CH 61 – USING THE GCF IN EQUATIONS AND FORMULAS

❑ Introduction

A while back we studied the Quadratic Formula and used it to solve quadratic equations such as $x^2 - 5x + 6 = 0$; we were also able to solve rectangle problems with a given area, and physics problems asking for the time required to drop a math teacher to the ground. ☺

This chapter's goal is to give us an alternate method for solving certain kinds of quadratic equations. It's called the ***factoring method***, and will allow us to apply our skills factoring the GCF. Since the Quadratic Formula always works on any quadratic equation, why do we need another method to solve a quadratic equation? The factoring method (when it works) is sometimes much easier and quicker to apply than the Quadratic Formula.

Consider the quadratic equation

$$(x)(x - 7) = 0$$

Can you see why this is a quadratic equation?

There are two solutions to this equation -- what are they? Before we present the formal process, here's what you should note: What would happen if we let $x = 0$ in the equation $(x)(x - 7) = 0$? We'd get

$$(\mathbf{0})(\mathbf{0} - 7) = (0)(-7) = 0 \quad \checkmark$$

And so we've stumbled upon a solution of the equation $(x)(x - 7) = 0$. Let's "stumble" one more time and choose $x = 7$ -- check it out:

$$(\mathbf{7})(\mathbf{7} - 7) = (7)(0) = 0 \quad \checkmark$$

Can you see how we stumbled across these two solutions, 0 and 7? Each solution was chosen so that one of the two factors would turn into zero. That way, the product of that zero factor with the other factor (no matter what it may be) would have to be zero.

EXAMPLE 1: **Solve for n: $3n(2n - 13) = 0$**

Solution: We have something ($3n$) times something else ($2n - 13$) whose product is 0. We saw above that if something times something is 0, then either the first something is 0 or the second something is 0. So we set each factor to 0 and get our two solutions for n:

> If $ab = 0$
>
> then $a = 0$
>
> or $b = 0$.

Setting the first factor, $3n$, to 0:

$$3n = 0$$

$$\Rightarrow \quad n = 0$$

Setting the second factor, $2n - 13$, to 0:

$$2n - 13 = 0$$

$$\Rightarrow \quad 2n = 13$$

$$\Rightarrow \quad n = \frac{13}{2}$$

Let's check each solution:

$n = 0$:

$$3(\mathbf{0})\big(2(\mathbf{0}) - 13\big) = 3(0)(-13) = 0 \quad \checkmark$$

$n = \frac{13}{2}$:

$$3\left(\frac{\mathbf{13}}{\mathbf{2}}\right)\left(2\left(\frac{\mathbf{13}}{\mathbf{2}}\right) - 13\right) = 3\left(\frac{\mathbf{13}}{\mathbf{2}}\right)(13 - 13) = 3\left(\frac{\mathbf{13}}{\mathbf{2}}\right)(0) = 0 \quad \checkmark$$

Thus, there are two solutions for n: $\boxed{0,\ \frac{13}{2}}$

Homework

1. Solve each quadratic equation by setting each factor to 0:

a. $x(x-3) = 0$

b. $a(a+4) = 0$

c. $y(2y-7) = 0$

d. $2c(c-1) = 0$

e. $3d(d+12) = 0$

f. $4w(2w+1) = 0$

g. $5u(3u-5) = 0$

h. $7z(z+99) = 0$

i. $13n(13n+12) = 0$

j. $b(7b+1) = 0$

k. $-5w(9w-2) = 0$

l. $-12a(13a-13) = 0$

❑ Solving Quadratic Equations

In the Introduction and the homework we solved quadratic equations using the following reasoning: Since the product of two factors was given to be 0, it followed that either of the factors could be 0. We then set each factor to 0 and solved the two resulting linear equations.

But what do we do if the expression on the left side of the equation is not in factored form? Simple -- we factor it ourselves and then proceed as above. Also, if the right side of the equation is not 0, we can remove whatever's over there in order to force a 0 on the right side.

We now have a formal process for solving a quadratic equation using the factoring method:

1) Write the equation so that it's in standard quadratic form.
2) Factor the quadratic expression.
3) Set each factor to zero.
4) Solve each linear equation.

EXAMPLE 2: **Solve for x: $6x^2 + 7x = 0$**

Solution: We first notice that one side of the equation is 0, which is necessary to use the factoring method.

$$6x^2 + 7x = 0 \quad \text{(the original equation)}$$

$$\Rightarrow \quad x(6x + 7) = 0 \quad \text{(factor out the } x\text{)}$$

$$\Rightarrow \quad \underline{x = 0} \quad \text{or} \quad 6x + 7 = 0 \quad \text{(set each factor to 0)}$$

$$6x = -7 \quad \text{(solve each equation)}$$

$$\underline{x = -\frac{7}{6}}$$

The two solutions are therefore $\boxed{0, -\frac{7}{6}}$

EXAMPLE 3: **Solve for y: $9y^2 = 6y$**

Solution: In this problem we don't have the 0 we need on one side of the equation. But we can make the right side into a 0 by subtracting $6y$ from each side of the equation and then proceeding to factor and set the factors to 0.

We start with the original equation: $9y^2 = 6y$

1) Make the right side 0 by subtracting $6y$ from each side of the equation: $9y^2 - 6y = 0$

2) Factor out the $3y$: $3y(3y - 2) = 0$

3) Set each factor to 0: $3y = 0$ or $3y - 2 = 0$

4) Solve each linear equation: $\underline{y = 0}$ or $3y = 2$

$$\underline{y = \frac{2}{3}}$$

We thus get two solutions to our quadratic equation: $\boxed{0, \frac{2}{3}}$

Homework

2. Solve each quadratic equation by factoring:

a. $3n^2 + 7n = 0$	b. $2x^2 - 8x = 0$	c. $3y^2 = y$
d. $7t^2 = 6t$	e. $5u^2 = 10u$	f. $7a^2 = -2a$
g. $7w^2 - 84w = 0$	h. $n^2 + 83n = 0$	i. $7x^2 = 7x$
j. $-5y^2 = 2y$	k. $13a^2 = -a$	l. $w^2 = 19w$
m. $13g^2 + 12g = 0$	n. $8n - 3n^2 = 0$	o. $23Q^2 = 19Q$

3. Solve each quadratric equation by factoring:

a. $3n^2 - 7n = 0$	b. $2x^2 + 10x = 0$	c. $7y^2 = y$
d. $7t^2 = -9t$	e. $10w^2 = 20w$	f. $17a^2 = -2a$
g. $7c^2 - 77c = 0$	h. $m^2 + 93m = 0$	i. $9x^2 = 9x$
j. $-7y^2 = 8y$	k. $14k^2 = -k$	l. $z^2 = 21z$
m. $13h^2 + 11h = 0$	n. $7n - 4n^2 = 0$	o. $29R^2 = 17R$

❑ Solving More Formulas

Do you remember how we solved for x in the formula (literal equation)

$$wx + y = A \ ?$$

We subtracted y from each side of the equation:

$$wx = A - y$$

and then we divided each side of the equation by w:

$$x = \frac{A - y}{w}$$ and we've isolated the x.

This is all fine and dandy when the unknown occurs only once in a formula. But how do we isolate something that occurs more than once in a formula? For example, how do we solve for x in the formula

$$ax - c = bx\,?$$

There's an x-term on each side of the equation. This x is going to be tough to isolate. What would you do if, instead of the symbols a, b, and c in the equation, they had been numbers?

For example, suppose the equation had been

$$7x - 10 = 4x$$

We subtract $4x$ from each side:	$7x - 4x - 10 = 0$
Then combine like terms:	$3x - 10 = 0$
Now add 10 to each side:	$3x = 10$
And last, divide each side by 3:	$x = \frac{10}{3}$

We follow a similar procedure for solving the formula $ax - c = bx$ for x.

The original formula:	$ax - c = bx$
Subtract bx from each side:	$ax - bx - c = 0$
Add c to each side:	$ax - bx = c$
	↓ Factoring out the GCF
How do we "combine the like terms" ax and $-bx$? Here's where <u>factoring</u> comes to the rescue -- by factoring x out of $ax - bx$, we get $x(a - b)$, which contains only one x, not two:	$x(a - b) = c$
Divide each side by $a - b$, and we're done:	$\boldsymbol{x = \dfrac{c}{a-b}}$

Note: There are no x's on the right side of the answer. Can you explain why this fact is so utterly important?

EXAMPLE 4: **Solve for n: $Qn - n + P = R$**

Solution:

$Qn - n + P = R$ (the original formula)

$\Rightarrow$ $Qn - n = R - P$ (subtract P from each side)

$\Rightarrow$ $n(Q - 1) = R - P$ (factor out the n)

$\Rightarrow$ $\dfrac{n(Q-1)}{Q-1} = \dfrac{R-P}{Q-1}$ (divide each side by $Q - 1$)

$\Rightarrow$ $\boxed{n = \dfrac{R-P}{Q-1}}$ (simplify)

EXAMPLE 5: **Solve for a: $c(a - d) + 3 = 5(e - a)$**

Solution:

$c(a - d) + 3 = 5(e - a)$ (the original formula)

$\Rightarrow$ $ac - cd + 3 = 5e - 5a$ (distribute)

$\Rightarrow$ $ac + 5a - cd + 3 = 5e$ (add $5a$ to each side)

$\Rightarrow$ $ac + 5a - cd = 5e - 3$ (subtract 3 from each side)

$\Rightarrow$ $ac + 5a = 5e - 3 + cd$ (add cd to each side)

Note: These steps were designed to get the variable a on one side of the equation, and the rest of the things on the other side.

$\Rightarrow$ $a(c + 5) = 5e - 3 + cd$ (factor out the a)

$\Rightarrow$ $\dfrac{a(c+5)}{\mathbf{c+5}} = \dfrac{5e-3+cd}{\mathbf{c+5}}$ (divide each side by $c + 5$)

$\Rightarrow$ $\boxed{a = \dfrac{5e-3+cd}{c+5}}$ (simplify)

Be sure you understand thoroughly why there must be no a's on the right side of the answer.

Homework

4. Solve each formula for n:

a. $cn + dn = 3$

b. $an - cn = d$

c. $Ln + n = M$

d. $tn = c - sn$

e. $rn = 3 + tn$

f. $m(n + 1) - Qn - R = 0$

g. $a(n + 3) + b(n + c) = R$

h. $an + bn + cn = d$

i. $an - n - a = 0$

j. $2(n + 1) + an = c$

5. Solve each formula for x:

a. $cx + 7x = 14$

b. $rx - ux = w + v$

c. $wx - x = w$

d. $ax - b = c - dx$

e. $u(x - a) + x = w$

f. $a(x + 1) + b(x - 1) = 0$

g. $mx - 3(x - w) = z + u$

h. $p(x - 3) = q(x + 2)$

i. $c(x + a) - a(x - 1) = a - b$

j. $a(b - x) + c(2 - x) = R - Q$

Review Problems

6. a. Solve for x: $-17x(34x + 14) = 0$

b. Solve for n: $-23n^2 + 46n = 0$

c. Solve for y: $6y^2 = 8y$

d. Solve for x: $ax = bx + c$

e. Solve for x: $a(x - 1) + x = b$

f. Solve for x: $b(x - c) + d(n - x) = T$

g. Solve for x: $gx - a = hx - b$

h. Solve for x: $c(x - d) + b = e(x + n) - w$

7. a. Solve for n: $15n(12n - 16) = 0$

b. Solve for n: $23n^2 - 69n = 0$

c. Solve for z: $6z^2 = 14z$

d. Solve for x: $cx = ax - b + c$

e. Solve for x: $b(x + 1) + x = w$

f. Solve for x: $a(x + c) + d(n + x) = z$

g. Solve for u: $eu + a = hu + b$

h. Solve for x: $c(x + ad) - b = e(x - m) + u$

Solutions

1. a. $0, 3$ b. $0, -4$ c. $0, \frac{7}{2}$ d. $0, 1$

e. $0, -12$ f. $0, -\frac{1}{2}$ g. $0, \frac{5}{3}$ h. $0, -99$

i. $0, -\frac{12}{13}$ j. $0, -\frac{1}{7}$ k. $0, \frac{2}{9}$ l. $0, 1$

2. a. $0, -\frac{7}{3}$ b. $0, 4$ c. $0, \frac{1}{3}$ d. $0, \frac{6}{7}$

e. $0, 2$	f. $0, -\frac{2}{7}$	g. $0, 12$	h. $0, -83$
i. $0, 1$	j. $0, -\frac{2}{5}$	k. $0, -\frac{1}{13}$	l. $0, 19$
m. $0, -\frac{12}{13}$	n. $0, \frac{8}{3}$	o. $0, \frac{19}{23}$	

3.

a. $0, \frac{7}{3}$	b. $0, -5$	c. $0, \frac{1}{7}$	d. $0, -\frac{9}{7}$
e. $0, 2$	f. $0, -\frac{2}{17}$	g. $0, 11$	h. $0, -93$
i. $0, 1$	j. $0, -\frac{8}{7}$	k. $0, -\frac{1}{14}$	l. $0, 21$
m. $0, -\frac{11}{13}$	n. $0, \frac{7}{4}$	o. $0, \frac{17}{29}$	

4. a. $cn + dn = 3 \Rightarrow n(c+d) = 3 \Rightarrow \frac{n\cancel{(c+d)}}{\cancel{c+d}} = \frac{3}{c+d} \Rightarrow n = \frac{3}{c+d}$

b. $n = \frac{d}{a-c}$

c. $n = \frac{M}{L+1}$

d. $tn = c - sn \Rightarrow tn + sn = c \Rightarrow n(t+s) = c \Rightarrow n = \frac{c}{t+s}$

e. $n = \frac{3}{r-t}$

f. $m(n+1) - Qn - R = 0 \Rightarrow mn + m - Qn - R = 0$

$\Rightarrow mn - Qn + m - R = 0 \Rightarrow mn - Qn = R - m$

$\Rightarrow n(m-Q) = R - m \Rightarrow n = \frac{R-m}{m-Q}$

g. $a(n+3) + b(n+c) = R \Rightarrow an + 3a + bn + bc = R$

$\Rightarrow an + bn = R - 3a - bc \Rightarrow n(a+b) = R - 3a - bc$

$\Rightarrow n = \frac{R-3a-bc}{a+b}$

h. $n(a+b+c) = d \Rightarrow n = \frac{d}{a+b+c}$

i. $n(a-1) = a \Rightarrow n = \dfrac{a}{a-1}$

j. $2n+2+an = c \Rightarrow n(2+a) = c-2 \Rightarrow n = \dfrac{c-2}{a+2}$

5. a. $x = \dfrac{14}{c+7}$ b. $x = \dfrac{w+v}{r-u}$ c. $x = \dfrac{w}{w-1}$

d. $x = \dfrac{c+b}{a+d}$ e. $x = \dfrac{w+au}{u+1}$ f. $x = \dfrac{b-a}{a+b}$

g. $x = \dfrac{z+u-3w}{m-3}$ h. $x = \dfrac{2q+3p}{p-q}$

i. $x = \dfrac{-b-ac}{c-a}$, or, upon multiplying top and bottom by –1, $\dfrac{b+ac}{a-c}$

j. $x = \dfrac{R-Q-2c-ab}{-a-c}$, or, upon multiplying top and bottom by –1, $\dfrac{Q+2c+ab-R}{a+c}$

6. a. $0, -\dfrac{7}{17}$ b. 0, 2 c. $0, \dfrac{4}{3}$ d. $\dfrac{c}{a-b}$

e. $\dfrac{b+a}{a+1}$ f. $\dfrac{T-dn+bc}{b-d}$ g. $\dfrac{a-b}{g-h}$ h. $\dfrac{en-w-b+cd}{c-e}$

7. a. $0, \dfrac{4}{3}$ b. 0, 3 c. $0, \dfrac{7}{3}$ d. $\dfrac{-b+c}{c-a}$

e. $\dfrac{w-b}{b+1}$ f. $\dfrac{z-ac-dn}{a+d}$ g. $\dfrac{b-a}{e-h}$

h. $\dfrac{-em+u-acd+b}{c-e}$

"Use what talents you possess. The woods would be very silent if no birds sang there except those that sang the best."

– Henry Van Dyke