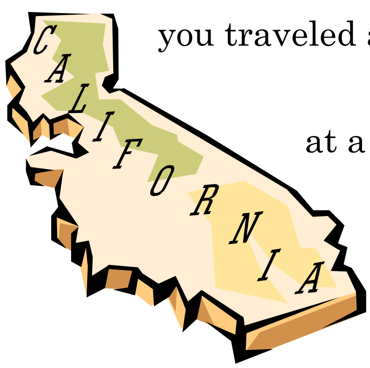
CH 29 – MOTION PROBLEMS, PART I

❑ *INTRODUCTION*

Whether it's the police pursuing a bank robber, or a physicist determining the velocity of a proton in a linear accelerator, the concepts of time, distance, and speed are at the heart of all science and technology.

If you travel from San Francisco to L.A., 400 miles away, and you travel for 8 hours at an average speed of 50 miles per hour, then



you traveled a **DISTANCE** of 400 miles

at a **RATE** (speed) of 50 mph

during a **TIME** interval of 8 hours.

Notice that in this example, if you multiply the rate by the time (50 mph × 8 hrs), you get the distance (400 mi). This idea always holds:

$$\text{Rate} \times \text{Time} = \text{Distance}$$

Homework

1. a. Moe traveled at a rate of 120 km/hr for 12 hours. Find Moe's distance.

   b. Larry flew a distance of 3000 miles in 6 hours. What was Larry's rate?

   c. Curly jogged 12 miles at a rate of 3 mph. How long was Curly jogging?

2. Which is the proper formula for distance?

   a. $d = rt$ b. $d = \frac{r}{t}$ c. $d = \frac{t}{r}$

3. Two skaters leave the skate park and skate in opposite directions, one at 10 mph and the other at 8 mph. After some time, they are 18 miles apart. If d_1 is the distance traveled by the first skater, and if d_2 is the distance traveled by the second skater, write an appropriate equation.

4. A woodpecker traveled from the maple tree to the oak tree at 13 mph, and then made a return trip at 19 mph. If d_1 is the distance it traveled to the oak tree, and if d_2 is the distance from the oak back to the maple, write an appropriate equation.

5. A 1096-mi trip took a total of 16 hours. The speed was 71 mph for the first part of the trip, and then decreased to 67 mph for the rest of the trip. If d_1 is the distance traveled on the first part of the trip, and if d_2 is the distance traveled on the second part of the trip, write an appropriate equation.

❑ OPPOSITE DIRECTIONS

<u>EXAMPLE 1:</u> **Mike and Sarah start from the burger stand and skate in opposite directions. Mike's speed is 5 less than 3 times Sarah's speed. In 10 hours they are 70 miles apart. Find the speed of both skaters.**

<u>Solution:</u> Let's organize all the information in a table using the basic $rt = d$ formula we're learning in this chapter. Down the first column are the names of our two skaters. Across the first row are the three components of motion, the rate (speed), the time, and the distance. We've written the formula to help us remember the basic relationship among these three concepts.

	Rate ×	Time =	Distance
Mike			
Sarah			

Since each skater's speed is being asked for (they're the unknowns), we'll let M stand for Mike's speed and S stand for Sarah's speed, and so these variables go into the Rate column.

As for the Time column, the problem states that each skater skated for exactly 10 hours, so each of their travel times is 10.

Since Distance = Rate × Time, the Distance column is simply the product of the Rate and Time columns for both Mike and Sarah.

	Rate ×	Time =	Distance
Mike	M	10	$10M$
Sarah	S	10	$10S$

Since there are two unknowns in this problem, it's likely we'll need two equations. Let's look at the rates first: From the phrase in the problem "Mike's speed is 5 less than 3 times Sarah's speed" we create the equation

$$M = 3S - 5 \qquad \text{[Equation 1]}$$

To determine the second equation, we have to picture where the skaters are going. They start in the same place and then proceed to skate in opposite directions and end up 70 miles from each other. Therefore, the sum of their individual distances must be 70. Well, Mike skated a distance of $10M$ miles while Sarah went $10S$ miles. So 70 must be the sum of $10M$ and $10S$:

$$10M + 10S = 70 \qquad \text{[Equation 2]}$$

Now substitute Equation 1 into Equation 2:

$$10(\mathbf{3S - 5}) + 10S = 70 \qquad \text{(replaced } M \text{ with } 3S - 5)$$

$$\Rightarrow \quad 30S - 50 + 10S = 70 \qquad \text{(distribute)}$$

$$\Rightarrow \quad 40S - 50 = 70 \qquad \text{(combine like terms)}$$

$$\Rightarrow \quad 40S = 120 \qquad \text{(add 50 to each side)}$$

$$\Rightarrow \quad \underline{S = 3} \qquad \text{(divide each side by 40)}$$

Recall that S stood for Sarah's speed, so we know for sure that Sarah skated 3 mph. To find Mike's speed we use Equation 1 and Sarah's speed:

$$\begin{aligned} M &= 3S - 5 \\ &= 3(\mathbf{3}) - 5 \\ &= 9 - 5 \\ &= \underline{4} \end{aligned}$$

> Note: We could have used the Addition Method to solve the system of equations:
>
> $$M = 3S - 5$$
> $$10M + 10S = 70$$

This shows that Mike skated at a rate of 4 mph. We now have the complete answer to the question:

> Mike's speed was 4 mph and
>
> Sarah's speed was 3 mph.

Homework

6. Two pedestrians leave from the same place and walk in opposite directions. The speed of one of the pedestrians is 5 mph less than 7 times the other. In 6 hours they are 354 miles apart. Find the speed of each pedestrian.

7. Two skaters leave from the same place and skate in opposite directions. The speed of one of the skaters is 8 mph less than 10 times the other. In 9 hours they are 819 miles apart. Find the speed of each skater.

8. Two joggers leave from the same place and jog in opposite directions. The speed of one of the joggers is 9 mph more than 5 times the other. In 7 hours they are 357 miles apart. Find the speed of each jogger.

9. Two pedestrians leave from the same place and walk in opposite directions. The speed of one of the pedestrians is 7 mph less than 9 times the other. In 10 hours they are 930 miles apart. Find the speed of each pedestrian.

10. Two skaters leave from the same place and skate in opposite directions. The speed of one of the skaters is 1 mph more than 7 times the other. In 9 hours they are 513 miles apart. Find the speed of each skater.

❑ *ROUND TRIP*

<u>EXAMPLE 2:</u> **It takes a helicopter a total of 13 hours to travel from the mountain to the valley at a speed of 30 mph and return at a speed of 35 mph. How long does it take to get from the mountain to the valley?**

Solution: We'll let x represent the time it takes to go from the mountain to the valley (since this is what's being asked for). Let's also choose y to stand for the time it takes to return from the valley to the mountain. The two rates (speeds) are given, and we are getting pretty good at knowing that each distance is the product of the rate and the time. So here's the table:

	Rate ×	**Time** =	**Distance**
to valley	30	x	$30x$
to mtn	35	y	$35y$

Since the <u>total</u> travel time is 13 hours, we get our first equation:

$$x + y = 13 \qquad \text{[Equation 1]}$$

Now what about the distances, $30x$ and $35y$? Wouldn't you agree that the distance from the mountain to the valley is the same as the distance from the valley to the mountain? That is, $30x$ and $35y$ must be equal:

$$30x = 35y \qquad \text{[Equation 2]}$$

To solve this system of two equations in two unknowns, let's take Equation 1 and solve it for y:

$$x + y = 13 \qquad \text{(Equation 1)}$$

$$\Rightarrow \quad y = 13 - x \qquad \text{(subtract } x \text{ from each side)}$$

We now replace the variable y in Equation 2 with the result just obtained:

$$30x = 35(\mathbf{13 - x})$$

$$\Rightarrow \quad 30x = 455 - 35x \qquad \text{(distribute)}$$

$$\Rightarrow \quad 65x = 455 \qquad \text{(add } 35x \text{ to each side)}$$

$$\Rightarrow \quad \underline{x = 7} \qquad \text{(divide each side by 65)}$$

Now, what did x represent? Go back to the table and see that x represented the travel time from the mountain to the valley.

Since this is exactly what was being asked for in the problem, we're done.

It takes 7 hours to travel from the mountain to the valley.

Homework

11. A helicopter traveled from the hospital to the battlefield at a speed of 22 mph and returned at a speed of 44 mph. If the entire trip took 18 hours, find the travel times to and from the battlefield.

12. A hang glider traveled from the oceanside to the mountain top at a speed of 18 mph and returned at a speed of 21 mph. If the entire trip took 13 hours, find the travel times to and from the mountain top.

13. A helicopter traveled from the hospital to the battlefield at a speed of 36 mph and returned at a speed of 24 mph. If the entire trip took 20 hours, find the travel times to and from the battlefield.

14. A tractor traveled from the wheat field to the chicken coop at a speed of 27 mph and returned at a speed of 36 mph. If the entire trip took 21 hours, find the travel times to and from the chicken coop.

15. A hang glider traveled from the oceanside to the mountain top at a speed of 34 mph and returned at a speed of 51 mph. If the entire trip took 15 hours, find the travel times to and from the mountain top.

❑ TWO-PART JOURNEY

EXAMPLE 3: **A limousine traveled at 29 mph for the first part of a 540-mile trip, and then increased its speed to 53 mph for the rest of the trip. How many hours were traveled at each rate if the total trip took 12 hours?**

Solution: The rates for each part of the trip are given, so just put them in the table in the right places. Let x be the travel time for the first part of the trip and let y be the travel time for the second part of the trip. Finally, the Distance column is the product of the Rate and Time columns.

	Rate ×	Time =	Distance
1st part	29	x	$29x$
2nd part	53	y	$53y$

The total travel is given to be 12 hours. Therefore,

$$x + y = 12$$

Since the total distance traveled was 540 miles, adding the distance of the 1st part of the trip plus the distance of the 2nd part of the trip should give a total of 540:

$$29x + 53y = 540$$

Solving the first equation for y gives $y = 12 - x$. Substituting $12 - x$ for y in the second equation gives:

$$29x + 53(\mathbf{12 - x}) = 540$$

$\Rightarrow$ $29x + 636 - 53x = 540$ (distribute)

$\Rightarrow$ $-24x + 636 = 540$ (combine like terms)

$\Rightarrow$ $-24x = -96$ (subtract 636)

$\Rightarrow$ $\underline{x = 4}$

This means that the first part of the limo trip took 4 hours. Using the equation $y = 12 - x$, we calculate the time for the rest of the trip as $y = 12 - x = 12 - \mathbf{4} = 8$. In short,

4 hours at 29 mph and 8 hours at 53 mph

Homework

16. A 1096-mi trip took a total of 16 hours. The speed was 71 mph for the first part of the trip, and then decreased to 67 mph for the rest of the trip. How many hours were traveled at each speed?

17. A 730-mi trip took a total of 11 hours. The speed was 68 mph for the first part of the trip, and then decreased to 59 mph for the rest of the trip. How many hours were traveled at each speed?

18. A 664-mi trip took a total of 12 hours. The speed was 30 mph for the first part of the trip, and then increased to 68 mph for the rest of the trip. How many hours were traveled at each speed?

19. A 489-mi trip took a total of 9 hours. The speed was 45 mph for the first part of the trip, and then increased to 59 mph for the rest of the trip. How many hours were traveled at each speed?

20. A 556-mi trip took a total of 14 hours. The speed was 38 mph for the first part of the trip, and then increased to 42 mph for the rest of the trip. How many hours were traveled at each speed?

Solutions

1. a. 1440 km b. 500 mi/hr c. 4 hrs

2. a. $d = rt$ or $rt = d$

3. $d_1 + d_2 = 18$

4. $d_1 = d_2$

5. $d_1 + d_2 = 1096$

6. 8 mph & 51 mph

7. 9 mph & 82 mph

8. 7 mph & 44 mph

9. 10 mph & 83 mph

10. 7 mph & 50 mph

11. 12 hrs & 6 hrs

12. 7 hrs & 6 hrs

13. 8 hrs & 12 hrs

14. 12 hrs & 9 hrs

15. 9 hrs & 6 hrs

16. 6 hrs & 10 hrs

17. 9 hrs & 2 hrs

18. 4 hrs & 8 hrs

19. 3 hrs & 6 hrs

20. 8 hrs & 6 hrs

"Formal education is but an incident in the lifetime of an individual. Most of us who have given the subject any study have come to realize that education is a continuous process ending only when ambition comes to a halt."

– *R.I. Rees*