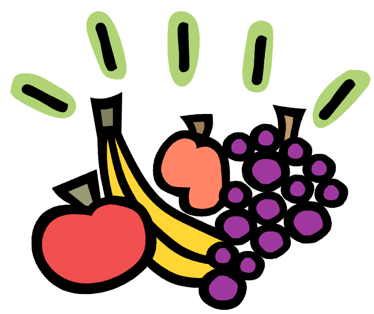
CH 23 – PRODUCING WIDGETS

❑ Introduction



Suppose we have 10 apples and 13 bananas, which of course means that we have 23 pieces of fruit in total. This can be written with the simple equation: $10 + 13 = 23$.

Assume we have a apples and b bananas, and also pretend that there are 12 pieces of fruit in total. Then we can write these facts as an equation: $a + b = 12$.

Suppose that yellow widgets cost $25 apiece to produce (called the *unit cost*), and that we will produce 40 yellow widgets. Then the total cost of production is given by the expression 25×40 (or 40×25).

Assume that the unit cost of maroon widgets is $14, and that we will produce m of the maroon widgets. Then the total cost of production is the product of 14 and m: $14m$.

If we make x widgets at $9 each and y widgets at $11 each, then

1) the total number of widgets made is $x + y$, and

2) the total production cost is $9x + 11y$

If we make w widgets at $12 each and q widgets at $26 each, for a total cost of $500, we can express this information in the equation $12w + 26q = 500$.

If y is designated to be three times whatever x is, then we can write $y = 3x$.

The phrase "The number of green widgets should be two times the number of blue widgets" can be translated into the equation $g = 2b$.

Homework

1. There were 20 vegetables: c carrots and t tomatoes. Write an equation to express this fact.

2. There were r red widgets and p purple widgets. If there were 150 widgets in all, write the equation which expresses this fact.

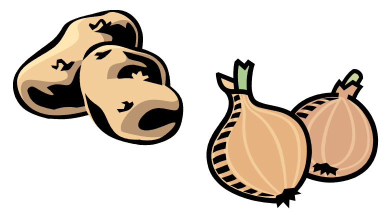
3. At a unit cost of \$49.75 per widget, what is the total cost to produce 100 widgets?

4. At \$3.75 per pound, what is the total cost to buy s pounds of top sirloin.

5. If the unit cost of a widget is \$500, and you produce w of them, find the total cost.

6. If 17 red widgets at a unit production cost of \$5 and 80 brown widgets at a unit production cost of \$1.75 are produced, calculate the total cost.

7. If you bought p pounds of potatoes at \$2 per pound and n pounds of onions at \$4 per pound, write an expression that represents the total cost.



8. In Problem 7, suppose the total cost was \$26. Write an equation which describes this situation. Dollar signs are generally omitted when writing an equation.

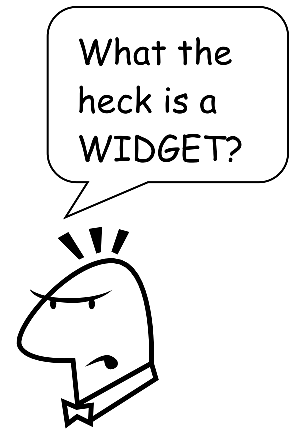
9. The factory spent a total of \$250 producing f flagons at \$7 apiece and d dragons at \$12 apiece. Write an equation which represents this situation.

10. Write an equation which expresses each statement:

a. x is 4 more than y.
b. T is 3 less than Z.
c. u is twice as big as v.
d. g is three times as big as h.
e. w is four times as large as s.
f. h is one-half as large as k.
g. E is one-third the size of m.
h. A is 12 less than B.
i. W is 1 less than Q.

❑ The Right Mixture

I suppose if we had an infinite amount of money, we could produce all the widgets in all the colors we would ever need, but of course this is absurd. We have constraints to worry about; the first will concern the number of widgets to be produced, and the second will be the amount of money we can spend producing them. Each of these constraints will be represented by an equation. Thus, each application we solve will have two equations that we must solve at the same time, for two different variables.



In this first application, we are told the total number of widgets to produce (some of one color and some of another) and the total amount of money we're allowed to spend.

EXAMPLE 1: **The unit cost of red widgets is $8 and the unit cost of green widgets is $10. Each production run can produce a total of 20 widgets, but we control how many of each color. The production budget is $168. How many widgets of each color should we manufacture?**

Solution: We're seeking two things in this problem: the number of red widgets and the number of green widgets to produce. Let's let

$\boldsymbol{r}$ represent the number of red widgets, and
$\boldsymbol{g}$ represent the number of green widgets.

We're given the unit costs of production: $8 for each red widget and $10 for each green widget. This means that it will cost $8r$ to produce the red widgets, and $10g$ to produce the green widgets.

Since we're seeking the values of two different variables, we expect to have two equations at our disposal. The first equation represents the fact that we will produce 20 widgets altogether.

Since r and g represent the number of widgets of each color, we get the equation

$$r + g = 20 \qquad \text{(20 widgets altogether)}$$

The total cost of production is the sum of the costs of the two colors of widgets -- this yields the equation

$$8r + 10g = 168 \qquad \text{(the total cost of \$168)}$$

We now have a system of two equations in two unknowns:

$$\begin{aligned} r + g &= 20 \\ 8r + 10g &= 168 \end{aligned}$$

$$\begin{array}{lcr} r + g = 20 & \xrightarrow{\text{times } -8} & -8r - 8g = -160 \\ 8r + 10g = 168 & \xrightarrow{\text{leave alone}} & 8r + 10g = 168 \\ \hline \text{Add the equations:} & & 0r + 2g = 8 \\ & & g = 4 \end{array}$$

This value of g tells us that we need to produce 4 green widgets. Since we need 20 widgets total, it's clear that we need to make 16 of the red widgets. More formally,

$$r + g = 20 \;\Rightarrow\; r + \mathbf{4} = 20 \;\Rightarrow\; r = 16$$

To summarize, the production schedule should be

16 red widgets & 4 green widgets

Check:

16 + 4 = 20 widgets ✓

16 @ \$8 = \$128

4 @ \$10 = \$ 40

\$168 ✓

In our second application, market research has determined that black widgets sell about twice as well as white widgets. To satisfy this demand, the production department has to solve the following problem:

EXAMPLE 2: **Assume that black widgets cost $3 apiece and white widgets cost $7 apiece to produce. Our goal is that the number of black widgets should be twice the number of white widgets, while our production budget is $260. How many widgets of each color should we produce to satisfy all the stated conditions?**

Solution: We'll let

$\boldsymbol{b}$ = the number of black widgets to produce
$\boldsymbol{w}$ = the number of white widgets to produce

One of the conditions stipulates that the number of black widgets must be twice the number of white widgets. This yields the equation

$$b = 2w$$ (# of blacks is twice # of whites)

Since we're producing b black widgets at a unit cost of $3, the cost of producing the black widgets is $3b$. Similarly, the cost of the white widgets is $7w$. The sum of these two costs must be $260, the given budget. This fact gives us the equation

$$3b + 7w = 260$$ (the total cost is $260)

We thus have a system of two equations in two unknowns:

$$\begin{aligned} b &= 2w \\ 3b + 7w &= 260 \end{aligned}$$

To solve this system by the addition method, a little rearranging is in order. In the first equation subtract $2w$ from each side, and leave the second equation alone:

$$\begin{aligned} b - 2w &= 0 \\ 3b + 7w &= 260 \end{aligned}$$

Now the variables are lined up.

Now we'll multiply the equations by appropriate numbers so that when we add the equations we'll eliminate the b:

$$b - 2w = 0 \xrightarrow{\text{times } -3} -3b + 6w = 0$$
$$3b + 7w = 260 \xrightarrow{\text{leave alone}} 3b + 7w = 260$$

Add the equations: $13w = 260$

$\Rightarrow \quad w = 20$

Since $b = 2w$, it follows that $b = 2(\mathbf{20}) = 40$. In conclusion, we instruct the factory to produce

20 white widgets & 40 black widgets

Check: 40 black is twice 20 white ✓

20 @ \$7 = \$140

40 @ \$3 = \$120

\$260 ✓

<u>ALTERNATE METHOD:</u> We can solve the system of equations in the previous example by the *Substitution Method*. Here is the system again:

$$b = 2w$$
$$3b + 7w = 260$$

Since $b = 2w$, we can substitute $2w$ for b in the second equation:

$$3(2w) + 7w = 260$$
$$\Rightarrow \quad 6w + 7w = 260$$
$$\Rightarrow \quad 13w = 260$$
$$\Rightarrow \quad w = \mathbf{20}$$
$$\Rightarrow \quad b = 2w = 2(20) = \mathbf{40}$$

And we get the same values for w and b.

<u>EXAMPLE 3:</u> **Assume that green widgets cost $5 apiece and yellow widgets cost $9 apiece to produce. Our goal is that we produce 8 more yellow widgets than green widgets. If we have a total of $352 to spend, how many widgets of each color should we manufacture?**

<u>Solution:</u> We'll set g and y represent the number of green and yellow widgets. Since we need "8 more yellow widgets than green widgets," we get

$$y = g + 8$$

The total cost to produce all the widgets gives us

$$5g + 9y = 352$$

Substitute the first equation into the second:

$$5g + 9(g + 8) = 352$$

$$\Rightarrow \quad 5g + 9g + 72 = 352$$

$$\Rightarrow \quad 14g = 280$$

$$\Rightarrow \quad g = 20$$

And using the first equation: $y = g + 8 = \mathbf{20} + 8 = 28$

20 green widgets & 28 yellow widgets

Notes: You should check this solution to make sure we have the right numbers of widgets. Also, we could have used the Addition Method to solve the system of equations; you might try this for practice.

Homework

11. The unit cost of red widgets is $6 and the unit cost of white widgets is $8. If the goal is to make 25 widgets for a total of $170, how many widgets of each color should we make?

12. The unit cost of maroon widgets is $5 and the unit cost of white widgets is $9. If the goal is that the number of white widgets should be 8 times the number of maroon widgets, and if the total budget is $462, how many widgets of each color should we make?

13. The unit cost of red widgets is $7 and the unit cost of yellow widgets is $6. If the goal is that the number of yellow widgets should be 75 more than the number of red widgets, and if the total budget is $645, how many widgets of each color should we make?

14. The unit cost of red widgets is $7 and the unit cost of maroon widgets is $8. If the goal is to make 23 widgets for a total of $183, how many widgets of each color should we make?

15. The unit cost of white widgets is $9 and the unit cost of black widgets is $6. If the goal is that the number of black widgets should be 7 more than the number of white widgets, and if the total budget is $57, how many widgets of each color should we make?

16. The unit cost of yellow widgets is $9 and the unit cost of white widgets is $8. If the goal is to make 35 widgets for a total of $305, how many widgets of each color should we make?

17. The unit cost of black widgets is \$9 and the unit cost of white widgets is \$11. If the goal is that the number of white widgets should be 8 times the number of black widgets, and if the total budget is \$2425, how many widgets of each color should we make?

18. The unit cost of green widgets is \$9 and the unit cost of black widgets is \$11. If the goal is to make 9 widgets for a total of \$85, how many widgets of each color should we make?

❑ A Graphical Approach to the Widget Production Problem

Each of the widget production problems was solved by generating two equations in two variables from the data given in the problem, followed by solving the resulting system of equations. But in more advanced math, business, and science problems, the system of equations may be unsolvable using algebra; the best solution we may get is an approximation. One of the approximation methods used is ***graphing***. We create the equations from the stated problem, graph each equation, and then estimate where the graphs cross each other (intersect).

<u>EXAMPLE 4:</u> **The unit cost of red widgets is \$13 and the unit cost of green widgets is \$25. Each production run can produce a total of 50 widgets, but we control how many of each color. The production budget is \$758. By graphing, determine how many widgets of each color we should manufacture.**

<u>Solution:</u> This problem leads to the system of equations:

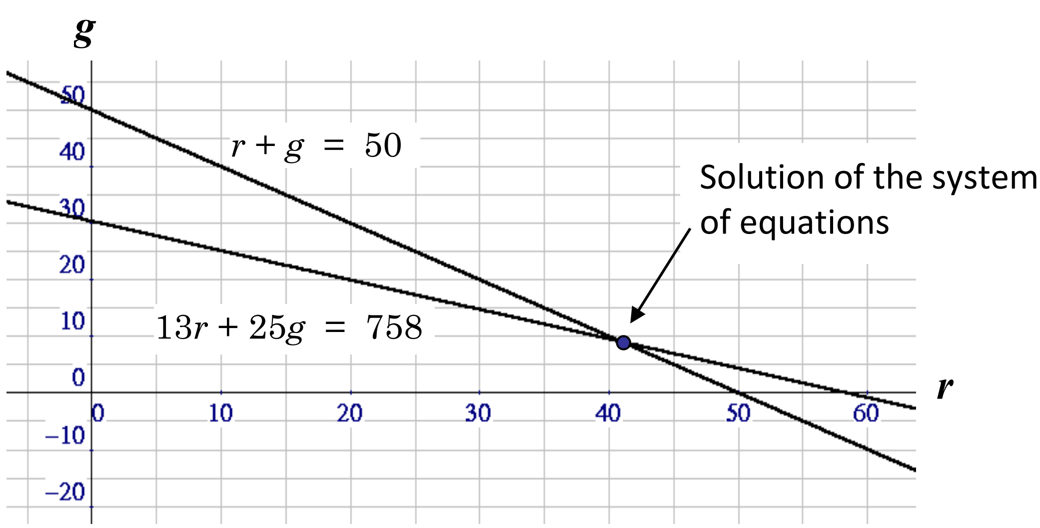
$$\begin{aligned} r + g &= 50 \\ 13r + 25g &= 758 \end{aligned}$$

To graph these two equations, we realize that each of them is a line, and we've learned that the simplest way to graph a line is to use the intercepts. For no particular reason, we will label the horizontal axis as r, and the vertical axis as g. This means that the ordered pairs will be written in the form (r, g).

$r + g = 50$: Setting $r = 0$ gives $g = 50$. Setting $g = 0$ gives $r = 50$. The intercepts are (0, 50) and (50, 0).

$13r + 25g = 758$: When $r = 0$, $g = 30.32$, and when $g = 0$, $r = 58.31$. The intercepts are therefore (0, 30.32) and (58.31, 0).

We now plot each line on the same grid, and then estimate the point of intersection of the two lines.



Focusing on the point of intersection, we estimate the r-coordinate to be about 41 and the g-coordinate to be about 9. That is, the point of intersection is about (41, 9). Our conclusion? We should make **41 red widgets and 9 green widgets**.

There's no homework for this section. All the problems in this chapter should be done using the *Addition or Substitution Method*.

Review Problems

19. The unit cost of black widgets is \$11 and the unit cost of maroon widgets is \$5. If the goal is that the number of maroon widgets should be 8 times the number of black widgets, and if the total budget is \$306, how many widgets of each color should we make?

20. The unit cost of yellow widgets is \$10 and the unit cost of maroon widgets is \$6. If the goal is that the number of maroon widgets should be 135 more than the number of yellow widgets, and if the total budget is \$1242, how many widgets of each color should we make?

21. The unit cost of green widgets is \$8 and the unit cost of black widgets is \$9. If the goal is to make 45 widgets for a total of \$389, how many widgets of each color should we make?

Solutions

1. $c + t = 20$	**2.** $r + p = 150$	**3.** 4975
4. $3.75s$	**5.** $500w$	**6.** 225
7. $2p + 4n$	**8.** $2p + 4n = 26$	**9.** $7f + 12d = 250$

10. a. $x = y + 4$ b. $T = Z - 3$ c. $u = 2v$

d. $g = 3h$ e. $w = 4s$ f. $h = \frac{1}{2}k$

g. $E = \frac{1}{3}m$ h. $A = B - 12$ i. $W = Q - 1$

11. 15 red and 10 white

12. 6 maroon and 48 white

13. 15 red and 90 yellow

14. 1 red and 22 maroon

15. 1 white and 8 black

16. 25 yellow and 10 white

17. 25 black and 200 white

18. 7 green and 2 black

19. 6 black and 48 maroon

20. 27 yellow and 162 maroon

21. 16 green and 29 black

"I'm a great believer in luck, and I find the harder I work, the more I have of it."

– *Thomas Jefferson*