CH 28 – PERCENT MIXTURE PROBLEMS, PART II

❑ INTRODUCTION

We continue our discussion of percent mixture problems. The only difference between this chapter's problems and the previous chapter's is that we are going to be asked to find the amounts of the two quantities that must be mixed together to achieve the desired outcome.

❑ EXAMPLES

EXAMPLE 1: **How many quarts each of a 62% poison solution and a 6% poison solution must a detective mix to get 14 quarts of a mixture that is 30% poison?**

Solution: Let x represent the quarts of the 62% poison.
Let y represent the quarts of the 6% poison.

	Quantity x	Concentration =	Amount
62% poison	x	62%	$0.62x$
6% poison	y	6%	$0.06y$
final mixture	14	30%	14×0.30

Looking at the Quantity column, we're mixing x quarts with y quarts to get a total of 14 quarts in the final mixture. It makes sense to say that the sum of x and y must be 14:

$$x + y = 14 \qquad \textbf{[Equation \#1]}$$

Consider the Concentration column. Does adding the concentrations together make any sense? Of course not: 62% + 6% $\neq$ 30%. In fact, our intuition tells us that the concentration of the final solution ought to be somewhere between the concentrations of the ingredients being mixed. But we need another equation -- since we have two variables x and y, we'll need two equations in order to find x and y.

Now look at the Amount column. Each ingredient being mixed together contains poison and water. Does it make sense that the actual amount of poison in the final solution must be the sum of the actual amounts of poison in the ingredients? This leads to the second equation:

$$0.62x + 0.06y = 14 \times 0.30$$

or, $0.62x + 0.06y = 4.2$ **[Equation #2]**

Equations 1 and 2 constitute a system of two equations in two variables, which we will solve using the Addition method.

$$\begin{array}{lll} x + y = 14 & (\text{times } -0.62) \Rightarrow & -0.62x - 0.62y = -8.68 \\ 0.62x + 0.06y = 4.2 & (\text{leave alone}) \Rightarrow & 0.62x + 0.06y = 4.2 \\ \hline & \text{Adding} \Rightarrow & 0 - 0.56y = -4.48 \\ & \Rightarrow & y = 8 \end{array}$$

Since y stood for the number of quarts of the 6% solution, we see that the detective needs 8 quarts of the 6% solution. Moreover, since $x + y = 14$, we see that $x + 8 = 14 \Rightarrow x = 6$, which means that the detective also needs 6 quarts of the 62% solution.

6 quarts of the 62% poison, & 8 quarts of the 6% poison

EXAMPLE 2: **A druggist wants to create 4 liters of a 94% anti-malaria medicine. How many liters each of pure anti-malaria medicine and a 92% anti-malaria medicine must she mix together?**

Solution:

Let x represent the liters of pure anti-malaria medicine.
Let y represent the liters of the 92% anti-malaria medicine.

Remembering that pure anti-malaria medicine has an anti-malaria concentration of 100%, we put all our information in the chart:

	Quantity x	Concentration =	Amount
pure medicine	x	100%	$1.00x$
92% medicine	y	92%	$0.92y$
final mixture	4	94%	4×0.94

One of our equations comes from the fact that the quantities x and y must add up to 4:

$$x + y = 4$$

The second equation comes from the fact that the amount of anti-malaria medicine in the 100% medicine plus the amount of anti-malaria medicine in the 92% medicine must equal the amount of anti-malaria medicine in the final mixture:

$$1.00x + 0.92y = 0.94 \times 4,$$

or, $$x + 0.92y = 3.76$$

Let's solve this system of equations via the addition method.

$$\begin{array}{lll} x + y = 4 & \text{(leave alone)} & \Rightarrow \quad x + y = 4 \\ x + 0.92y = 3.76 & \text{(times } -1\text{)} & \Rightarrow \quad -x - 0.92y = -3.76 \\ \hline & \text{Adding} & \Rightarrow \quad 0 + 0.08y = 0.24 \\ & & \Rightarrow \quad y = 3 \end{array}$$

Since $y = 3$, and $x + y = 4$, it follows that $x = 1$. We now know how many liters of each medicine she must mix together.

> 3 liters of the 92% medicine, &
> 1 liter of the pure medicine

EXAMPLE 3: **How many fluid ounces each of pure water and a 10% albuterol inhalant must an allergist mix to get 5 fluid ounces of an inhalant that is 6% albuterol?**

Solution: This is the same scenario as the two previous examples, so we can get right to it; note, however, that pure water contains 0% albuterol (that is, there is no albuterol in pure water).

	Quantity	x Concentration	= Amount
pure water	x	0%	$0x$
10% albuterol	y	10%	$0.10y$
final mixture	5	6%	5×0.06

The two equations are $x + y = 5$ and $0x + 0.10y = 5 \times 0.06$, or $0.10y = 0.3$. Setting up the system of equations neatly gives

$$\begin{aligned} x + y &= 5 \\ 0.10y &= 0.3 \end{aligned}$$

The second equation can be easily solved for y, so we don't need the Addition method. Dividing each side of the second equation by 0.10, we get $y = 3$. Using $x + y = 5$, we find that $x = 2$. Thus,

$$\boxed{\begin{array}{l}\text{2 ounces of water, \&} \\ \text{3 ounces of the 10\% inhalant}\end{array}}$$

Homework

1. How many pounds each of a 97% poison solution and a 67% poison solution must a detective mix to get 15 pounds of a solution that is 91% poison?

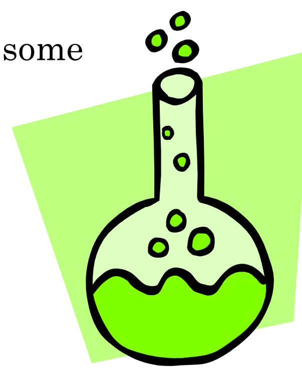
2. A druggist wants to create 24 ounces of a 92% alcohol medicine. How many ounces each of pure alcohol medicine and a 76% alcohol medicine must he mix together?

3. How many ounces each of pure water and an 80% albuterol inhalant must an allergist mix to get 20 ounces of an inhalant that is 64% albuterol?

4. A druggist wants to mix some 59% alcohol medicine with some 21% alcohol medicine. How many mL of each substance must she use to get a 19-mL mixture that is 57% alcohol?

5. A detective wants to mix some pure poison with some 20% poison solution. How many pounds of each substance must she use to get a 25-pound mixture that is 52% poison?

6. How many liters each of a 99% solution and an 88% solution must be mixed together to get 22 liters of a mixture whose concentration is 97%?



7. How many liters each of a pure solution and a 12% solution must be mixed together to get 88 liters of a mixture whose concentration is 42%?

8. How many liters each of a 24% solution and a 68% solution must be mixed together to get 22 liters of a mixture whose concentration is 44%?

9. How many liters each of pure water and a 57% solution must be mixed together to get 38 liters of a mixture whose concentration is 42%?

10. How many liters each of a 79% solution and a 70% solution must be mixed together to get 36 liters of a mixture whose concentration is 76%?

11. How many liters each of pure water and a 48% solution must be mixed together to get 72 liters of a mixture whose concentration is 14%?

12. How many liters each of a 9% solution and a 94% solution must be mixed together to get 68 liters of a mixture whose concentration is 69%?

13. How many liters each of pure water and a 20% solution must be mixed together to get 100 liters of a mixture whose concentration is 9%?

14. How many liters each of a 73% solution and a 97% solution must be mixed together to get 12 liters of a mixture whose concentration is 93%?

15. How many liters each of a pure solution and a 58% solution must be mixed together to get 48 liters of a mixture whose concentration is 93%?

Solutions

1. 12 pounds of the 97% solution and 3 pounds of the 67% solution

2. 16 ounces of pure alcohol and 8 ounces of the 76% alcohol medicine

3. 4 ounces of water and 16 ounces of the 80% albuterol inhalant

4. 18 mL of the 59% alcohol medicine and 1 mL of the 21% alcohol medicine

5. 10 pounds of pure poison and 15 pounds of the 20% poison solution

6. 18 L of the 99% solution and 4 L of the 88% solution

7. 30 L of the pure solution and 58 L of the 12% solution

8. 12 L of the 24% solution and 10 L of the 68% solution

9. 10 L of pure water and 28 L of the 57% solution

10. 24 L of the 79% solution and 12 L of the 70% solution

11. 51 L of pure water and 21 L of the 48% solution

12. 20 L of the 9% solution and 48 L of the 94% solution

13. 55 L of pure water and 45 L of the 20% solution

14. 2 L of the 73% solution and 10 L of the 97% solution

15. 40 L of the pure solution and 8 L of the 58% solution

"When you do the common things in life in an uncommon way, you will command the attention of the world."

- George Washington Carver (1864-1943)