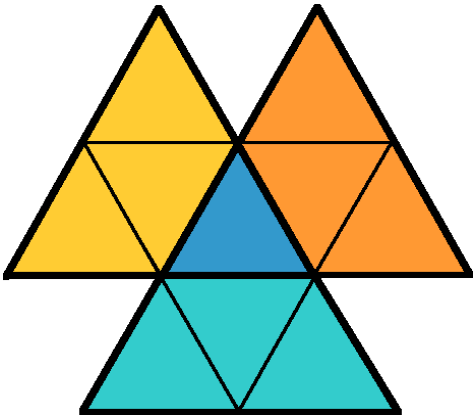
CH 32 – TRIANGLES AND PERIMETER

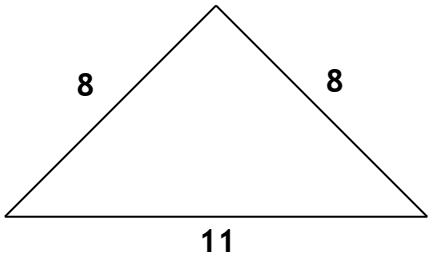
❑ Introduction



We continue our study of using algebra to solve geometry problems, but now we focus exclusively on triangles.

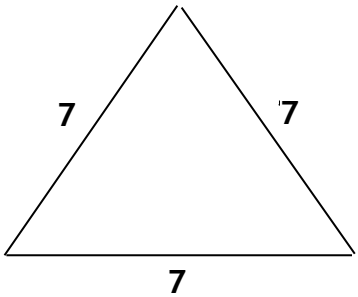
❑ Terminology

A triangle with <u>at least</u> two equal sides is called an ***isosceles triangle***.



An Isosceles Triangle

An ***equilateral triangle*** has three equal sides.



An Equilateral Triangle, (which is also isosceles)

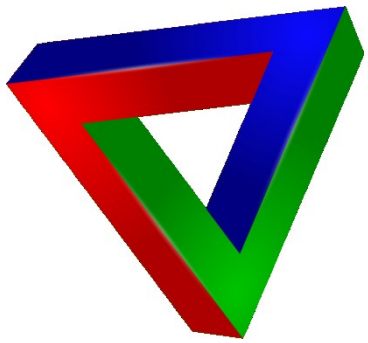
Since an isosceles triangle must have <u>at least</u> two equal sides, it follows that an equilateral triangle is <u>also</u> isosceles, since it has three equal sides, which is <u>at least</u> two equal sides. Thus, based on our definitions, every equilateral triangle is isosceles, but certainly not every isosceles triangle is equilateral.

Homework

1. Two sides of an isosceles triangle are 14 and 20. The third side must be either _____ or _____.
2. One side of an equilateral triangle is 52. What are the other two sides of the triangle?
3. Two sides of a triangle are 23 and 99. If the perimeter is 200, find the length of the third side.
4. Two sides of an isosceles triangle are 17 and 17. Must the third side be different from 17?

5. True/False:
   a. Every equilateral triangle is isosceles.
   b. Every isosceles triangle is equilateral.

6. Translate to algebra (and simplify if possible):
   a. 3 more than n
   b. 13 less than a
   c. 7 more than twice T
   d. 12 less than 3 times Q
   e. 5 more than $n + 8$
   f. 10 less than $3a + 6$
   g. 3 more than $4x - 13$
   h. 8 less than $2b - 20$
   i. 17 more than $9y + 18$
   j. 23 less than $5w + 20$

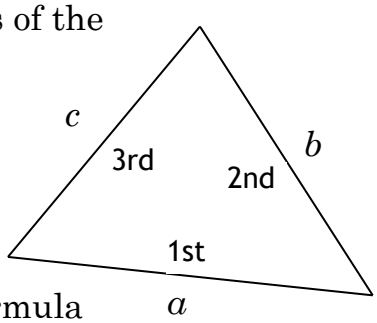


❑ Perimeter Problems

EXAMPLE 1: **The first side of a triangle is 2 more than twice the second side, while the third side is 3 more than the second side. If the perimeter is 21, what is the length of each side?**

Solution: First, notice that this problem deals with the perimeter of a triangle, so the main formula we'll deal with is the fact that the perimeter is the sum of the lengths of the three sides.

We start by giving names to the three sides of the triangle:



a = 1st side
b = 2nd side
c = 3rd side

The first equation we might write is the formula for the perimeter of the triangle:

$a + b + c = 21$ (the perimeter is 21)

Here's the dilemma in a nutshell: too many variables for a single equation. Let's read the question again, looking for more information.

Check out the phrase: *The first side of the triangle is 2 more than twice the second.* We can translate this to Algebra like this:

$a = 2b + 2$

Similarly, *the third side is 3 more than the second side* becomes

$c = b + 3$

Let's rewrite the perimeter formula we formed above, and then substitute our little formulas above for a and c:

$$\begin{array}{l} a + b + c = 21 \\ \downarrow \quad \downarrow \quad \downarrow \\ \boxed{2b+2} + \boxed{b} + \boxed{b+3} = 21 \end{array}$$

$\Rightarrow \quad 2b + 2 + b + b + 3 = 21$

$\Rightarrow \quad 4b + 5 = 21$ (combine like terms)

$\Rightarrow \quad 4b = 16$ (subtract 5 from each side)

$\Rightarrow \quad \underline{b = 4}$ (divide each side by 4)

Now that we have b, the second side, we deduce that:

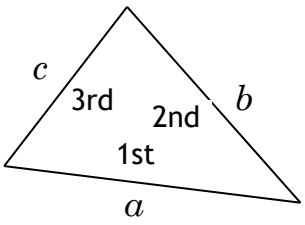
The first side is $a = 2b + 2 = 2(4) + 2 = \underline{10}$.
The second side is, of course, $b = \underline{4}$.
The third side is $c = b + 3 = 4 + 3 = \underline{7}$.

We're done. The sides of the triangle are

10, 4, and 7

EXAMPLE 2: **The perimeter of a triangle is 38. Its first side is 3 less than 2 times the third side, and its second side is 6 less than the first side. What are the lengths of its sides?**

Solution: As in the previous example, let's label the 1st, 2nd, and 3rd sides as a, b, and c.



A perimeter of 38 means that

$$a + b + c = 38$$

And we have the same problem . . . too many variables.

The phrase *Its first side is 3 less than 2 times the third side* translates to

$$a = 2c - 3$$

The phrase *its second side is 6 less than the first side* gives us the additional equation:

$$b = a - 6$$

We rewrite the perimeter formula, and then substitute:

$$a + b + c = 38$$

$$\Rightarrow \quad \boxed{2c - 3} + \boxed{a - 6} + \boxed{c} = 38 \qquad \text{(substitute)}$$

$$\Rightarrow \quad 3c - 9 + a = 38 \qquad \text{(combine like terms)}$$

We have a little problem here. Our perimeter equation has <u>two</u> variables in it, a and c. We need to substitute again, using the fact that $a = 2c - 3$:

$$\Rightarrow \quad 3c - 9 + \boxed{2c - 3} = 38 \qquad \text{(substitute } 2c - 3 \text{ for } a\text{)}$$

$$\Rightarrow \quad 5c - 12 = 38 \qquad \text{(combine like terms}$$

$$\Rightarrow \quad 5c = 50 \qquad \text{(add 12 to each side)}$$

$$\Rightarrow \quad c = \mathbf{10} \qquad \text{(divide each side by 5)}$$

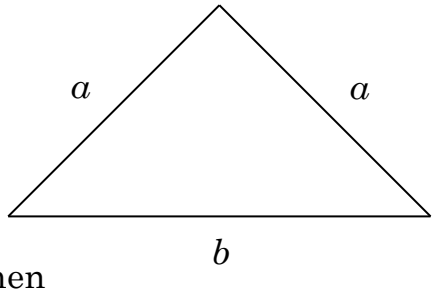
We can now find all three sides:

The third side is $c = \mathbf{10}$.
The first side is $a = 2c - 3 = 2(\mathbf{10}) - 3 = 17$.
The second side is $a - 6 = \mathbf{17} - 6 = 11$.

The three sides of the triangle are

17, 11, and 10

EXAMPLE 3: **An isosceles triangle has a perimeter of 24. If its third side is 4 less than 2 times one of the equal sides, what is the longest side of the triangle?**



Solution: An isosceles triangle has (at least) two equal sides, but we certainly can't assume that all three sides are equal. So we'll let a represent the length of each of the equal sides. Then the third side, call it b, must be expressed as

$$b = 2a - 4$$

Since the sum of these three quantities must equal the given perimeter, our equation is

	$a + a + b = 24$	(perimeter formula)
$\Rightarrow$	$a + a + 2a - 4 = 24$	(since $b = 2a - 4$)
$\Rightarrow$	$4a - 4 = 24$	(combine like terms)
$\Rightarrow$	$4a = 28$	(add 4 to each side)
$\Rightarrow$	$a = \mathbf{7}$	(divide each side by 4)

So the two equal sides, a, are each 7, while the third side, b, is $2(\mathbf{7}) - 4 = 10$. Hence, the longest side of the triangle (go back and read what's being asked for) has length

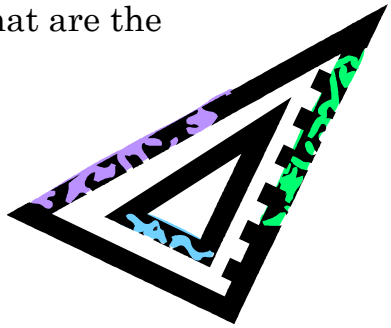
$$\boxed{10}$$

Homework

7. An isosceles triangle has a perimeter of 30. If its third side is 2 less than 2 times one of the equal sides, what are the lengths of its sides?

8. The perimeter of a triangle is 59. Its first side is 4 less than 2 times the second, and its third side is 13 less than the first side. What are the lengths of its sides?

9. An isosceles triangle has a perimeter of 9. If its third side is 3 less than 2 times one of the equal sides, what are the lengths of its sides?



10. The perimeter of a triangle is 40. Its first side is 4 less than 2 times the second, and its third side is 2 less than the first side. What are the lengths of its sides?

11. An isosceles triangle has a perimeter of 18. If its third side is 2 less than 2 times one of the equal sides, what are the lengths of its sides?

12. The perimeter of a triangle is 12. Its first side is 1 less than 2 times the second, and its third side is 1 less than the first side. What are the lengths of its sides?

13. An isosceles triangle has a perimeter of 47. If its third side is 2 more than one of the equal sides, what are the lengths of its sides?

14. The perimeter of a triangle is 32. Its first side is 3 less than the second, and its third side is 11 less than twice the first side. What are the lengths of its sides?

15. The third side of a triangle is 16 more than 3 times the first side, while the second side is 8 more than the third side. If the perimeter is 187, what is the length of each side?

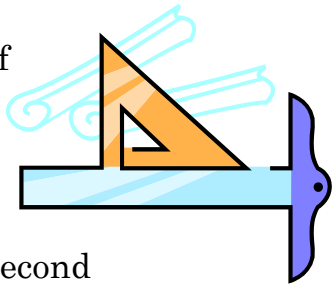
16. The second side of a triangle is 15 less than the third side, while the third side is 4 less than 3 times the first side. If the perimeter is 117, what is the length of each side?

17. The second side of a triangle is 14 more than 3 times the third side, while the first side is 12 more than 3 times the third side. If the perimeter is 166, what is the length of each side?

18. The second side of a triangle is 5 less than 2 times the first side, while the third side is 2 less than 2 times the first side. If the perimeter is 48, what is the length of each side?

19. An isosceles triangle has a perimeter of 91. If its third side is 57 less than 2 times one of the equal sides, what are the lengths of its sides?

Review Problems



20. An isosceles triangle has a perimeter of 80. If its third side is 24 less than 2 times one of the equal sides, what are the lengths of its sides?

21. The first side of a triangle is 2 less than the second side, while the second side is 9 more than 4 times the third side. If the perimeter is 205, what is the length of each side?

22. The first side of a triangle is 14 less than 4 times the third side, while the second side is 16 less than 4 times the third side. If the perimeter is 159, what is the length of each side?

23. The third side of a triangle is 3 less than the first side, while the first side is 12 less than 2 times the second side. If the perimeter is 58, what is the length of each side?

24. The third side of a triangle is 11 more than 3 times the first side, while the second side is 7 less than 4 times the first side. If the perimeter is 108, what is the length of each side?

25. The third side of a triangle is 3 less than the second side, while the second side is 6 more than 3 times the first side. If the perimeter is 121, what is the length of each side?

26. An isosceles triangle has a perimeter of 139. If its third side is 41 less than 2 times one of the equal sides, what are the lengths of its sides?

27. The second side of a triangle is 12 less than 2 times the first side, while the third side is 10 less than the second side. If the perimeter is 81, what is the length of each side?

28. The second side of a triangle is 16 less than the third side, while the third side is 17 less than 3 times the first side. If the perimeter is 97, what is the length of each side?

Solutions

1. 14, 20

2. 52 and 52

3. 78

4. No

5. a. T b. F

6. a. $n + 3$ b. $a - 13$
c. $2T + 7$ d. $3Q - 12$
e. $(n + 8) + 5 = n + 13$ f. $(3a + 6) - 10 = 3a - 4$

g. $(4x - 13) + 3 = 4x - 10$ h. $(2b - 20) - 8 = 2b - 28$
i. $(9y + 18) + 17 = 9y + 35$ j. $(5w + 20) - 23 = 5w - 3$

7. 8, 8, 14 **8**. 28, 16, 15 **9**. 3, 3, 3

10. 16, 10, 14 **11**. 5, 5, 8 **12**. 5, 3, 4

13. 15, 15, 17 **14**. 9, 10, 13 **15**. 21, 79, 87

16. 20, 56, 41 **17**. 20, 74, 72 **18**. 11, 17, 20

19. 37, 37, 17 **20**. 26, 26, 28 **21**. 21, 93, 91

22. 21, 70, 68 **23**. 17, 22, 19 **24**. 13, 50, 45

25. 16, 54, 51 **26**. 45, 45, 49 **27**. 23, 34, 24

28. 21, 46, 30

"A life spent making mistakes is not only more honorable, but more useful than a life spent doing nothing."

– George Bernard Shaw