CH 64 – REDUCING QUADRATIC FRACTIONS

❑ *INTRODUCTION*

Now that we're getting proficient at factoring, we can knock some ugly fractions down to size. Let's review a few reducing problems to get us ready for this chapter.

First Fraction: Let's reduce the fraction $\frac{39}{52}$. By prime factoring the top and bottom, we get $\frac{39}{52} = \frac{3 \cdot 13}{2 \cdot 2 \cdot 13} = \frac{3 \cdot \cancel{13}^{1}}{2 \cdot 2 \cdot \cancel{13}_{1}} = \mathbf{\frac{3}{4}}$. Once the top and bottom have been factored (i.e., written as a product), we can divide out (or cancel) the common factor of 13.

Second Fraction: Applying the same logic to the fraction $\frac{ax}{a^2 + 7a}$, we can reduce it by writing $\frac{ax}{a^2 + 7a} = \frac{ax}{a(a + 7)} = \frac{\cancel{a}x}{\cancel{a}(a + 7)} = \boldsymbol{\frac{x}{a + 7}}$.

And a Third Fraction: $\frac{LT - QT}{AL - AQ} = \frac{\overbrace{T(L - Q)}^{\text{top factored}}}{\underbrace{A(L - Q)}_{\text{bottom factored}}} = \frac{T\cancel{(L - Q)}^{1}}{A\cancel{(L - Q)}_{1}} = \boldsymbol{\frac{T}{A}}$

And don't forget that when something is factored, it consists of a single term (where muliplication is the final operation). Thus, $a(b - c)$ is factored, but $nx + ny$ is not factored.

❑ *Reducing Quadratic Fractions*

Whether we're reducing $\frac{39}{52}$ (like the one above) or something like what you're about to see, the process is the same:

1. Factor top and bottom.
2. Divide out any common factors.

EXAMPLES: **Reduce each fraction to lowest terms:**

A. $\dfrac{x^2+5x+6}{x^2-4} = \dfrac{(x+3)(x+2)}{(x+2)(x-2)} = \dfrac{(x+3)\cancel{(x+2)}}{\cancel{(x+2)}(x-2)} = \mathbf{\dfrac{x+3}{x-2}}$

B. $\dfrac{6n^2+5n-21}{3n^2+22n+35} = \dfrac{(3n+7)(2n-3)}{(3n+7)(n+5)} = \dfrac{\cancel{(3n+7)}(2n-3)}{\cancel{(3n+7)}(n+5)} = \mathbf{\dfrac{2n-3}{n+5}}$

C. $\dfrac{x+5}{2x^2+7x-15} = \dfrac{x+5}{(2x-3)(x+5)} = \dfrac{\cancel{x+5}}{(2x-3)\cancel{(x+5)}} = \mathbf{\dfrac{1}{2x-3}}$

D. $\dfrac{3w^2-2w-5}{3w-5} = \dfrac{(3w-5)(w+1)}{3w-5} = \dfrac{\cancel{(3w-5)}(w+1)}{\cancel{3w-5}} = \mathbf{w+1}$

E. $\dfrac{a^2-25}{a^2+5a+6} = \dfrac{(a+5)(a-5)}{(a+3)(a+2)}$ which contains no common factors.

Therefore, this fraction is **not reducible**.

Homework

1. Verify that the fraction $\frac{ab+ac}{ad}$ reduces to $\frac{b+c}{d}$ by letting $a = 2$, $b = 3$, $c = 4$, and $d = 6$ in both fractions.

2. Prove that the fraction $\frac{x+z}{yz}$ does not reduce to $\frac{x}{y}$ by choosing numerical values for x, y, and z. Also prove to yourself that $\frac{x+z}{yz}$ does not reduce to $\frac{x+1}{y}$, either.

3. Reduce each fraction to lowest terms:

a. $\frac{n^2-9}{n^2+6n+9}$ b. $\frac{4a^2+4a+1}{2a^2+5a+2}$ c. $\frac{c+7}{c^2-49}$

d. $\frac{4w^2-9}{2w-3}$ e. $\frac{6x^2+11x-7}{2x^2+17x-9}$ f. $\frac{h^2+3h+2}{h^2-3h+2}$

g. $\frac{3k^2-17k+1}{3k^2-17k+1}$ h. $\frac{100-w^2}{w^2+10w}$ i. $\frac{10x^2+11x-6}{5x^2-12x+4}$

j. $\frac{x^2+7x+10}{x^2+3x+2}$ k. $\frac{n^2-9}{n^2+4n+3}$ l. $\frac{y-7}{y^2-14y+49}$

m. $\frac{w^2-81}{w+9}$ n. $\frac{m^2+10m+25}{m^2-25}$ o. $\frac{x^2+2x+1}{x^2-4}$

p. $\frac{6a^2+13a-5}{9a^2+12a-5}$ q. $\frac{k^2-6k+7}{7-6k+k^2}$ r. $\frac{16u^2+34u-15}{2u^2+3u-5}$

Review Problems

4. Reduce: $\dfrac{6a^2-5a-21}{3a^2-4a-7}$

5. Reduce: $\dfrac{x^2-9}{x^2-4}$

6. Reduce: $\dfrac{10a^2+29a-21}{5a^2-38a+21}$

7. Reduce: $\dfrac{x-3}{x^2-9}$

8. Reduce: $\dfrac{n^2+14n+49}{(n+7)^2}$

Solutions

1. Note that this does not constitute a complete proof.

2. Be sure you don't let either the y or the z be zero. How come?

3.

a.	$\dfrac{n-3}{n+3}$	b.	$\dfrac{2a+1}{a+2}$	c.	$\dfrac{1}{c-7}$	d.	$2w+3$
e.	$\dfrac{3x+7}{x+9}$	f.	Not reducible	g.	1	h.	$\dfrac{10-w}{w}$
i.	$\dfrac{2x+3}{x-2}$	j.	$\dfrac{x+5}{x+1}$	k.	$\dfrac{n-3}{n+1}$	l.	$\dfrac{1}{y-7}$
m.	$w-9$	n.	$\dfrac{m+5}{m-5}$	o.	Not reducible	p.	$\dfrac{2a+5}{3a+5}$
q.	1	r.	$\dfrac{8u-3}{u-1}$				

4. $\dfrac{2a+3}{a+1}$ **5.** Not reducible **6.** $\dfrac{2a+7}{a-7}$

7. $\dfrac{1}{x+3}$ **8.** 1

❑ *To ∞ and Beyond*

1. Consider the fraction

$$\frac{3}{x-7}$$

Let's choose a number for x. We see that if $x = 4$, for instance, then the value of the fraction is

$$\frac{3}{\mathbf{4}-7} = \frac{3}{-3} = -1$$

And x can also take on the value 0, since $\frac{3}{\mathbf{0}-7} = \frac{3}{-7} = -\frac{3}{7}$. In fact, x can be any number at all . . . with one exception. What is it?

2. Consider the fraction

$$\frac{x^2-4}{x^2-11x+30}$$

Find the two values of x which can never be used in this fraction.

"Education is what survives when what has been learned has been forgotten."

– **B. F. Skinner** (1904 - 1990)