CH 90 – VARIATION

❑ INTRODUCTION

Let's start right off with an example.

Let D = number of hard Drives sold
C = the Capacity of the drive, in gigabtyes (GB)
P = selling Price

Now let's make up a formula that will illustrate the ideas of this chapter:

$$D = \frac{5C}{P}$$

① What happens to drives sold D when the capacity C is increased? Let's assume that last month the capacity was 200 GB and that the price was \$100. The number of drives sold was

$$D = \frac{5C}{P} = \frac{5 \cdot 200}{100} = \underline{10 \text{ drives}}$$

Now increase the capacity to 600 GB; the number of drives sold will be

$$D = \frac{5C}{P} = \frac{5 \cdot 600}{100} = \underline{30 \text{ drives}}$$

Make sense? Increasing the capacity (making the drives better) should <u>increase</u> sales.

② Now let's see what happens if we increase the price. Assume that last month the price P was \$150 when the capacity was 450 GB. We calculate the number of drives sold:

$$D = \frac{5C}{P} = \frac{5 \cdot 450}{150} = \underline{15 \text{ drives}}$$

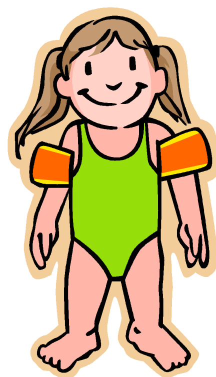
Let's predict future sales if we increase the price to \$225:

$$D = \frac{5C}{P} = \frac{5 \cdot 450}{225} = \underline{10 \text{ drives}}$$

Make sense? Increasing the price produced a decrease in sales.

Without going into the details, we could also demonstrate that if the capacity C goes down, so will the number of drives sold. And if the price P goes down, D will go up.

❑ DIRECT AND INVERSE VARIATION



Direct Variation What happens to the sale of bathing suits, B, when the temperature, T, goes up? As the temperature rises, so does the sale of bathing suits. We say that B is ***directly proportional*** to T, or that B ***varies directly*** as T. A formula of this type might be

$$B = 3T$$

For example, if the temperature is 80°, then 240 suits will be sold. But if the temperature rises to 100°, then 300 units will be sold. When it gets cold again, B will decrease. Whatever T does, B does the **same**.

Inverse Variation What happens to the air pressure, P, as you increase your elevation, E? As your elevation goes up, the air pressure goes down. We say that P is ***inversely proportional*** to E, or that P ***varies inversely*** as E. An example might be the formula

$$P = \frac{2000}{E}$$

For instance, if $E = 200$, then $P = 10$. But if E is increased to 500, then the pressure P is reduced to 4. When you return to lower altitudes, the pressure goes back up. Whatever E does, P does the **reverse**.

See the numbers "3" and "2000" in our two formulas above? These numbers are called the ***constants of proportionality*** (or ***constants of variation***).

In the following definitions, the letter $\boldsymbol{k}$ is the positive constant of proportionality.

$y = kx$	is read "y is ***directly proportional to*** x," or "y ***varies directly as*** x," and means: If x increases, then y increases; and if x decreases, then y decreases. In other words, y does whatever x does.
$y = \frac{k}{x}$	is read "y is ***inversely proportional to*** x," or "y ***varies inversely as*** x," and means: If x increases, then y decreases; and if x decreases, then y increases. In other words, y does the reverse of what x does.

Homework

1. Consider the *direct variation* $D = 12Q$. We can pretend that Q is the quality of a car and D is the demand for that car.

   a. Find the value of D if Q is 40.
   b. Double the Q to 80 and recalculate D.
   c. When the quality increased, what happened to the demand?
   d. Now reduce Q to 3 and recalculate D.
   e. As the quality decreased, what happened to the demand?

2. Consider the *inverse variation* $P = \frac{200}{S}$. Let's assume that S is supply and P is price.

a. Find the value of P if $S = 10$.
b. Quadruple the S to 40 and recalculate P.
c. When the supply increased, what happened to the price?
d. Now reduce S to 5 and recalculate P.
e. As the supply decreased, what happened to the price?

❑ EXTENSIONS OF THE BASIC VARIATION FORMULAS

To extend the usefulness of problems in variation, we can add exponents and square roots to our direct and inverse variation formulas; for example, a fact of physics is that the kinetic energy of an object (energy of motion) varies directly as the square of its velocity, which can be written $E = kv^2$.

We can also combine direct and inverse variation into a single formula. For example, a chemistry principle states that "the volume of a gas is directly proportional to its temperature and inversely proportional to its pressure." This is summarized by the formula $V = \frac{kT}{P}$.

<u>EXAMPLE 1:</u> **Translate each variation statement into an algebraic formula, using *k* as the constant of variation:**

A. z varies <u>directly</u> as the cube of T. $z = kT^3$

B. R is <u>inversely</u> proportional to the square root of V. $R = \frac{k}{\sqrt{V}}$

C. P varies directly as the square of Q, and inversely as the square root of R. $P = \frac{kQ^2}{\sqrt{R}}$

D. B varies directly as the product of the square root of A and the cube of C. $B = k\sqrt{A}\,C^3$

E. y varies directly as the product of x and z, and inversely as the fourth power of w. $y = \frac{kxz}{w^4}$

Homework

3. Translate each variation statement into an algebraic formula, using ***k*** as the constant of proportionality:

a. b varies inversely as the 6th power of t.

b. n varies directly as the square root of b.

c. p is inversely proportional to the square of s.

d. v is directly proportional to c.

e. p varies inversely as the cube of x.

f. u is directly proportional to the square of t.

g. t varies directly as the square of h.

h. L is inversely proportional to the cube of s.

i. w varies directly as the 9th power of u.

j. R varies directly as the product of w and y.

k. g is directly proportional to the cube of c.

l. h varies directly as c and inversely as y.

m. x varies directly as the cube of t and inversely as the cube of h.

n. u is directly proportional to the product of c and the cube of r, and inversely proportional to m.

o. y varies directly as the square of v and inversely as the cube of d.

p. n varies directly as the product of t and the square of d, and inversely as b.

q. m is directly proportional to w and inversely proportional to z.

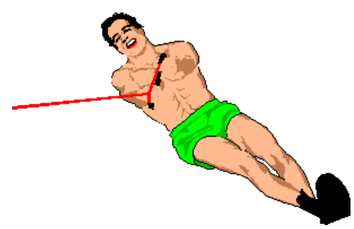
r. A varies directly as the product of u and the square of x, and inversely as p.

s. n is directly proportional to r and inversely proportional to v.

t. c varies directly as the cube of u and inversely as the cube of z.

❑ APPLICATIONS

EXAMPLE 2: **The number of bathing suits sold is directly proportional to the outside temperature, and inversely proportional to the selling price. The number of suits sold is 1,600 when the temperature is 80° and the selling price is \$50. Find the number of suits sold when the temperature rises to 95° and the price is reduced to \$40.**



Solution: We'll start by letting

B = bathing suits sold

T = temperature

P = price

The first sentence of the problem, "The number of bathing suits sold is directly proportional to the outside temperature, and inversely proportional to the selling price," gives us our variation formula:

$$B = \frac{kT}{P}$$

The second sentence "The number of suits sold is 1,600 when the temperature is 80° and the selling price is $50," allows us to find the value of *k*:

$$1{,}600 = \frac{k(80)}{50} \qquad \text{(substitute the given values)}$$

$$\Rightarrow \quad 80k = 80{,}000 \qquad \text{(multiply each side by 50)}$$

$$\Rightarrow \quad k = 1{,}000 \qquad \text{(divide each side by 80)}$$

The third sentence, "Find the number of suits sold when the temperature rises to 95° and the price is reduced to $40," (with the value $k = 1{,}000$ we just calculated) gives us all the parts needed to compute the number of bathing suits sold under the new set of conditions:

$$B = \frac{kT}{P} \qquad \text{(our variation formula)}$$

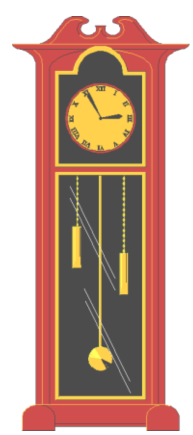
$$\Rightarrow \quad B = \frac{1{,}000T}{P} \qquad \text{(the constant } k \text{ is 1,000)}$$

$$\Rightarrow \quad B = \frac{1{,}000(95)}{40} \qquad \text{(use the new values of } T \text{ and } P\text{)}$$

$$\Rightarrow \quad B = \frac{95{,}000}{40}$$

$$\Rightarrow \quad \boxed{B = 2{,}375 \text{ bathing suits}}$$

<u>EXAMPLE 3:</u> **The period of the pendulum (the amount of time for one full swing) in a grandfather clock varies directly as the <u>square root</u> of its length. If the period is 50π when the length is 25, find the period when the length is 49.**



Solution: The first sentence gives us our variation formula:

$$P = k\sqrt{L}$$

Substituting a period of 50π and a length of 25 gives:

$$50\pi = k\sqrt{25}$$

$\Rightarrow \quad 50\pi = 5k$ (the positive square root of 25 is 5)

$\Rightarrow \quad k = 10\pi$ (divide each side by 5)

Now we rewrite our variation formula using the *k* we just found:

$$P = k\sqrt{L} = 10\pi\sqrt{L}$$

Finally, find the period when the length is 49:

$$P = 10\pi\sqrt{49}$$

$\Rightarrow \quad P = 10\pi(7)$

$\Rightarrow \quad \boxed{P = 70\pi}$

Homework

4. The area of a rectangle is directly proportional to its length. If the area is 247 when the length is 19, find the area when the length is 11.

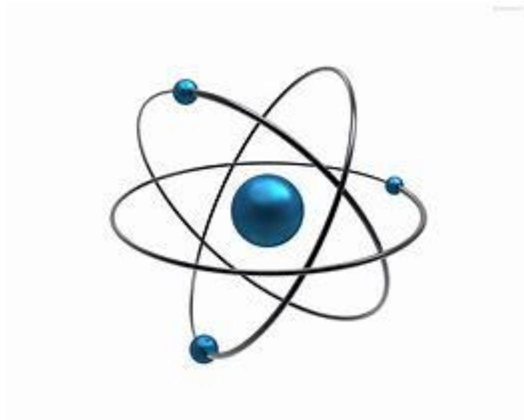
5. The circumference of a circle is directly proportional to its radius. If the circumference is 40π when the radius is 20, then what is the circumference when the radius is 8?

6. The current in a circuit varies directly as the voltage. If the current is 336 when the voltage is 21, find the current when the voltage is 35.

In a variation problem, DON'T forget "***k***", the *constant of variation*.

7. The area of a circle varies directly as the square of its radius. If the area is 9π when the radius is 3, then what is the area when the radius is 6?

8. The number of electrons varies directly as the square of the energy level. If the number of electrons is 32 when the energy level is 4, then how many electrons are there when the energy level is 3?



9. The energy density is directly proportional to the fourth power of the temperature. If the energy is 512 when the temperature is 4, then what is the energy when the temperature is 5?

10. The density of an object varies inversely as the object's volume. If the density is 22 when the volume is 18, then what is the density when the volume is 6?

11. The acceleration of an object is inversely proportional to the object's mass. If the acceleration is 9 when the mass is 19, then what is the acceleration when the mass is 1?

12. The force of gravity varies inversely as the square of the distance between the objects. If the force is 4 when the distance is 8, then what is the force when the distance is 1?

13. The period of a pendulum varies directly as the square root of its length. If the period is 22π when the length is 121, then what is the period when the length is 49?

14. The velocity of an object is directly proportional to the square root of its kinetic energy. If the velocity is 524 when the kinetic energy is 4, then what is the velocity when the kinetic energy is 25?

15. The potential energy of an object varies directly as the product of its mass and its height. If the potential energy is 80 when the mass is 4 and the height is 2, find the potential energy if the mass is 14 and the height is 13.

16. The fluid force on an object is directly proportional to the product of its area and its depth. If the fluid force is 1920 when the area

is 10 and the depth is 24, find the fluid force if the area is 23 and the depth is 5.

17. The volume of a gas varies directly as its temperature, and inversely as its pressure. If the volume is 12 when the temperature is 9 and the pressure is 12, find the volume when the temperature is 5 and the pressure is 16.

18. The electric field is directly proportional to the charge, and inversely proportional to the area. If the electric field is 13 when the charge is 13 and the area is 9, find the electric field when the charge is 15 and the area is 3.

Review Problems

19. Translate the variation statement into an algebraic formula, using k as the constant of variation: "E varies directly as the product of x and the cube of y, and inversely as the square root of z."

20. The volume of a gas varies directly as its temperature, and inversely as its pressure. If the volume is 80 when the temperature is 40 and the pressure is 5, find the volume when the temperature is 30 and the pressure is 6.

21. The current in a circuit varies directly as the voltage. If the current is 336 when the voltage is 21, find the current when the voltage is 35.

22. The acceleration of an object is inversely proportional to the object's mass. If the acceleration is 9 when the mass is 19, then what is the acceleration when the mass is 1?

23. The potential energy of an object varies directly as the product of its mass and its height. If the potential energy is 462 when the mass is 6 and the height is 7, find the potential energy if the mass is 24 and the height is 17.

24. The electric field is directly proportional to the charge, and inversely proportional to the area. If the electric field is 10 when the charge is 15 and the area is 3, find the electric field when the charge is 12 and the area is 6.

25. The kinetic energy of a particle varies directly as the product of its mass and the square of its velocity. Assume that a particle of mass 10 and traveling at a velocity of 8 has a kinetic energy of 320. Find the kinetic energy of a particle with mass 7 and velocity 10.

Solutions

1. a. 480 b. 960 c. It increased d. 36 e. It decreased

2. a. 20 b. 5 c. It decreased d. 40 e. It increased

3. a. $b = \frac{k}{t^6}$ b. $n = k\sqrt{b}$ c. $p = \frac{k}{s^2}$ d. $v = kc$

e. $p = \frac{k}{x^3}$ f. $u = kt^2$ g. $t = kh^2$ h. $L = \frac{k}{s^3}$

i. $w = ku^9$ j. $R = kwy$ k. $g = kc^3$ l. $h = \frac{kc}{y}$

m. $x = \frac{kt^3}{h^3}$ n. $u = \frac{kcr^3}{m}$ o. $y = \frac{kv^2}{d^3}$ p. $n = \frac{ktd^2}{b}$

q. $m = \frac{kw}{z}$ r. $A = \frac{kux^2}{p}$ s. $n = \frac{kr}{v}$ t. $c = \frac{ku^3}{z^3}$

4. 143 **5.** 16π **6.** 560 **7.** 36π

8. 18 **9.** 1,250 **10.** 66 **11.** 171

12. 256 **13.** 14π **14.** 1,310 **15.** 1,820

16. 920 **17.** 5 **18.** 45 **19.** $E = \frac{ky^3}{\sqrt{z}}$

20. 50

21. In electronics, current is denoted by the letter i.

$i = kV \Rightarrow 336 = k \cdot 21 \Rightarrow k = 16 \Rightarrow i = 16V$,
so when $V = 35$, $i = 16 \cdot 35 = 560$.

22. $a = 171$

23. $E = kmh \Rightarrow 462 = k \cdot 6 \cdot 7 \Rightarrow k = 11 \Rightarrow E = 11mh$
$\Rightarrow E = 11(24)(17) = 4488$

24. $F = \frac{kC}{A}$; $k = 2$; $F = 4$

25. $E = kmv^2$ $E = 350$

"It is never too late to be what you might have been."

- George Eliot