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# CH 20 – NUMBER WORD PROBLEMS

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## □ Terminology

To **double** a number means to multiply it by 2.

When  $n$  is doubled, it becomes  $2n$ .

The double of 12 is  $2(12) = 24$ .

To **square** a number means to multiply it by **itself**.

When  $n$  is squared, it becomes  $n^2$ .

The square of 12 is  $12^2 = 144$ .

To **triple** a number means to multiply it by 3.

The result of tripling  $x$  is  $3x$ .

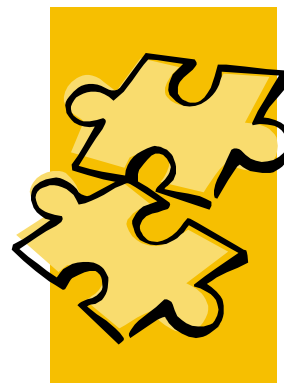
To **quadruple** a number means to multiply it by 4.

To **quintuple** a number means to multiply it by 5.

10 is added to  $n$                        $n + 10$

7 is subtracted from  $x$                $x - 7$

13 is added to twice  $m$              $2m + 13$



Phrases such as “the result is,” “the final result is,” “the answer is,” and “is the same as” translate to the **equal sign** in the equation. In fact, the word “is” alone usually (but not always) represents the equal sign.

## Homework

1. Beginners frequently confuse *doubling* and *squaring*. Let’s clear this up right now.
 

<p>a. What is the double of 15?</p> <p>c. What’s 9 squared?</p> <p>e. 5 doubled gives ____.</p> <p>g. <math>(20)(2) =</math></p> <p>i. Doubling 25 gives ____.</p> <p>k. The square of <math>y</math> is ____.</p>	<p>b. What is the square of 15?</p> <p>d. What’s 9 doubled?</p> <p>f. 5 squared gives ____.</p> <p>h. <math>20^2 =</math></p> <p>j. Squaring 25 gives ____.</p> <p>l. The double of <math>y</math> is ____.</p>
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2. Now we add to the confusion. Consider the number 0. Notice that 0 doubled is  $2(0) = 0$ . Also, 0 squared is  $0^2 = 0 \times 0 = 0$ . In other words, 0 is a special number whose double is the same as its square. Find the only other number whose double is equal to its square.

### □ I’m Thinking of a Number

**EXAMPLE 1:** I’m thinking of a number. If I double it and add 10, the result is 64. What is the number?

**Solution:** Let  $n$  represent the unknown number. Doubling it produces the expression  $2n$ . Adding 10 to that gives the expression  $2n + 10$ . The phrase “the result is” becomes an equal sign, and so the equation we need to solve is



$$\begin{aligned}
2n + 10 &= 64 && \text{(English to Algebra)} \\
\Rightarrow 2n + 10 - \mathbf{10} &= 64 - \mathbf{10} && \text{(subtract 10 from each side)} \\
\Rightarrow 2n &= 54 && \text{(simplify)} \\
\Rightarrow \frac{2n}{\mathbf{2}} &= \frac{54}{\mathbf{2}} && \text{(divide each side by 2)} \\
\Rightarrow n &= 27 && \text{(simplify)}
\end{aligned}$$

Since  $n$  stood for the number we were seeking, we can conclude that the number asked for in the question is

27
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**EXAMPLE 2:**    **I'm thinking of a number. If 7 is subtracted from the number, and then that result is multiplied by 10, the final result is 150. What is the number?**

**Solution:** For variety, we'll select  $x$  to represent the unknown number. First 7 is subtracted from  $x$ ; this can be written  $x - 7$  (not the other way around!). Next, that quantity is to be multiplied by 10. How shall we indicate that 10 must be multiplied by the quantity  $x - 7$ ? We use parentheses:  $10(x - 7)$ . And, of course, the phrase "the final result is" produces our equal sign, yielding us the equation

$$\begin{aligned}
10(x - 7) &= 150 && \text{(English to Algebra)} \\
\Rightarrow 10x - 70 &= 150 && \text{(distribute)} \\
\Rightarrow 10x - 70 + \mathbf{70} &= 150 + \mathbf{70} && \text{(add 70 to each side)} \\
\Rightarrow 10x &= 220 && \text{(simplify)} \\
\Rightarrow \frac{10x}{\mathbf{10}} &= \frac{220}{\mathbf{10}} && \text{(divide each side by 10)} \\
\Rightarrow x &= 22 && \text{(simplify)}
\end{aligned}$$

We conclude that my number was 

22
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**EXAMPLE 3:** Solve each “I’m thinking of a number” problem:

- A. I’m thinking of a number. If I add 3 to the number, and then divide that sum by 2, the result is 7. What is the number?

**Solution:** Translating the words into algebra gives the equation

$$\begin{aligned} \frac{n+3}{2} &= 7 \\ \Rightarrow \frac{n+3}{2}[2] &= 7[2] && \text{(multiply each side by 2)} \\ \Rightarrow n+3 &= 14 && \text{(simplify)} \\ \Rightarrow n &= 11 && \text{(subtract 3 from each side)} \end{aligned}$$

- B. I’m thinking of a number. If I divide the number by 9, and then subtract 5 from that quotient, the answer is 3. Find the number.

**Solution:** In this problem we divide first, and then subtract. Our equation to solve is therefore

$$\begin{aligned} \frac{x}{9} - 5 &= 3 \\ \Rightarrow \frac{x}{9} &= 8 && \text{(add 5 to each side)} \\ \Rightarrow \frac{x}{9}[9] &= 8[9] && \text{(multiply each side by 9)} \\ \Rightarrow x &= 72 && \text{(simplify)} \end{aligned}$$

- C. I’m thinking of a number. If I subtract 5 from the number, and then divide that difference by 3.1, the result is  $-2.7$ . What is the number?

**Solution:** Except for the ugliness of the numbers, this is just like part A above:

$$\begin{aligned} \frac{y-5}{3.1} &= -2.7 \\ \Rightarrow y-5 &= (-2.7)(3.1) && \text{(multiply each side by 3.1)} \\ \Rightarrow y-5 &= -8.37 && \text{(simplify)} \\ \Rightarrow y &= -3.37 && \text{(add 5 to each side)} \end{aligned}$$

## Homework

Solve each of the following “I’m thinking of a number” problems by creating and solving an appropriate equation:

3.
  - a. If I add 7 to the number, the result is 53.
  - b. If I subtract 9 from the number, the result is 33.
  - c. If I double the number, the result is 23.
  - d. If I triple the number, the result is 72.
  - e. If I multiply the number by 7, the result is 861.
  - f. If I multiply the number by 8, the result is 100.
  - g. If I divide the number by 19, the result is 17.
  - h. If I divide the number by 3, the result is 287.
  
4.
  - a. If I double it and add 17, the result is 50.
  - b. If I triple it and subtract 9, the result is 0.
  - c. If I multiply it by 7 and then add 50, the result is  $-3$ .
  - d. If I add 3 to the number, and then multiply that result by 9, the final result is 34.
  - e. If I subtract 5 from the number, and then multiply that result by 17, the final result is 20.

- f. If 30 is added to the number, and then that result is multiplied by 3, the final result is 99.
- g. If 17 is subtracted from the number, and then that result is multiplied by  $-5$ , the result is 30.
5. a. If I add 7 to the number, and then divide that sum by 3, the result is 20.
- b. If I subtract 9 from the number, and then divide that difference by 5, the result is  $-10$ .
- c. If I divide the number by 3, and then add 13 to the quotient, the result is 12.
- d. If I divide the number by 12, and then subtract 50 from the quotient, the result is  $-23$ .

### ▣ More Number Problems

The phrase “7 more than  $x$ ” represents the situation where 7 has been added to  $x$ , and therefore can be represented by the expression  $x + 7$ .

The phrase “9 less than  $w$ ” represents the situation where 9 has been subtracted from  $w$ , and thus can be written as the expression  $w - 9$ .

Here are some more translations from English to Algebra:

the sum of $A$ and $B$	$A + B$
the difference of $x$ and $y$	$x - y$
the product of $a$ and $b$	$ab$
the quotient of $m$ and $n$	$\frac{m}{n}$ (or $m/n$ )
a number $p$ is divided by 7	$\frac{p}{7}$
10 more than $z$	$z + 10$

8 less than $a$	$a - 8$
3 more than twice $n$	$2n + 3$
5 less than 11 times $y$	$11y - 5$
the sum of 4 times $a$ and 4 times $b$	$4a + 4b$
4 times the sum of $a$ and $b$	$4(a + b)$
the difference of twice $q$ and 3	$2q - 3$
twice the difference of $q$ and 3	$2(q - 3)$
a number $n$ increased by 12	$n + 12$
a number $w$ decreased by 7	$w - 7$
100 less than the square of $n$	$n^2 - 100$
23 less than the double of $u$	$2u - 23$
the square of the sum of $x$ and $y$	$(x + y)^2$
the difference of the squares of $A$ and $B$	$A^2 - B^2$

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## Homework

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6. Translate from English to Algebra:
- |                                    |                                  |
|------------------------------------|----------------------------------|
| a. $n$ doubled                     | b. $x$ squared                   |
| c. $a$ tripled                     | d. $b$ quadrupled                |
| e. $t$ quintupled                  | f. 3 added to $u$                |
| g. 9 subtracted from $x$ .         | h. twice $Q$                     |
| i. 7 added to 3 times $n$          | j. 2 subtracted from 4 times $z$ |
| k. 9 less than $w$                 | l. 4 more than $x$               |
| m. 18 more than $T$                | n. 27 less than $P$              |
| o. 3 more than twice $x$           | p. 3 less than 10 times $y$      |
| q. 1 more than the triple of $t$ . |                                  |

- r. 6 less than the quadruple of  $n$
  - s. the quotient of  $x$  and 3
  - t. the product of  $a$  and  $b$
  - u. 3 times  $a$  added to 4 times  $a$
  - v. 9 times  $R$
  - w. 6 times the sum of  $x$  and  $y$
  - x. 9 times the difference of  $A$  and  $B$
  - y. 12 times the sum of  $x$  and 20
  - z. 7 times the difference of  $n$  and 200
- 7.
- a. Translate “the square of the sum of  $u$  and  $w$ ” to Algebra.
  - b. Translate “the sum of the squares of  $u$  and  $w$ ” to Algebra.
  - c. Find a counterexample to the statement that the expressions in part a. and part b. are equal.

**EXAMPLE 4:**      **Eight times a number is 6 less than twice the number. Find the number.**



**Solution:**      Let  $x$  represent the number. We’ll translate the English sentence into algebra like this:

$$\begin{array}{ccc} \text{Eight times a number} & \text{is} & \text{6 less than twice the number} \\ \downarrow & & \downarrow \\ 8x & = & 2x - 6 \end{array}$$

In other words, the equation we must solve is

$$\begin{aligned} 8x &= 2x - 6 \\ \Rightarrow 8x - 2x &= 2x - 2x - 6 && \text{(subtract } 2x \text{ from each side)} \\ \Rightarrow 6x &= -6 && \text{(simplify)} \\ \Rightarrow \frac{6x}{6} &= \frac{-6}{6} && \text{(divide each side by 6)} \\ \Rightarrow x &= -1 && \text{(simplify)} \end{aligned}$$

Therefore, the number we’re searching for is -1



**EXAMPLE 5:** Find a number given that 3 times the difference of the number and 1 is 13 more than the number.

**Solution:** If we let  $n$  represent the unknown number, we can translate as follows:

“Three times difference of the number and 1”  $\Rightarrow 3(n - 1)$

“is”  $\Rightarrow =$

“13 more than the number”  $\Rightarrow n + 13$

Putting it all together:

$$\begin{aligned} 3(n - 1) &= n + 13 && \text{(the equation we need to solve)} \\ \Rightarrow 3n - 3 &= n + 13 && \text{(distribute)} \\ \Rightarrow 2n - 3 &= 13 && \text{(subtract } n \text{ from each side)} \\ \Rightarrow 2n &= 16 && \text{(add 3 to each side)} \\ \Rightarrow n &= 8 && \text{(divide each side by 2)} \end{aligned}$$

The number we're seeking is 8

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## Homework

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8. Seven times a number is 2 more than 9 times the number. What is the number?
9. Nine more than 2 times a number is equal to 9 more than 6 times the number. Find the number.
10. Six times the difference of a number and 3 is 13 less than the number. Find the number.
11. Find a number given that 5 times the sum of the number and 4 is 32 more than the number.

12. If 3 times a number is added to 4 times the number, the result is 700. Find the number.
13. Eight times a number is 3 more than 5 times the number. What is the number?
14. Nine less than 5 times a number is equal to 7 more than 8 times the number. Find the number.
15. Five times the difference of a number and 4 is 18 less than the number. Find the number.
16. Find a number given that 6 times the sum of the number and 7 is 8 less than the number.
17. Find a number given that 7 times the sum of the number and 9 is the same as 201 more than the number.
18. Find a number given that 10 times the difference of the number and 10 is the same as 64 less than the number.
19. Find a number given that 6 times the difference of the number and 7 is the same as 43 more than the number.
20. Find a number given that 5 times the sum of the number and 3 is the same as 141 less than the number.
21. Find a number given that 9 times the sum of the number and 8 is the same as 224 more than the number.
22. Find a number given that 9 times the difference of the number and 7 is the same as 263 less than the number.
23. 47 less than 8 times a number is equal to 26 less than the number. What is the number?
24. 5 less than 3 times a number is equal to 11 more than the number. What is the number?
25. 43 more than 12 times a number is equal to 122 less than the number. What is the number?

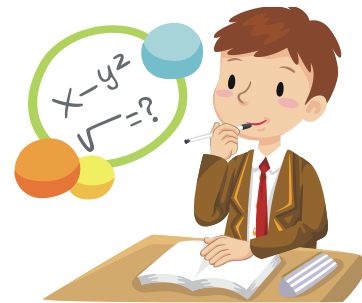
26. Find a number given that 12 times the sum of the number and 4 is the same as 499 more than the number.
27. 5 more than 6 times a number is equal to 50 more than the number. What is the number?
28. 47 more than 14 times a number is equal to 229 more than the number. What is the number?
29. Find a number given that 8 times the difference of the number and 1 is the same as 162 less than the number.

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## Review Problems

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30. I'm thinking of a number. If 10 is subtracted from the number, and then that result is multiplied by 11, the final answer is  $-165$ . What is the number?
31. I'm thinking of a number. If  $-2$  is subtracted from the number, and then that result is multiplied by  $-11$ , the final answer is  $-165$ . What is the number?
32. I'm thinking of a number. If I divide the number by  $-5$ , and then add  $-6$ , the final result will be  $-17$ . What is the number?
33. I'm thinking of a number. If 114 is subtracted from the number, and then that result is divided by  $-11$ , the final answer is 10. What is the number?
34. I'm thinking of a number. If  $-46$  is subtracted from the number, and then that result is divided by  $-8$ , the final answer is  $-5$ . What is the number?



35. Seventeen times a number is 2 less than 10 times the number. What is the number?
36. Nine less than 7 times a number is equal to 19 more than 4 times the number. Find the number.
37. Three times the difference of a number and 5 is 15 less than the number. Find the number.
38. Find a number given that 15 times the sum of the number and 4 is 32 less than the 6 times the number.

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## Solutions

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1. a. 30      b. 225      c. 81      d. 18      e. 10      f. 25  
 g. 40      h. 400      i. 50      j. 625      k.  $y^2$       l.  $2y$
2. I'd rather not give it away just yet.
3. a.  $n + 7 = 53$ ;  $n = 46$       b.  $n - 9 = 33$ ;  $n = 42$   
 c.  $2n = 23$ ;  $n = \frac{23}{2}$       d.  $3n = 72$ ;  $n = 24$   
 e.  $7n = 861$ ;  $n = 123$       f.  $8n = 100$ ;  $n = \frac{25}{2}$   
 g.  $\frac{n}{19} = 17$ ;  $n = 323$       h.  $\frac{n}{3} = 287$ ;  $n = 861$
4. a.  $2n + 17 = 50$ ;  $n = \frac{33}{2}$       b.  $3n - 9 = 0$ ;  $n = 3$   
 c.  $7n + 50 = -3$ ;  $n = -\frac{53}{7}$       d.  $9(n + 3) = 34$ ;  $n = \frac{7}{9}$   
 e.  $17(n - 5) = 20$ ;  $n = \frac{105}{17}$       f.  $3(n + 30) = 99$ ;  $n = 3$   
 g.  $-5(n - 17) = 30$ ;  $n = 11$
5. a.  $\frac{n+7}{3} = 20$ ;  $n = 53$       b.  $\frac{n-9}{5} = -10$ ;  $n = -41$

c.  $\frac{n}{3} + 13 = 12; n = -3$       d.  $\frac{n}{12} - 50 = -23; n = 324$

**6.**  $-5$       **7.**  $13$       **8.**  $-1$       **9.**  $4$       **10.**  $-6$

**11.** a.  $2n$       b.  $x^2$       c.  $3a$       d.  $4b$   
 e.  $5t$       f.  $u + 3$       g.  $x - 9$       h.  $2Q$   
 i.  $3n + 7$       j.  $4z - 2$       k.  $w - 9$       l.  $x + 4$   
 m.  $T + 18$       n.  $P - 27$       o.  $2x + 3$       p.  $10y - 3$   
 q.  $3t + 1$       r.  $4n - 6$       s.  $\frac{x}{3}$       t.  $ab$   
 u.  $4a + 3a$       v.  $9R$       w.  $6(x + y)$       x.  $9(A - B)$   
 y.  $12(x + 20)$       z.  $7(n - 200)$

**12.** a.  $(u + w)^2$   
 b.  $u^2 + w^2$   
 c. Pick a pair of numbers for  $u$  and  $w$  and show that you get a different answer for each expression.

**13.**  $7x = 9x + 2; x = -1$

**14.**  $2x + 9 = 6x + 9; x = 0$

**15.**  $6(x - 3) = x - 13; x = 1$

**16.**  $5(x + 4) = x + 32; x = 3$

**17.**  $100$       **18.**  $1$       **19.**  $-\frac{16}{3}$       **20.**  $\frac{1}{2}$       **21.**  $-10$

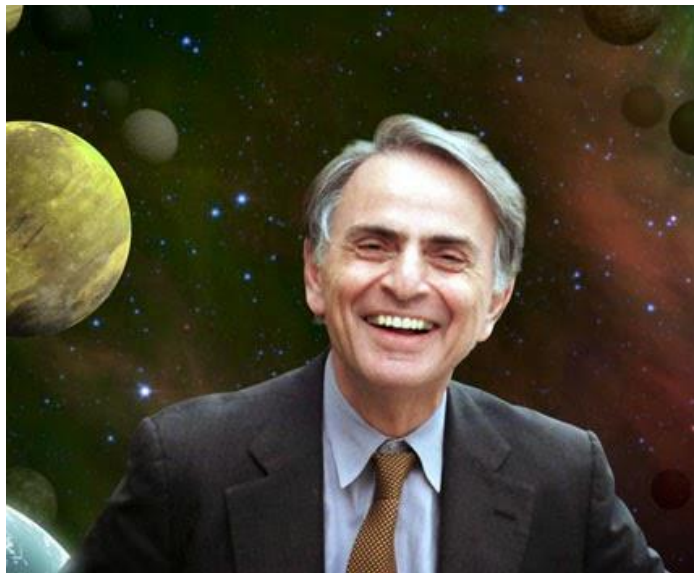
**22.**  $23$       **23.**  $4$       **24.**  $17$       **25.**  $-39$       **26.**  $19$

**27.**  $-25$       **28.**  $3$       **29.**  $8$       **30.**  $-15$       **31.**  $41$

**32.**  $9$       **33.**  $14$       **34.**  $-22$       **35.**  $-\frac{2}{7}$       **36.**  $\frac{28}{3}$

**37.**  $0$       **38.**  $-\frac{92}{9}$

“It is far better to grasp the Universe as it really is than to persist in delusion, however satisfying and reassuring.”



– Carl Sagan (1934 – 1996)