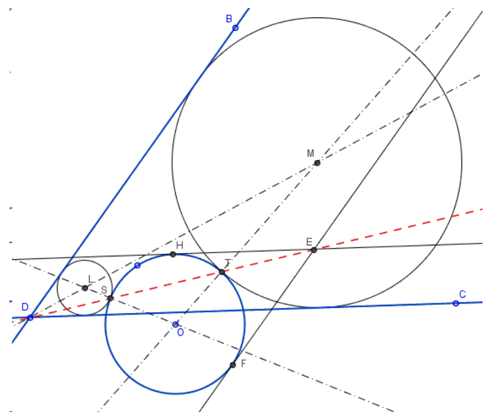
CH 36 – GRAPHING LINES

❑ Introduction



Consider the equation

$$y = 2x - 1$$

If we let $x = 10$, for instance, we can calculate the corresponding value of y:

$$y = 2(\mathbf{10}) - 1 = 20 - 1 = 19$$

Thus, a solution of the equation is the pair $x = 10$, $y = 19$, which we can write as the ordered pair (10, 19). Another solution to this equation is (0, –1); in fact, there are an infinite number of solutions. Our goal now? Make a graph (a picture) of ALL the solutions of the equation.

❑ Graphing a Line

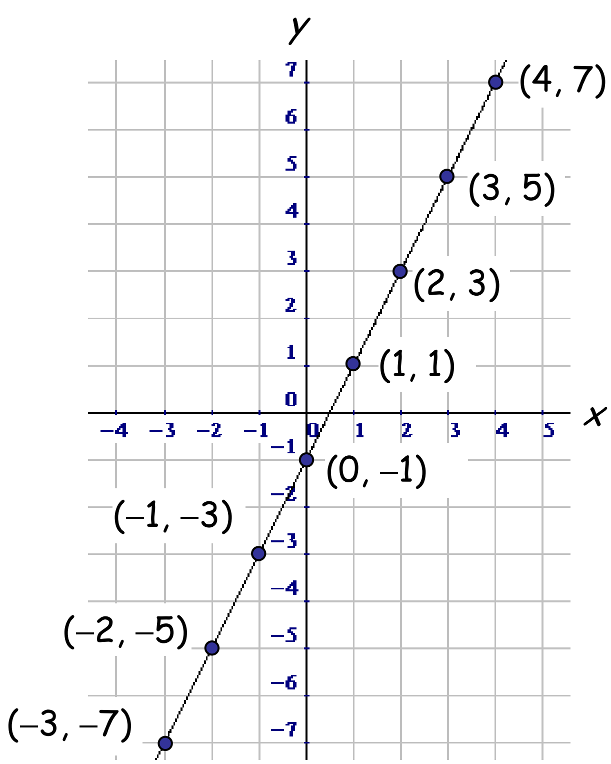
<u>EXAMPLE 1:</u> **Graph the line $y = 2x - 1$.**

<u>Solution:</u> The most important aspect of graphing is to learn where the x-values come from. They basically come from your head -- you get to make them up. Certainly, in later courses this process will become more sophisticated, but for now just conjure them up from your imagination.

I'm going to choose x-values of –3, –2, –1, 0, 1, 2, 3, and 4. For each of these x-values, I will calculate the corresponding y-value using the given formula, $y = 2x - 1$. Organizing all this information in a table is useful:

x	$y = 2x - 1$	(x, y)
−3	2(**−3**) − 1 = −6 − 1 = −7	(−3, −7)
−2	2(**−2**) − 1 = −4 − 1 = −5	(−2, −5)
−1	2(**−1**) − 1 = −2 − 1 = −3	(−1, −3)
0	2(**0**) − 1 = 0 − 1 = −1	(0, −1)
1	2(**1**) − 1 = 2 − 1 = 1	(1, 1)
2	2(**2**) − 1 = 4 − 1 = 3	(2, 3)
3	2(**3**) − 1 = 6 − 1 = 5	(3, 5)
4	2(**4**) − 1 = 8 − 1 = 7	(4, 7)

Now I will plot each of the eight points just calculated in 2-space, that is, on an *x*-*y* coordinate system. Then the points will be connected with the most reasonable graph, in this case a straight line.



Notice that each (*x*, *y*) point is just one of the many solutions of the equation *y* = 2*x* − 1. Thus, each time we plot one of these points, we're plotting a solution of the equation. Also, whatever graph we get when we're done plotting all possible points is simply a picture of all of the solutions of the equation. So, in essence, we'll have a graph of the equation *y* = 2*x* − 1.

Notice that the graph passes through every quadrant except the second; also notice that as we move from left to right (that is, as the x's grow larger) the graph rises.

Final Comments on Example 1:

We could let $x = 1{,}000{,}000$ for this equation, in which case y would equal $2(\mathbf{1{,}000{,}000}) - 1 = 1{,}999{,}999$. That is, the point (1000000, 1999999) is on the line. Can we graph it? Not with the scale we've selected for our graph. But if we traveled along the line – up and up and up and way up – we would eventually run into the point (1000000, 1999999).

We could also have used fractions like 2/7 for x. This would have given us the point (2/7, –3/7), which is in Quadrant IV. This point, too, is definitely on the line.

And we may even have used a number like π for x, in which case we would obtain the point $(\pi, 2\pi - 1)$. This point is impossible to plot precisely, but I guarantee that it's in the first quadrant (both coordinates are positive), and it's on the line we've drawn.

In short, every solution of the equation $y = 2x - 1$ is a point on the line, and every point on the line is a solution of the equation.

EXAMPLE 2: **Graph the line $y + x = 2$.**

Solution: The equation of the line will be easier to work with if we first solve for y by subtracting x from each side of the equation:

$$y = 2 - x$$

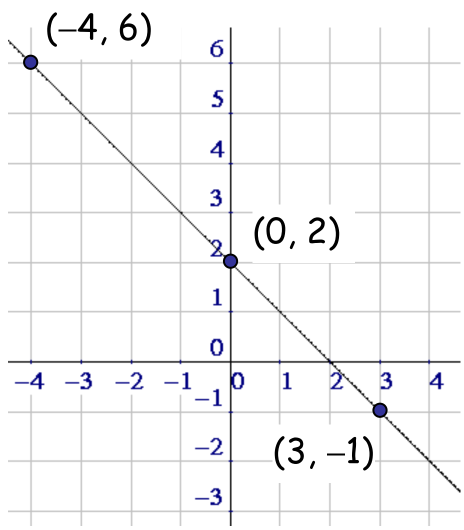
It's also better form (you'll see why later) if we put the $-x$ term right after the equal sign. The line is now

$$\mathbf{y = -x + 2}$$

If $x = 0$, then $y = -\mathbf{0} + 2 = 2$. Thus, (0, 2) is on the line.

If $x = 3$, then $y = -\mathbf{3} + 2 = -1$. So (3, –1) is on the line.

If $x = -4$, then $y = -(-4) + 2 = 4 + 2 = 6$. Therefore, $(-4, 6)$ is on the line. If we plot these three points, and then connect them together, we get a graph like the following:



Notice that the graph passes through every quadrant except the third. Also note that as we move from left to right (that is, as the *x*'s grow larger) the graph falls.

Note: Of course it's true that two different points completely determine a line, and so some students plot exactly two points, connect them with a straight line, and they're done. Warning: You're taking a big gamble when you plot just two points -- if you make an error with either point, you'll get the wrong line, and there will be no way for you to know that you've goofed. Even worse, what if the equation isn't even a line in the first place? Plotting just two points (even if they're both correct) will not suffice to graph a curve that is not a simple straight line (like a circle).

The more points you plot, the better!

Homework

1. a. Does the point $(3, 19)$ lie on the line $y = 6x + 1$?

   b. Does the point $(-1, 5)$ lie on the line $y = -2x + 4$?

   c. Does the point $(4, -3)$ lie on the line $2x + 5y = 3$?

   d. Does the point $(-2, -4)$ lie on the line $x - 3y = 10$?

2. Graph each of the following lines:

a. $y = x$ b. $y = -x$ c. $y = 2x$

d. $y = -3x$ e. $y = 1.5x$ f. $y + 0.5x = 0$

3. Graph each of the following lines:

a. $y = x + 3$ b. $y = x - 2$ c. $y = -x + 1$

d. $y = 2x + 3$ e. $y = 3x - 4$ f. $y = -2x - 1$

g. $y + 2x = 1$ h. $y - x = 2$ i. $y - 2x + 1 = 0$

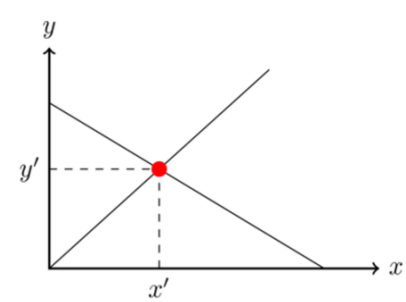
j. $y = 0.5x - 1$ k. $y = -1.5x - 2$ l. $y = -3x - 3$

❑ The Graphing Method for Two Equations in Two Variables

Consider the system of two equations in two variables:

$$\begin{aligned} x + y &= 7 \\ -2x + y &= 1 \end{aligned}$$

We have learned two methods to solve this system: the *Addition Method*, where we add the equations together (after multiplying the equations by some appropriate numbers) to eliminate one of the variables. The other was termed *Substitution*, where we solved one of the equations for some letter, and then substituted that result into the other equation.



Now that we know how to graph straight lines, we're going to learn a third method to solve a system of two equations in two variables. Each equation in the system above is a line; assuming the lines intersect (and assuming they're not the same line), there will be <u>one</u> point of intersection. Since that point of intersection lies on both lines, the coordinates (the x and y) of that point must satisfy both of the equations. Does that make any sense?

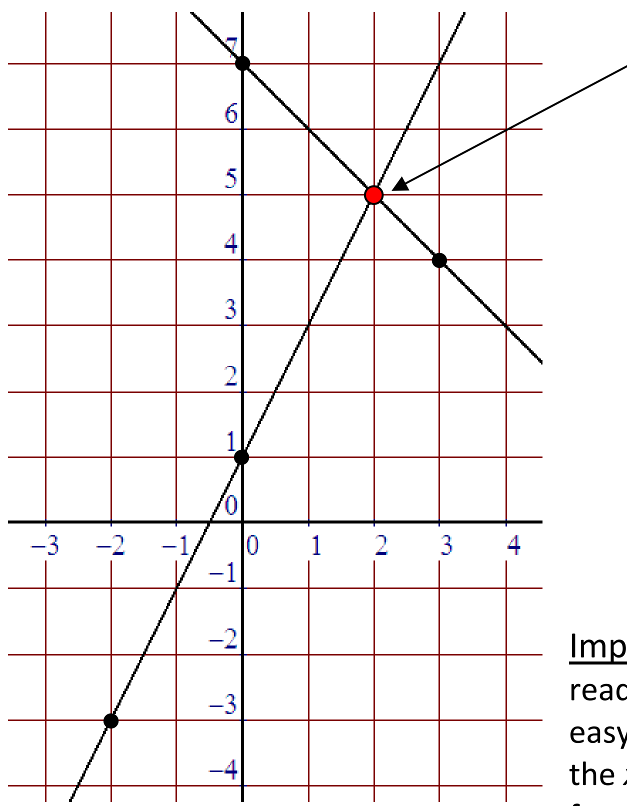
EXAMPLE 3: **Solve the system of equations by graphing.**

$$\begin{aligned} x + y &= 7 \\ -2x + y &= 1 \end{aligned}$$

Solution: Each equation is a line, so let's graph each of them and then turn our attention to their point of intersection.

Line #1: Solve for y to get $y = -x + 7$. If we let $x = 3$, then $y = 4$, and so (3, 4) is on the line. If we choose $x = 0$, then $y = 7$, in which case (0, 7) is on the line. We'll use these two points for Line #1.

Line #2: Solve for y to get $y = 2x + 1$. If $x = 0$, then $y = 1$, and if $x = -2$, then $y = -3$. So we'll use the points (0, 1) and (–2, –3) for Line #2.



The point of intersection of the two lines appears to be the point (2, 5). This means that the solution of the given system of equations is

$x = 2, y = 5$

Important Note: Since we're reading points on a graph, it's very easy to misread them (imagine if the x- and y-coordinates were fractions). Thus, the graphing method is only an approximation, but it's a great method when no algebraic methods exist.

Review Problems

4. a. If $y = -7x + 10$, and if $x = 10$, then $y =$ ___.
   b. If $y = 13x$, and if $x = \pi$, then $y =$ ___. (give the exact answer)
   c. Describe the line $y = x$.
   d. Describe the line $y = -x$.

5. Graph: $y = -3x - 1$

6. Sketch each line and determine the only quadrant that the line does not pass through:

   a. $y = 2x + 7$ b. $y = 3x - 9$ c. $y = -2x + 7$

   d. $y = -5x + 1$ e. $y = -x - 9$ f. $y = \pi x - \sqrt{2}$

7. Determine whether or not the graph of the given equation passes through the origin [the point (0, 0)].

   a. $y = x + 1$ b. $y = x$ c. $y = x - 5$

   d. $y = 7x$ e. $y = x^2$ f. $y = x^2 + 1$

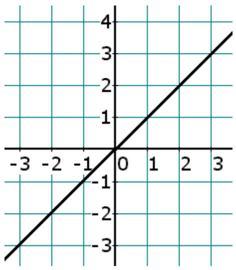
   g. $y = 7 - x$ h. $y = -x$ i. $y = x^3$

   j. $y = \frac{x}{3}$ k. $y = \frac{x+1}{2}$ l. $y = 0.3x^2$

Solutions

1. a. Yes b. No c. No d. Yes

2. a.



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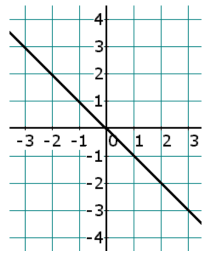
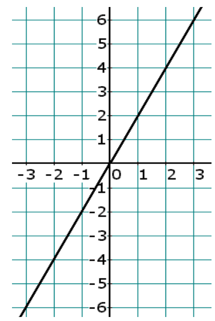


c.



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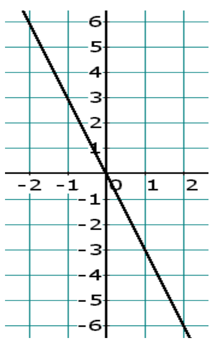

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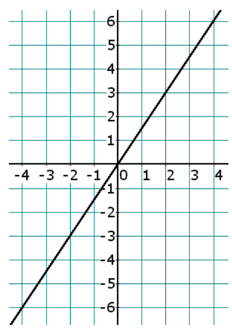

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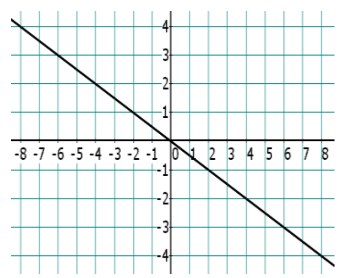
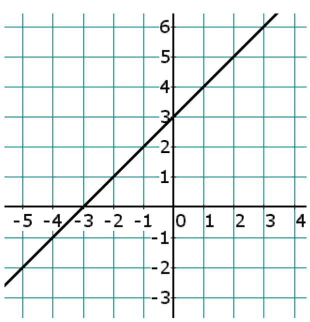


3. a.



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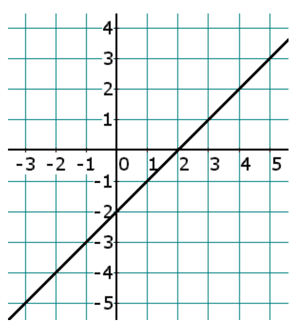

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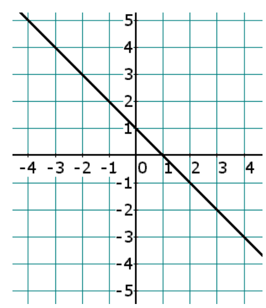

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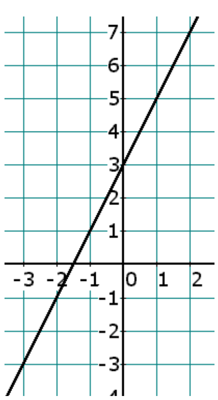

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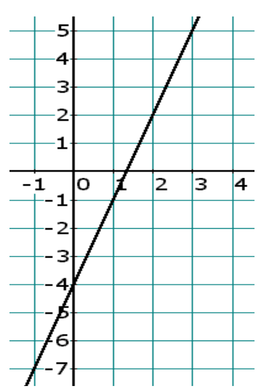

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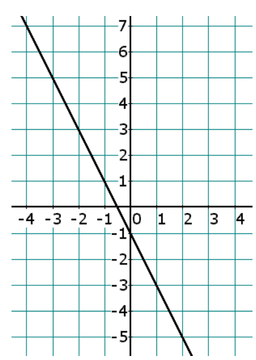
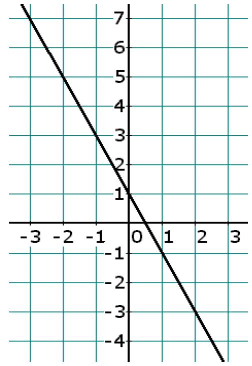


g.



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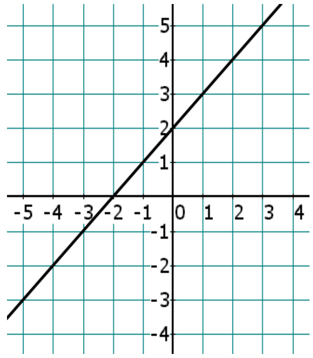
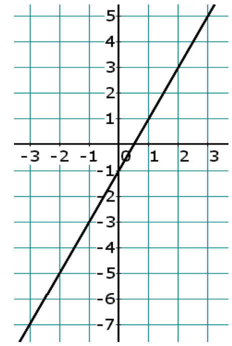
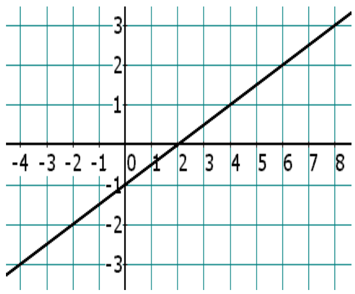


i.



j.



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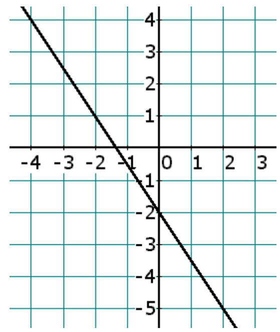
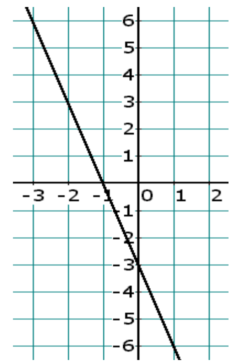
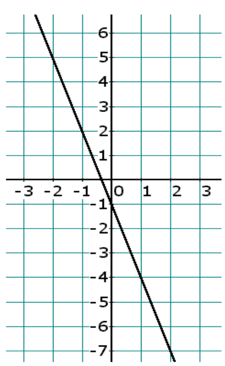


l.



4.
a. –60
b. 13π
c. diagonal, thru the origin, going thru Quadrants I and III
d. diagonal, thru the origin, going thru Quadrants II and IV

5.



6. a. IV   b. II   c. III   d. III   e. I   f. II

7. a. No   b. Yes   c. No   d. Yes   e. Yes   f. No
g. No   h. Yes   i. Yes   j. Yes   k. No   l. Yes

❑ To ∞ and Beyond!

A. Graph: $y = x^2$

B. Graph: $x^2 + y^2 = 25$ Think 0's & 5's; then think 3's & 4's.

"To the world you might be
just one person,
but to one person,
you might just be the world."

– Theodore Geisel (Dr. Seuss)