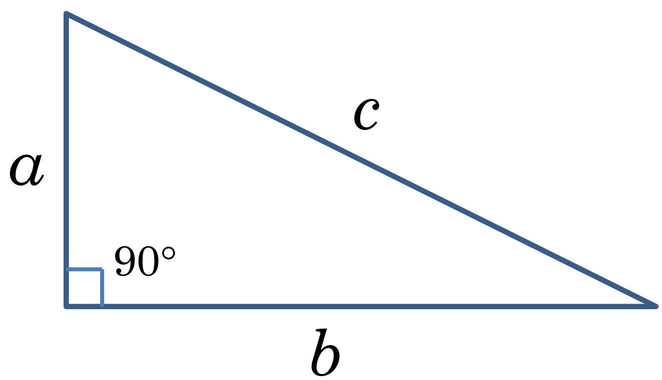
CH 34 – MORE PYTHAGOREAN THEOREM AND RECTANGLES

❑ Recalling The Pythagorean Theorem

$$a^2 + b^2 = c^2$$



The 90° angle is called the **right** angle of the right triangle. The other two angles of the right triangle are called the **acute** angles, because each of them must be less than 90°. The **legs** are *a* and *b*, and the **hypotenuse** is *c*.

EXAMPLE 1: **One leg of a right triangle is 7, while the hypotenuse is 25. Find the length of the other leg.**

Solution: Prior to this chapter we were always given the two legs and asked to find the hypotenuse. This problem throws us a curve, but we now have the skills needed to solve it. Since the sum of the squares of the legs must equal the square of the hypotenuse, we start with the Pythagorean Theorem,

$$a^2 + b^2 = c^2$$ *a* and *b* are legs; *c* is the hypotenuse

Putting in a leg of 7 and a hypotenuse of 25, the formula becomes

$$a^2 + \mathbf{7}^2 = \mathbf{25}^2 \quad \text{(the leg can be either } a \text{ or } b\text{)}$$

$$\Rightarrow \quad a^2 + 49 = 625 \quad \text{(square the numbers)}$$

$$\Rightarrow \quad a^2 + 49 - \mathbf{49} = 625 - \mathbf{49} \quad \text{(subtract 49 from each side)}$$

$$\Rightarrow \quad a^2 = 576 \quad \text{(simplify each side)}$$

$$\Rightarrow \quad a = 24 \quad \text{(calculator or guessing)}$$

We conclude that the other leg has a length of $\boxed{24}$

Check: If we're claiming that a right triangle can have legs of 24 and 7 and a hypotenuse of 25, then these three values had better satisfy the Pythagorean Theorem. Let's see if they do:

$$(\text{leg})^2 + (\text{leg})^2 \stackrel{?}{=} (\text{hypotenuse})^2$$

$$24^2 + 7^2 \stackrel{?}{=} 25^2$$

$$576 + 49 \stackrel{?}{=} 625$$

$$625 = 625 \checkmark$$

EXAMPLE 2: **Find the hypotenuse of a right triangle whose legs are 5 and 7.**

Solution:

$$a^2 + b^2 = c^2 \quad \text{(the Pythagorean Theorem)}$$

$$\mathbf{5}^2 + \mathbf{7}^2 = c^2 \quad \text{(substitute the known values)}$$

$$25 + 49 = c^2 \quad \text{(square each leg)}$$

$$74 = c^2 \quad \text{(simplify)}$$

Is there a whole number whose square is 74? No, because $8^2 = 64$, which is smaller than 74, while $9^2 = 81$, which is bigger than 74. We see, therefore, that the solution is somewhere between the consecutive numbers 8 and 9; but where between 8 and 9?

$8^2 = 64$ ✓
$what^2 = 74$??
$9^2 = 81$ ✓

Finding a number whose square is 74 is the kind of problem that has plagued and enticed mathematicians, scientists, and philosophers for literally thousands of years. They'd really be mad if they knew that we can find an excellent approximation of this number using a cheap calculator. Enter the number 74 followed by the *square root* key (or the other way around, depending on the calculator), which is labeled something like $\sqrt{x}$. Thus, the hypotenuse is $\sqrt{74}$ (read: *the positive square root of 74*), and your calculator should have the result 8.602325267, or something close to that.

But your calculator doesn't tell the whole story. The fact is, the square root of 74 has an infinite number of digits following the decimal point, and they never have a repeating pattern (very similar to the number π). Thus, we'll have to round off the answer to whatever's appropriate for the problem. For this problem we'll round to the third digit past the point.

The hypotenuse is 8.602

Notice that $8.602^2 = 73.994404$, which is quite close to 74.

EXAMPLE 3: **The hypotenuse of a right triangle is 15 and one of its legs is 4. Find the length of the other leg (rounded to 3 digits).**

Solution: Let's get right to it:

$$a^2 + b^2 = c^2 \Rightarrow \mathbf{4}^2 + b^2 = \mathbf{15}^2 \Rightarrow 16 + b^2 = 225$$

$$\Rightarrow \quad b^2 = 209 \quad \Rightarrow \quad b = \boxed{14.457} \quad \text{(approximately)}$$

This result was found using a calculator; if no calculator is allowed, then the answer would be

The leg is between 14 and 15.

Homework

1. In each problem, one leg and the hypotenuse of a right triangle are given (l = leg and h = hypotenuse). Find the other leg -- NO calculator.

   a. $l = 3;\ h = 5$ b. $l = 5;\ h = 13$ c. $l = 6;\ h = 10$

   d. $l = 9;\ h = 41$ e. $l = 12;\ h = 13$ f. $l = 24;\ h = 25$

2. In each problem, one leg and the hypotenuse of a right triangle are given (l = leg and h = hypotenuse). Use your calculator to find the other leg, rounded to 3 digits. If no calculator is allowed, tell what two consecutive whole numbers the answer is between.

   a. $l = 3;\ h = 10$ b. $l = 2;\ h = 9$ c. $l = 5;\ h = 6$

   d. $l = 4;\ h = 20$ e. $l = 7;\ h = 19$ f. $l = 13;\ h = 55$

3. A triangle has sides of 6, 10, and 8. Prove that it's a *right triangle*. Hint: See the Check in Example 1.

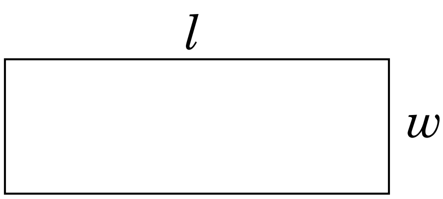
4. A triangle has sides of 5, 9, and 7. Prove that it's <u>not</u> a *right triangle*.

   [Note: Whether you're trying to prove that a triangle is – or is not – a right triangle, logic dictates that you must assume that the longest of the three given sides is the hypotenuse.]

❑ More on Rectangles and Perimeters

EXAMPLE 4: **The length of a rectangle is 4 less than 3 times its width. If the perimeter is 32, find the dimensions of the rectangle.**

Solution: Ignoring details for a moment, we see that the problem is talking about the perimeter of a rectangle:



$$P = 2l + 2w$$

Since the length is 4 less than 3 times the width, we can write

$$l = 3w - 4$$

We also know that the perimeter is 32; that is, $P = 32$. Thus, the general formula

$$P = 2l + 2w$$

becomes

$32 = 2\boxed{(3w - 4)} + 2w$ (since $P = 32$ and $l = 3w - 4$)

$\Rightarrow$ $32 = 6w - 8 + 2w$ (distribute)

$\Rightarrow$ $32 = 8w - 8$ (combine like terms)

$\Rightarrow$ $40 = 8w$ (add 8 to each side)

$\Rightarrow$ $w = \mathbf{5}$ (divide each side by 8)

$\Rightarrow$ length $= 3w - 4 = 3(\mathbf{5}) - 4 = 15 - 4 = \mathbf{11}$

Therefore, the dimensions of the rectangle are $\boxed{\text{11 by 5}}$

Homework

5. The length of a rectangle is 30 more than 2 times the width. If the perimeter is 66, find the dimensions of the rectangle.

6. Find the dimensions of a rectangle with a perimeter of 44 if its length is 6 more than 3 times its width.

7. The width of a rectangle is 23 less than 2 times the length. If the perimeter is 62, find the dimensions of the rectangle.

8. The width of a rectangle is 18 less than 2 times the length. If the perimeter is 48, find the dimensions of the rectangle.

9. Find the dimensions of a rectangle with a perimeter of 42 if its width is 15 less than 2 times its length.

10. The width of a rectangle is 26 less than 3 times the length. Find the dimensions of the rectangle if its perimeter is 28.

11. The length of a rectangle is 539 less than 6 times its width. If the perimeter is 2254, find the dimensions of the rectangle.

12. The length of a rectangle is 30 more than 5 times the width. If the perimeter is 120, find the dimensions of the rectangle.

Review Problems

13. The hypotenuse of a right triangle is 65 and one of its legs is 60. Find the other leg.

14. Use a calculator (if allowed) to find the leg of a right triangle if its hypotenuse is 89 and one of its legs is 1. Round to the 3rd digit.

15. The perimeter of a rectangle is 100. If the length is 10 more than 7 times the width, find the dimensions of the rectangle.

16. The width of a rectangle is 14 less than 3 times the length. Find the dimensions of the rectangle if its perimeter is 20.

17. Verify that a triangle with sides 24, 10, and 26 is a right triangle.

18. Is a triangle with sides 15, 12, and 8 a right triangle?

Solutions

1. a. 4 b. 12 c. 8 d. 40
e. 5 f. 7

2. a. 9.539 b. 8.775 c. 3.317 d. 19.596
e. 17.664 f. 53.442

3. Verify that $6^2 + 8^2 = 10^2$.

4. Show that $5^2 + 7^2 \neq 9^2$. Notice that even though the 9 was listed second in the list of the three sides of the triangle, we used the 9 as the potential hypotenuse.

5. 32 by 1 **6.** 18 by 4 **7.** 18 by 13 **8.** 14 by 10

9. 12 by 9 **10.** 10 by 4 **11.** 889 by 238 **12.** 55 by 5

13. 25 **14.** 88.99 **15.** 45 by 5 **16.** 6 by 4

17. Verify that $10^2 + 24^2 = 26^2$.

18. No; be sure you assume that the hypotenuse would be the 15.

"It is possible to store the mind with a million facts and still be entirely uneducated."

– Alec Bourne, M.D.