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# CH 22 – SYSTEMS OF LINEAR EQUATIONS

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## □ Introduction

Planning production is the key to any manufacturing business. We have various constraints, which are restrictions that limit our options. For example, we might need to produce 20 widgets today, but how many red widgets and how many green widgets can we afford to make? Or perhaps we should produce the red and green widgets in a ratio that's in line with our customers' demands. The next chapter will solve these kinds of business problems, but solving such a problem will lead to two equations with two variables. And so this is the subject of the current chapter.



## □ Two Equations in Two Variables

Here's an example of a *system of two equations in two variables*:

$$\begin{aligned}x + y &= 10 \\2x - 3y &= 5\end{aligned}$$

The *two equations* are easy to see, as are the *two variables*. The term *system* refers to the fact that these two equations are tied together -- our final solution must be a pair of numbers, one of  $x$  and one for  $y$ , which satisfy both equations.

For example, in the system above, the values  $x = 8$  and  $y = 2$  will satisfy the first equation [ $8 + 2 = 10$ ], but will not satisfy the second equation [ $2(8) - 3(2) = 16 - 6 = 10$ , not 5]. Therefore,  $x = 8$  and  $y = 2$  is not a solution to the *system* of equations.

But  $x = 7$  and  $y = 3$  is a solution of the system of equations. Here's why:

$$7 + 3 = 10 \quad \checkmark \quad \text{and} \quad 2(7) - 3(3) = 14 - 9 = 5 \quad \checkmark$$

## Homework

1. Consider the system of equations:
 
$$\begin{aligned} a + b &= 9 \\ a - b &= 7 \end{aligned}$$
  - a. Show that  $a = 5$  and  $b = 4$  is a solution of the first equation, but is not a solution of the system.
  - b. Show that  $a = 20$  and  $b = 13$  is a solution of the second equation, but is not a solution of the system.
  - c. Show that  $a = 8$  and  $b = 1$  is a solution of the system.

2. Try to solve the system
 
$$\begin{aligned} u + w &= 12 \\ u - w &= 0 \end{aligned}$$
 by guessing.

Now try the system
 
$$\begin{aligned} 3x - 17y &= 200 \\ -5x - 12y &= 29 \end{aligned}$$
 by guessing (just kidding!).

I hope this problem motivates you to study the next section carefully.

### □ The Addition Method

To solve the applications in future chapters, we need to be able to solve two equations in two variables. A method that works very well in many cases is called the **Addition Method**. We multiply one or both equations by appropriate numbers (whatever that means), **add** the resulting equations to eliminate a variable, and then solve for the variable that survived. The *elimination method* is another term used to describe this procedure.





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## Homework

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3. Solve each system using the Addition Method, and be sure you practice checking your solution (your pair of numbers) in both of the original equations:

a. 
$$\begin{aligned} 2x + y &= 5 \\ -2x + 7y &= 19 \end{aligned}$$

b. 
$$\begin{aligned} 5a - 3b &= 5 \\ 10a + 4b &= -40 \end{aligned}$$

c. 
$$\begin{aligned} -2u - 3v &= -16 \\ -7u + 8v &= -56 \end{aligned}$$

d. 
$$\begin{aligned} 7x + 12y &= -24 \\ 6x - 7y &= 14 \end{aligned}$$

e. 
$$\begin{aligned} 3m - 2n &= 34 \\ -6m + n &= -62 \end{aligned}$$

f. 
$$\begin{aligned} -3s - 3t &= -24 \\ 10s + 8t &= 64 \end{aligned}$$

g. 
$$\begin{aligned} 2c - 3d &= 13 \\ 5c + 6d &= -8 \end{aligned}$$

h. 
$$\begin{aligned} -5w - 4x &= -20 \\ 20w + 3x &= 15 \end{aligned}$$

i. 
$$\begin{aligned} -5x - 4n &= -8 \\ 11x + 6n &= -2 \end{aligned}$$

j. 
$$\begin{aligned} 2w - 4a &= 6 \\ -3w + 9a &= -12 \end{aligned}$$

k. 
$$\begin{aligned} 2n - 3y &= -2 \\ 8n - 11y &= -2 \end{aligned}$$

l. 
$$\begin{aligned} 4c + 9y &= 4 \\ -5c - 11y &= 12 \end{aligned}$$

m. 
$$\begin{aligned} 5g - 2h &= -6 \\ 4g + 2h &= 3 \end{aligned}$$

n. 
$$\begin{aligned} -4w + 3h &= -1 \\ -3w + 4h &= 5 \end{aligned}$$

o. 
$$\begin{aligned} -3w + 4m &= 6 \\ -3w - m &= 1 \end{aligned}$$

p. 
$$\begin{aligned} 3a + 3q &= 1 \\ -5a + 5q &= 6 \end{aligned}$$



**Notes:** First, if you'd like to check our solution, substitute the values of  $x$  and  $y$  into both of the original equations. Second, what if none of the variables have a coefficient of 1? Well, you'll just have to pick a variable and then deal with whatever fractions may arise. In the real world, however, it's probably best to solve the system using the Addition Method.

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## Homework

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4. Solve each system using the Substitution Method, and be sure you practice checking your solution (your pair of numbers) in both of the original equations:

a. $2x + y = 5$	b. $3m - 2n = 34$	c. $-3w + 4m = 6$
$-2x + 7y = 19$	$-6m + n = -62$	$-3w - m = 1$

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## Review Problems

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5. Consider the system of equations  $x - y = 12$   
 $x + y = 8$
- a. Show that  $x = 15$  and  $y = 3$  is a solution of the first equation, but is not a solution of the system.
- b. Show that  $x = 7$  and  $y = 1$  is a solution of the second equation, but is not a solution of the system.
- c. Show that  $x = 10$  and  $y = -2$  is a solution of the system.
6. Solve the system of equations by Addition:  $7x - 3y = 8$   
 $-9x + 5y = -8$
7. Solve the system of equations by Substitution:  $7a + 4b = -1$   
 $a - 3b = -18$

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# Solutions

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1. a.  $5 + 4 = 9$  ✓ But  $5 - 4 = 1 \neq 7$ .  
 b.  $20 - 13 = 7$  ✓ But  $20 + 13 = 33 \neq 9$ .  
 c.  $8 + 1 = 9$  ✓ And  $8 - 1 = 7$  ✓
2. Both variables = 6, since  $6 + 6 = 12$ , and  $6 - 6 = 0$ .
3. a.  $x = 1, y = 3$

Complete Check:

$$2x + y = 5$$

$$-2x + 7y = 19$$

$$2(1) + 3 = 5$$

$$-2(1) + 7(3) = 19$$

$$2 + 3 = 5$$

$$-2 + 21 = 19$$

$$5 = 5 \quad \checkmark$$

$$19 = 19 \quad \checkmark$$

b.  $a = -2, b = -5$

c.  $u = 8, v = 0$

d.  $x = 0, y = -2$

e.  $m = 10, n = -2$

f.  $s = 0, t = 8$

g.  $c = 2, d = -3$

h.  $w = 0, x = 5$

i.  $x = -4, n = 7$

j.  $w = 1, a = -1$

k.  $n = 8, y = 6$

l.  $c = -152, y = 68$

m.  $g = -\frac{1}{3}, h = \frac{13}{6}$

n.  $w = \frac{19}{7}, h = \frac{23}{7}$

o.  $w = -\frac{2}{3}, m = 1$

p.  $a = -\frac{13}{30}, q = \frac{23}{30}$

4. a.  $x = 1, y = 3$

b.  $m = 10, n = -2$

c.  $w = -\frac{2}{3}, m = 1$

5. a.  $15 - 3 = 12$  ✓

But  $15 + 3 = 18 \neq 8$

b.  $7 + 1 = 8$  ✓

But  $7 - 1 = 6 \neq 12$

c.  $10 - (-2) = 12$  ✓

And  $10 + (-2) = 8$  ✓✓

6.  $x = 2, y = 2$

7.  $a = -3, b = 5$

**“To educate a man in mind and not in morals is to educate a menace to society.”**

*Theodore Roosevelt* (1858–1919)

