CH 67.5 – BEYOND SQUARE ROOTS

❑ *ALL KINDS OF ROOTS*

The number 9 has two ***square roots***, 3 and –3. This is because $3^2 = 9$ and $(-3)^2 = 9$. The positive square root of 9 (the 3) is denoted $\sqrt{9}$, and the negative square root of 9 (the –3) is written $-\sqrt{9}$. In other words, $\sqrt{9} = 3$, and only 3, while $-\sqrt{9} = -3$.

In analyzing $\sqrt{-25}$, the "positive" square root of –25, we discover that we cannot find an answer for this problem, since the square of a real number can never be negative. If there is an answer to $\sqrt{-25}$, it lies outside $\mathbb{R}$, the set of real numbers.

Consider the number 8. Since $2^3 = 8$, we can say that 2 is a ***cube root*** of 8. In fact, it's the only cube root of 8, simply because there's no other real number whose cube is 8. Perhaps a little surprising is that we can calculate the cube root of a negative number without leaving $\mathbb{R}$. For example, $\sqrt[3]{-27}$ equals –3, since $(-3)^3 = -27$.

The number 16 has two ***fourth roots***. The positive fourth root is $\sqrt[4]{16} = 2$, and the negative fourth root is $-\sqrt[4]{16} = -2$. After all, both 2 and –2 raised to the fourth power result in 16. However, just like square roots, $\sqrt[4]{-1}$ is not a real number.

The ***fifth root*** of 32 is 2; that is, $\sqrt[5]{32} = 2$. This is because $2^5 = 32$. Like cube roots, we can calculate the fifth root of a negative number. For example, $\sqrt[5]{-243}$ equals –3, since $(-3)^5 = -243$.

Homework

1. Find the square root(s) of

   a. 100 b. 15 c. 0 d. –36 e. 1

2. Find the cube root(s) of

   a. 64 b. –125 c. 0 d. 20 e. 1

3. Find the fourth root(s) of

   a. 81 b. 0 c. –625 d. 25 e. 1

4. Find the fifth root(s) of

   a. 1 b. 0 c. –243 d. 29 e. 32

5. Evaluate each radical:

   a. $\sqrt{169}$ b. $\sqrt{225}$ c. $\sqrt[3]{8}$ d. $\sqrt[3]{27}$ e. $\sqrt[3]{-125}$

   f. $\sqrt[4]{625}$ g. $\sqrt[4]{1}$ h. $\sqrt[4]{-16}$ i. $\sqrt[5]{-32}$ j. $\sqrt[5]{0}$

   k. $\sqrt[3]{64}$ l. $\sqrt[3]{216}$ m. $\sqrt[3]{-64}$ n. $-\sqrt[5]{-1}$ o. $\sqrt[4]{16}$

   p. $-\sqrt[4]{81}$ q. $\sqrt[3]{-1}$ r. $-\sqrt[4]{-1}$ s. $\sqrt[5]{243}$ t. $\sqrt{0}+\sqrt[3]{0}$

❑ *SIMPLIFYING MORE ROOTS*

Assume that x and y represent non-negative numbers. Then

$$\sqrt[n]{xy} = \sqrt[n]{x}\,\sqrt[n]{y}$$

The root of a product
is the product of the roots

Another useful rule for radicals is the following. If x represents a non-negative number, then

$$\sqrt[n]{x^n} = x$$

The nth root cancels the nth power

[The special case of simplifying $\sqrt{x^2}$, where x is negative, will not be dealt with in the chapter.]

EXAMPLE 4: **Simplify each radical expression:**

A. $\sqrt{200} = \sqrt{100 \times 2} = \sqrt{100} \times \sqrt{2} = 10\sqrt{2}$

B. $\sqrt[3]{81} = \sqrt[3]{27 \times 3} = \sqrt[3]{27} \times \sqrt[3]{3} = 3\sqrt[3]{3}$

C. $\sqrt[4]{1250} = \sqrt[4]{625 \cdot 2} = \sqrt[4]{625} \cdot \sqrt[4]{2} = 5\sqrt[4]{2}$

D. $\sqrt[3]{2250}$ Sometimes the radicand (the 2250) is too big to easily see if there's a perfect cube in it. So let's try a slightly different approach. We factor the 2250 into primes to get

$$2250 = 2 \cdot 3^2 \cdot 5^3$$

Clearly we can take the cube root of 5^3 (it's 5), but there are not enough of the other factors to take their cube roots. So we can write

$$\sqrt[3]{2250} = \sqrt[3]{2 \cdot 3^2 \cdot 5^3} = \sqrt[3]{5^3} \cdot \sqrt[3]{2 \cdot 3^2} = 5\sqrt[3]{18}$$

Homework

6. Simplify each radical:

a. $\sqrt{288}$	b. $\sqrt[3]{54}$	c. $\sqrt[3]{16}$	d. $\sqrt[3]{250}$
e. $\sqrt[4]{32}$	f. $\sqrt[4]{243}$	g. $\sqrt[4]{162}$	h. $\sqrt[4]{1}$
i. $\sqrt[3]{-54}$	j. $\sqrt[4]{-16}$	k. $\sqrt[5]{64}$	l. $\sqrt[5]{486}$
m. $\sqrt[3]{135}$	n. $\sqrt[4]{162}$	o. $\sqrt[3]{189}$	p. $\sqrt[5]{96}$
q. $\sqrt[3]{128}$	r. $\sqrt[4]{1250}$	s. $\sqrt[3]{250}$	t. $\sqrt[3]{432}$
u. $\sqrt[5]{320}$	v. $\sqrt[3]{48}$	w. $\sqrt[4]{648}$	x. $\sqrt[5]{2673}$

Practice Problems

7. Simplify each square root:

a. $\sqrt{250}$	b. $\sqrt{56}$	c. $\sqrt{112}$	d. $\sqrt{400}$
e. $\sqrt{144}$	f. $\sqrt{76}$	g. $-\sqrt{288}$	h. $\sqrt{8}$
i. $\sqrt{4}$	j. $-\sqrt{54}$	k. $\sqrt{300}$	l. $\sqrt{9}$
m. $\sqrt{49}$	n. $\sqrt{196}$	o. $\sqrt{475}$	p. $\sqrt{72}$
q. $\sqrt{240}$	r. $\sqrt{100}$	s. $\sqrt{52}$	t. $\sqrt{96}$
u. $\sqrt{98}$ v. $-\sqrt{2}$	w. $\sqrt{648}$	x. $\sqrt{500}$	y. $\sqrt{-3}$

8. Simplify each radical:

a. $\sqrt[3]{108}$	b. $\sqrt[4]{405}$	c. $\sqrt[5]{192}$	d. $\sqrt[3]{-500}$
e. $\sqrt[4]{-32}$	f. $\sqrt[5]{-486}$	g. $\sqrt[3]{3000}$	h. $\sqrt[4]{567}$
i. $\sqrt[3]{648}$	j. $\sqrt[5]{320}$	k. $\sqrt[3]{81}$	l. $\sqrt[3]{250}$
m. $\sqrt[3]{-56}$	n. $\sqrt[4]{48}$	o. $\sqrt[4]{-405}$	p. $\sqrt[4]{768}$
q. $\sqrt[5]{128}$	r. $\sqrt[5]{1215}$		

Solutions

1.	a. ± 10	b. $\pm\sqrt{15}$	c. 0	d. Not real	e. ± 1	
2.	a. 4	b. -5	c. 0	d. $\sqrt[3]{20}$	e. 1	
3.	a. ± 3	b. 0	c. Not real	d. $\pm\sqrt[4]{25}$	e. ± 1	
4.	a. 1	b. 0	c. -3	d. $\sqrt[5]{29}$	e. 2	
5.	a. 13	b. 15	c. 2	d. 3	e. -5	
	f. 5	g. 1	h. Not real	i. -2	j. 0	
	k. 4	l. 6	m. -4	n. 1	o. 2	
	p. -3	q. -1	r. Not real	s. 3	t. 0	
6.	a. $12\sqrt{2}$	b. $3\sqrt[3]{2}$	c. $2\sqrt[3]{2}$	d. $5\sqrt[3]{2}$	e. $2\sqrt[4]{2}$	f. $3\sqrt[4]{3}$
	g. $3\sqrt[4]{2}$	h. 1	i. $-3\sqrt[3]{2}$	j. Not real	k. $2\sqrt[5]{2}$	l. $3\sqrt[5]{2}$
	m. $3\sqrt[3]{5}$	n. $3\sqrt[4]{2}$	o. $3\sqrt[3]{7}$	p. $2\sqrt[5]{3}$	q. $4\sqrt[3]{2}$	r. $5\sqrt[4]{2}$
	s. $5\sqrt[3]{2}$	t. $6\sqrt[3]{2}$	u. $2\sqrt[5]{10}$	v. $2\sqrt[3]{6}$	w. $3\sqrt[4]{8}$	x. $3\sqrt[5]{11}$

7. a. $5\sqrt{10}$ b. $2\sqrt{14}$ c. $4\sqrt{7}$ d. 20 e. 12
f. $2\sqrt{19}$ g. $-12\sqrt{2}$ h. $2\sqrt{2}$ i. 2 j. $-3\sqrt{6}$
k. $10\sqrt{3}$ l. 3 m. 7 n. 14 o. $5\sqrt{19}$
p. $6\sqrt{2}$ q. $4\sqrt{15}$ r. 10 s. $2\sqrt{13}$ t. $4\sqrt{6}$
u. $7\sqrt{2}$ v. $-\sqrt{2}$ w. $18\sqrt{2}$ x. $10\sqrt{5}$ y. Not real

8. a. $5\sqrt{11}$ b. $7\sqrt{19}$ c. $4\sqrt{13}$ d. $7\sqrt{7}$ e. $-3\sqrt{7}$
f. $-5\sqrt{7}$ g. $6\sqrt{10}$ h. $2\sqrt{11}$ i. $5\sqrt{15}$ j. $2\sqrt{17}$
k. $9\sqrt{10}$ l. $4\sqrt{5}$ m. $3\sqrt{11}$ n. $6\sqrt{7}$ o. $4\sqrt{10}$
p. $3\sqrt{17}$ q. $-6\sqrt{5}$ r. $14\sqrt{3}$ s. $3\sqrt{15}$ t. $8\sqrt{3}$
u. $5\sqrt{17}$ v. $4\sqrt{11}$ w. $2\sqrt{23}$ x. $12\sqrt{5}$ y. Not real

9. a. $3\sqrt[3]{4}$ b. $3\sqrt[4]{5}$ c. $2\sqrt[5]{6}$ d. $-5\sqrt[3]{4}$ e. Not real
f. $-3\sqrt[5]{2}$ g. $10\sqrt[3]{3}$ h. $3\sqrt[4]{7}$ i. $6\sqrt[3]{3}$ j. $2\sqrt[5]{10}$
k. $3\sqrt[3]{3}$ l. $5\sqrt[3]{2}$ m. $-2\sqrt[3]{7}$ n. $2\sqrt[4]{3}$ o. Not real
p. $4\sqrt[4]{3}$ q. $2\sqrt[5]{4}$ r. $3\sqrt[5]{5}$

"An educational system isn't worth a great deal if it teaches young people how to make a living– but doesn't teach them how to make a life."

Unknown