting $x = 5$, prove your partner wrong.

3. Simplify each expression by combining like terms:

a. $10n + 7n$ b. $8x - 8x$ c. $14z + 14z$

d. $19a + a$ e. $32r - 20r$ f. $8x + 7y$

g. $77n + 77$ h. $24z - 24z$ i. $12Q - 11Q$

j. $-8x + 5x$ k. $-9t - t$ l. $12a - 20a$

m. $-9 + 9L$ n. $-7e - 7e$ o. $-12k - 13k$

4. Simplify each expression by combining like terms:

a. $3x + 7x - 8$ b. $9 + 2g - g$ c. $7x - 3x + 10$

d. $7 + 5x - 3$ e. $12d - 13d + d$ f. $4 - 7x + 9x$

g. $7y + 3y - 10y$ h. $-x - 2x - 3x$ i. $7w + 2w - 20w$

j. $-6r + 3r - r$ k. $30 - 3q + 3q$ l. $9u - 12 - 12u$

5. Simplify each expression by combining like terms:

a. $7x - 3 + 8x - 5$ b. $17 - n - n - 1$

c. $20q - 8q + 8 - 8$ d. $-1 - x - 1 - x$

e. $40i - 8 - 39i - 8$ f. $4v + 7 - 7v - 34$

g. $33w + 99 - 99 - 33w$ h. $-7x + 6x + 2x - 8 + 8$

Review Problems

6. Simplify by combining like terms:

   $7n + 2n - 8 - 1 + 5n - n - n + 9 - 13n + 20n$

7. Now your study partner is insisting that adding $3n$ and $2n$ should come out $5n^2$. That is, he claims that $3n + 2n = 5n^2$. Prove him wrong again.

Solutions

1.

a.	21 feet	b.	9 lbs	c.	28 apples	d.	\$357
e.	2 meters	f.	20 widgets	g.	3 cm	h.	10 iPads
i.	€10	j.	50 m^2				

2. We will compare the expression $3x + 7$ with the expression $10x$ specifically when $x = 5$:

$3x + 7 = 3(\mathbf{5}) + 7 = 15 + 7 = 22$

$10x = 10(\mathbf{5}) = 50$; not the same

Therefore, $3x + 7 \neq 10x$. By the way, $3x + 7x$ would equal $10x$.

3.

a.	$17n$	b.	0	c.	$28z$	d.	$20a$	e.	$12r$	f.	As is
g.	As is	h.	0	i.	Q	j.	$-3x$	k.	$-10t$	l.	$-8a$
m.	As is	n.	$-14e$	o.	$-25k$						

4.

a.	$10x - 8$	b.	$9 + g$	c.	$4x + 10$	d.	$5x + 4$	e.	0
f.	$4 + 2x$	g.	0	h.	$-6x$	i.	$-11w$	j.	$-4r$
k.	30	l.	$-3u - 12$						

5. a. $15x - 8$ b. $16 - 2n$ c. $12q$ d. $-2 - 2x$
e. $i - 16$ f. $-3v - 27$ g. 0 h. x

6. $19n$

7. We'll show him the error of his ways with a numerical counterexample; we'll let $n = 10$. On the one hand,

$$3n + 2n = 3(\mathbf{10}) + 2(\mathbf{10}) = 30 + 20 = 50.$$

On the other hand,

$$5n^2 = 5(\mathbf{10}^2) = 5(100) = 500.$$

Hence, $3n + 2n \neq 5n^2$.

P.S. Get a new study partner!

"In the middle of difficulty lies opportunity."

--Albert Einstein (1879-1955)