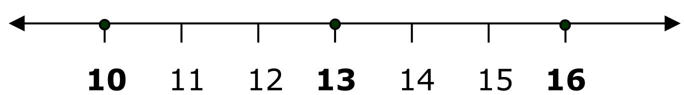
CH 5 – MIDPOINT, DISTANCE, INTERCEPTS, AND SLOPE

❑ *MIDPOINT ON THE LINE*

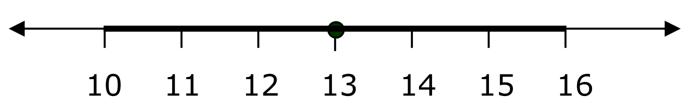
Here's a question for you: What number is *midway* between 10 and 16? You probably know that the number is 13. Why? Because 13 is 3 away from 10 and 13 is also 3 away from 16.



Now we need a simple way to find the number that is midway between any two numbers, even when the numbers are not nice. Notice this: If we take the **average** (officially called the *mean*) of 10 and 16 -- by adding the two numbers and dividing by 2 -- we get

$$\frac{10+16}{2} = \frac{26}{2} = \mathbf{13}, \text{ the midway number.}$$

Let's rephrase what we've done with some new terminology. Consider the *line segment* connecting 10 and 16 on the number line:



We can now refer to the 13 as the ***midpoint*** of the line segment connecting 10 and 16.

What is the *midpoint* of the line segment connecting –2.8 and 14.6? Just calculate the average of –2.8 and 14.6:

$$\frac{-2.8+14.6}{2} = \frac{11.8}{2} = \mathbf{5.9}$$

> When you see the term ***midpoint***, think *average*!

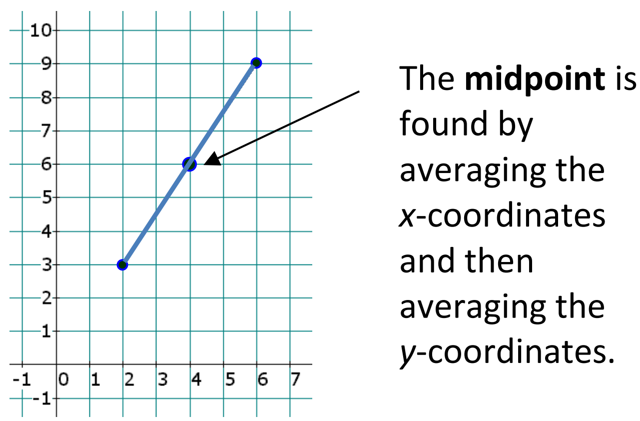
Homework

1. Find the **midpoint** of the line segment connecting the two given numbers on a number line:

a. 10 and 20	b. 13 and 22	c. –8 and –26
d. –3 and 7	e. –7 and 6	f. π and $-\pi$
g. –21 and –99	h. 0 and 43	i. –50 and 0
j. –44 and 19	k. –41 and 88	l. $3x$ and $-x$

❑ *MIDPOINT IN THE PLANE*

Now for the real question. Consider the two points (2, 3) and (6, 9) in the plane and the line segment that connects them. We need to figure out what point is the *midpoint* of the line segment connecting the two points. Recall the advice given above: When you see midpoint, think *average*. So the x-coordinate of the midpoint is the average of the x-coordinates of the two endpoints:



$$x = \frac{2+6}{2} = \frac{8}{2} = 4$$

And the y-coordinate of the midpoint is the average of the y-coordinates of the two endpoints:

$$y = \frac{3+9}{2} = \frac{12}{2} = 6$$

We conclude that the midpoint is **(4, 6)**. That's all there is to it. Now let's do a complete example without plotting any points or drawing any segments.

<u>EXAMPLE 1:</u> **Find the midpoint of the line segment connecting the points (–42, –33) and (90, –10).**

<u>Solution:</u> The x-coordinate of the midpoint is found by averaging the x-coordinates of the two given points:

$$x = \frac{-42+90}{2} = \frac{48}{2} = 24$$

The y-coordinate of the midpoint is found by averaging the y-coordinates of the two given points:

$$y = \frac{-33+(-10)}{2} = \frac{-43}{2} = -\frac{43}{2}$$

The midpoint is therefore the point

$$\boxed{\left(24, -\frac{43}{2}\right)}$$

Homework

2. Find the **midpoint** of the line segment connecting the given pair of points:

a. (–2, 5) and (2, 7)
b. (0, 1) and (0, 6)
c. (–5, 8) and (–5, –8)
d. (–2, 7) and (5, –3)
e. (–9, 2) and (–13, –40)
f. (0, 0) and (–6, –9)
g. (5, 4) and (5, 4)
h. (14, 0) and (0, –9)

i. (8, 8) and (–19, –19)

j. (π, 0) and (–π, 0)

k. $(0, \sqrt{2})$ and $(0, -\sqrt{2})$

l. (a, b) and (c, d)

m. (a, b) and (a, –b)

n. (3a, 3b) and (–3a, b)

❑ DISTANCE ON THE LINE

Our plan now is to create a formula that will give us the ***distance*** between two points on a line. Consider the two points 10 and 17 on a line. Is it pretty clear that the distance between them is 7? If it's not really obvious, you can simply subtract the smaller number from the larger one:

$$larger - smaller = 17 - 10 = \mathbf{7}$$

This formula works perfectly for <u>any</u> two numbers on a line:

The distance between –7 and 5 = 5 – (–7) = 5 + 7 = **12**.
[Note that 5 is larger than –7.]

The distance between –9 and –20 = –9 – (–20) = –9 + 20 = **11**.
[Note that –9 is larger than –20.]

Now comes the real problem; remember, this is an Algebra class, so we need a formula for the distance between <u>any</u> two numbers on a line. In other words, we need a formula for the distance between the numbers a and b on a line, when we MAY NOT KNOW which one of them, a or b, is the larger one. Here's the secret: use *absolute value*. This way, if we were to "accidentally" subtract in the wrong direction, and end up with a negative distance (which DOESN'T EXIST), the absolute value will automatically convert the negative number into a positive number.

So, in short, if a and b are any two numbers on the number line, we don't care which one is bigger. We calculate the distance between them by using the formula

$$d = |a - b|$$

Homework

3. Find the **distance** between the given pair of points on the number line:

   a. 7 and 2 b. –2 and 9 c. –3 and –3

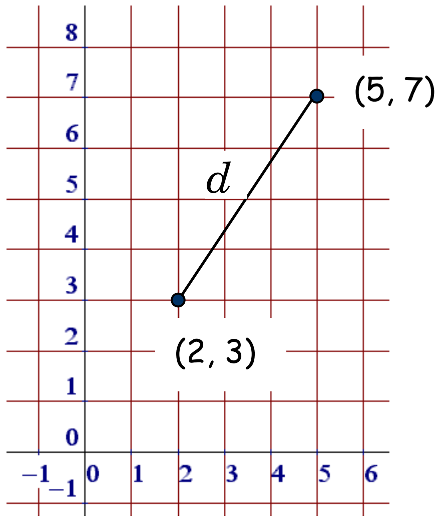
   d. 99 and –99 e. –5 and –13 f. –20 and –4

❑ *DISTANCE IN THE PLANE*

Now we add a dimension to the previous section and ask: How do we find the **distance** between two points in the **plane**? If the Earth were flat, it would be like asking how far apart two cities are if we knew the latitude and longitude of each city.

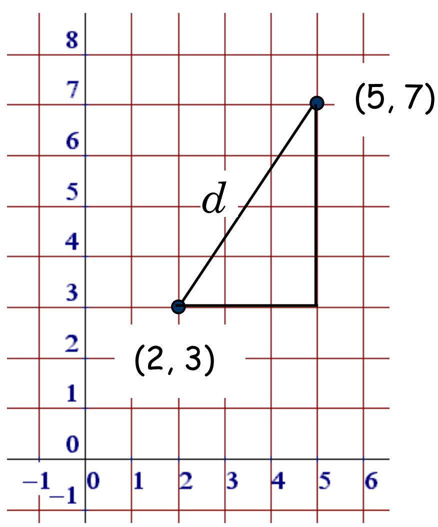
<u>EXAMPLE 2:</u> **Find the distance between the points (2, 3) and (5, 7) in the plane.**

<u>Solution:</u> Let's draw a picture and see what we can see. We'll plot the two given points and connect them with a straight line segment. The distance between the two points, which we'll call d, is simply the length of that line segment.

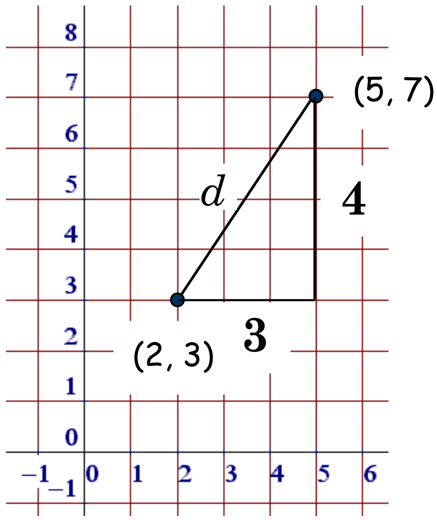


How far is it between the two points (2, 3) and (5, 7)? Equivalently, what is the length of the line segment connecting the two points?

Looks pretty, but now what do we do? Well, here comes the interesting part. If we're creative enough, we can see that the segment connecting the two points could be thought of as the hypotenuse of a right triangle, as long as we sketch in a pair of legs to create that triangle. Let's do that:



Sure enough, we've constructed a right triangle where d is the length of the hypotenuse. If we can determine the lengths of the legs, then we can use the Pythagorean Theorem (see the Prologue) to find the length of the hypotenuse. By counting squares along the base of the triangle, we see that one leg is 3. Similarly, the other leg (the height) is 4.



We've created a right triangle whose legs are 3 and 4, and whose hypotenuse is precisely the distance between the two given points.

Since the square of the hypotenuse is equal to the sum of the squares of the legs, we can write the equation

$$d^2 = 3^2 + 4^2 \qquad \text{(Pythagorean Theorem: } \text{hyp}^2 = \text{leg}^2 + \text{leg}^2\text{)}$$

$$\Rightarrow \quad d^2 = 9 + 16 \qquad \text{(square the legs)}$$

$$\Rightarrow \quad d^2 = 25 \qquad \text{(add)}$$

$$\Rightarrow \quad d = 5 \qquad \text{(since } \sqrt{25} = 5\text{)}$$

Notice that $d = -5$ also satisfies the equation $d^2 = 25$, since $(-5)^2 = 25$. But does a negative value of d make sense? No, since distance can never be negative. We conclude that

$$\boxed{\text{The distance between the two points is 5.}}$$

Homework

4. By plotting the two given points in the plane and using the Pythagorean Theorem, find the **distance** between the given pair of points.

a. (2, 3) and (4, 7)	b. (0, 3) and (1, 4)
c. (2, –1) and (–3, 4)	d. (–1, 3) and (–2, 5)
e. (–2, –3) and (–4, –6)	f. (2, 3) and (2, 7)
g. (–1, 5) and (–1, 1)	h. (4, 0) and (8, 0)
i. (4, 6) and (0, 0)	j. (7, 8) and (7, 8)

❑ *INTERCEPTS*

Consider the line $2x - 3y = 12$. We can find one point on the line very easily by letting $x = 0$. This produces

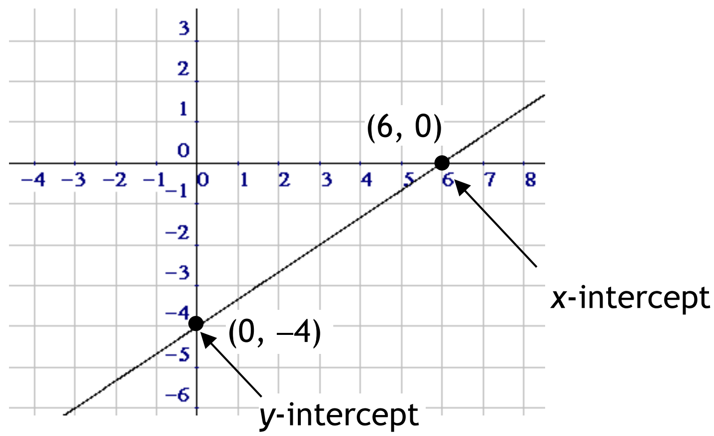
$$2(\mathbf{0}) - 3y = 12$$
$$\Rightarrow \quad 0 - 3y = 12$$
$$\Rightarrow \quad -3y = 12$$
$$\Rightarrow \quad y = -4$$

This shows us that the point **(0, –4)** is on the line.

Now let's set y to 0. We obtain

$$2x - 3(\mathbf{0}) = 12 \;\Rightarrow\; 2x - 0 = 12 \;\Rightarrow\; 2x = 12 \;\Rightarrow\; x = 6$$

We conclude that the point **(6, 0)** is also on the line. Since two points suffice to construct a line (although plotting more than two is an excellent idea), we'll graph our line now using the points (0, –4) and (6, 0):



Notice that the point (6, 0), although certainly on the line $2x - 3y = 12$, also lies on the x-axis. We call the point (6, 0) in this example the ***x-intercept*** of the line. Similarly, we call the point (0, –4) the ***y-intercept*** of the line.

Looking back at the calculations, we see that the $x = 6$ was found by setting y to 0, and that the $y = -4$ was found by setting x to 0. Here's a summary of this easy way to find the intercepts of a line (or any graph):

To find x-intercepts, set $y = 0$.

To find y-intercepts, set $x = 0$.

Of course, a graph like a circle or some other curve may have more than one x-intercept or more than one y-intercept. And when we get to horizontal and vertical lines, you'll see that they may have only one kind of intercept, or they may possibly have an infinite number of intercepts!

<u>EXAMPLE 3:</u> **Find the x-intercept and the y-intercept of the line $3x - 7y = 42$.**

<u>Solution:</u> To find the x-intercept of this line (of any graph, in fact) we set $y = 0$ and solve for x:

$$3x - 7y = 42 \qquad \text{(the line)}$$

$$\Rightarrow \quad 3x - 7(\mathbf{0}) = 42 \qquad \text{(set } y = 0\text{)}$$

$$\Rightarrow \quad 3x - 0 = 42 \qquad \text{(anything times 0 is 0)}$$

$$\Rightarrow \quad 3x = 42 \qquad \text{(simplify)}$$

$$\Rightarrow \quad \frac{3x}{3} = \frac{42}{3} \qquad \text{(divide each side by 3)}$$

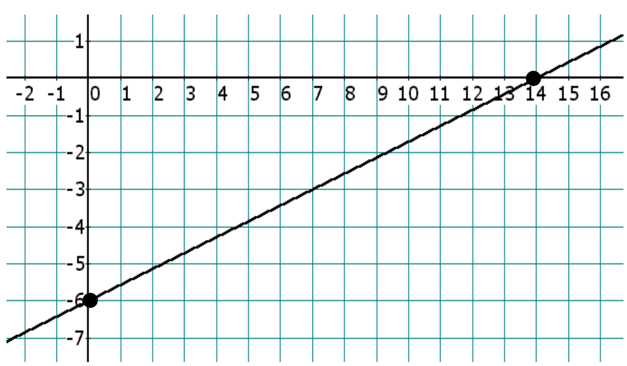
$$\Rightarrow \quad x = 14$$

The x-intercept is therefore $\boxed{(14, 0)}$

Setting $x = 0$ to find the y-intercept gives

$3(\mathbf{0}) - 7y = 42 \;\Rightarrow\; -7y = 42 \;\Rightarrow\; y = -6$

and so the y-intercept is $\boxed{(0, -6)}$



The x-intercept is (14, 0), the point where the line intercepts the x-axis.

The y-intercept is (0, –6) the point where the line intercepts the y-axis.

Homework

5. Each of the following points (except one) is an intercept. Is it an *x*-intercept or a *y*-intercept?

a. $(0, \pi)$ b. $(-99, 0)$ c. $(0, 0)$

d. $(3\pi, 0)$ e. $(0, -3.7)$ f. $(7, 7)$

6. Find the ***x*-intercept** and the ***y*-intercept** of each line -- be sure that every intercept you write consists of an ordered pair (i.e., two coordinates, one of which must be 0):

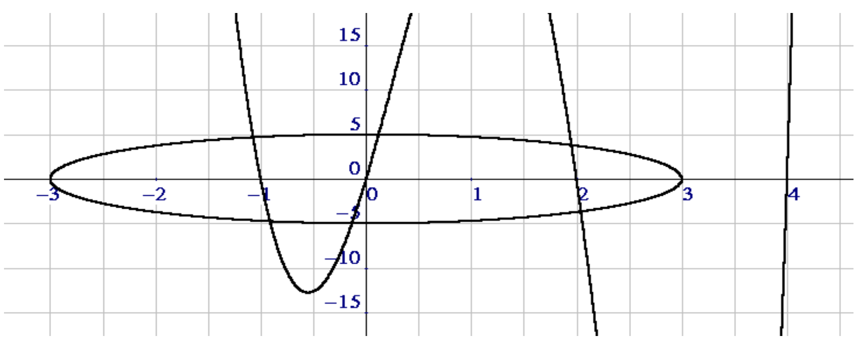
a. $2x + y = 12$ b. $3x - 4y = 24$ c. $-4x + 7y = 28$

d. $x - 7y = 7$ e. $y - 3x = 12$ f. $6x + 5y = 60$

g. $4x - 3y = 2$ h. $8y + 3x = 1$ i. $3x - 9y = 0$

j. $y = 7x - 3$ k. $y = -9x + 2$ l. $y = -x - 5$

7. Find <u>all</u> the intercepts of the following graph:



8. ** Find all the intercepts of the graph of $x^2 + y^2 = 25$.

9. ** Find all the intercepts of the graph of $y = x^2 - 9$.

❑ Slope

A trucker is keenly aware of the *grade*, or angle, of the road on which a truck travels -- it determines the speed limit and the proper gear that the truck needs to be in. A roofer is concerned with the *pitch*, or steepness, of a roof. A construction worker needs to make sure that a wheelchair ramp has the right *angle* with the street or the sidewalk. All of these ideas are examples of the concept "steepness."

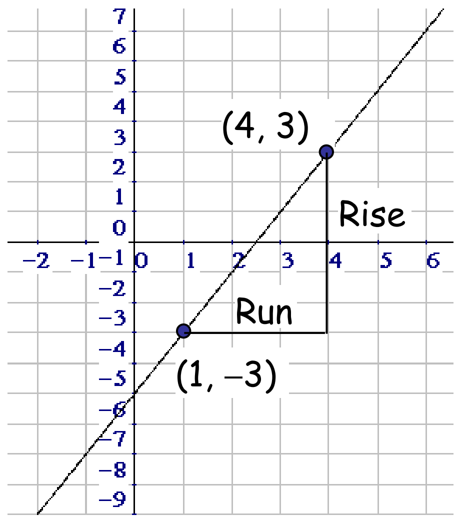
We'll use the term ***slope*** to represent steepness, and give it the letter m (I don't know why -- maybe m for mountain?). Our definition of slope in this course and all future math courses (and chemistry and economics courses) is as follows:

$$\boxed{m = \frac{\text{rise}}{\text{run}}}$$

As we'll see shortly, a **rise** is a vertical (up/down) change, while a **run** is a horizontal (left/right) change. Slope is defined as the ratio of the rise to the run; we can also say that slope is the quotient of the rise and the run. Let's do an example where we place a line in the Cartesian coordinate system and analyze the slope of that line.

<u>EXAMPLE 4:</u> **Graph the line $y = 2x - 5$ and determine its slope.**

<u>Solution:</u> If we let $x = 1$, then $y = -3$, so the point $(1, -3)$ is on the line. And if we let $x = 4$, then $y = 3$, giving us the point $(4, 3)$. We could calculate more points for our line, but let's cut to the chase and graph the line given the two points just computed.



Notice that we've constructed a right triangle using the line segment between the two given points as the hypotenuse. The rise and run are then just the lengths of the legs of the triangle. Counting squares from left to right along the bottom of the triangle, we see that the run is **3**. Counting squares up the side of the triangle yields a rise of **6** . Using the slope formula, we can calculate the slope of the line:

$$m = \frac{\text{rise}}{\text{run}} = \frac{6}{3} = \boxed{2}$$

<u>Alternate Solution:</u> In the solution above, we essentially started at the point (1, –3), moved 3 units to the <u>right</u> (producing a run of 3), moved 6 units <u>up</u> (producing a rise of 6), finally arriving at the point (4, 3).

Let's reverse our journey; we'll start at the point (4, 3) and travel to the point (1, –3). First we could move 3 units to the <u>left</u>; this produced a run of –3. Then we go <u>down</u> 6 units, giving a rise of –6. This time our calculation of slope is

$$m = \frac{\text{rise}}{\text{run}} = \frac{-6}{-3} = \boxed{2}$$

which is, of course, the same slope as before.

Homework

10. For each pair of points, plot them on a grid, find the rise and the run, and then use the formula for slope to calculate the **slope** of the line connecting the two points:

a. (2, 3) and (4, 7) b. (–3, 0) and (0, 6)

c. (1, –3) and (–2, 5) d. (2, 2) and (7, 7)

e. (–3, –3) and (0, 0) f. (–1, –2) and (3, –5)

g. (1, 1) and (–2, 3) h. (1, 4) and (0, 0)

i. (–3, –2) and (1, –3) j. (–1, 3) and (1, –3)

k. (–4, 5) and (0, 0) l. (–1, –1) and (4, –2)

11. Find the **slope** of the given line by graphing the line and then using the rise and run. You may, of course, use any two points on the line to calculate the rise and the run:

a. $y = x + 3$ b. $y = 2x - 1$ c. $y = -2x + 3$

d. $y = 3x + 1$ e. $y = -3x - 2$ f. $y = -x + 2$

g. $x + 2y = 4$ h. $2x - 3y = 1$ i. $3x - y = 3$

j. $-3x + 2y = 6$ k. $2x + 5y = 10$ l. $3x - 4y = -8$

❑ *A New View of Slope*

Finding the slope, $m = \frac{\text{rise}}{\text{run}}$, of a line by plotting two points and counting the squares to determine the rise and the run works fine only when it's convenient to plot the points. Consider the line connecting the points (π, 2000) and $(3\pi, -5000)$. Certainly these points determine a line, and that line has some sort of slope. But plotting these points is really not feasible -- we need a simpler way to calculate slope.

Notice from Example 4 that the *run* is simply the difference in the x-values and the *rise* is the difference in the y-values. Thus, another way to write the formula for slope is

$$m = \frac{\text{change in } y}{\text{change in } x}$$

But there's even a cooler way to write our formula for m. The natural world is filled with changes. In slope, we've seen changes in x and y in the notions of rise and run. In chemistry, there are changes in the volume and pressure of a gas. In nursing, there are changes in body temperature and blood pressure, and in economics there are changes in supply and demand. This concept occurs so often that there's a special notation for a "change" in something. We use the Greek capital letter delta, Δ, to represent a change in something. A change in volume might be denoted by ΔV and a change in time by Δt.

The Delta Airlines logo

And so now we can redefine ***slope*** as

$$\boxed{m = \frac{\Delta y}{\Delta x}}$$

Slope is the *ratio* of the change in *y* to the change in *x*.

Now we're ready to find the slope using the points mentioned at the beginning of this section: $(\pi, 2000)$ and $(3\pi, -5000)$.

<u>EXAMPLE 5:</u> **Find the slope of the line connecting the points (π, 2000) and (3π, –5000).**

<u>Solution:</u> A simple ratio will give us the slope:

$$m = \frac{\Delta y}{\Delta x} = \frac{2{,}000-(-5{,}000)}{\pi - 3\pi} = \frac{7{,}000}{-2\pi} = \frac{\cancel{2}\cdot 3{,}500}{-\cancel{2}\pi} = -\frac{\mathbf{3{,}500}}{\pi}$$

In the last step of this calculation we used the fact that a positive number divided by a negative number is negative. Also, we could obtain an approximate answer by dividing 3,500 by 3.14 -- then attaching the negative sign -- to get about $-1{,}114.65$.

Be careful! You must subtract in <u>the same order</u>. In the problem above, we subtracted from left to right for both the top and the bottom of the slope formula. You could also have subtracted from right to left on the top, as long as you do the subtraction from right to left in the bottom.

Notice that there's no need to plot points and count squares on a grid. We've turned the geometric concept of slope into an arithmetic problem. As we did when we used $\frac{\text{rise}}{\text{run}}$, try reversing the order of the subtractions above to make sure you get the same slope.

Homework

12. Use the formula $m = \dfrac{\Delta y}{\Delta x}$ to find the **slope** of the line connecting each pair of points:

a. (2, 3) and (4, 7)
b. (–3, 0) and (0, 6)
c. (1, –3) and (–2, 5)
d. (2, 2) and (7, 7)
e. (–3, –3) and (0, 0)
f. (–1, –2) and (3, –5)
g. (1, 1) and (–2, 3)
h. (1, 4) and (0, 0)
i. (–3, –2) and (1, –3)
j. (–1, 3) and (1, –3)
k. (–4, 5) and (0, 0)
l. (–1, –1) and (4, –2)

Practice Problems

13. Find the **midpoint** of the line segment connecting the given pair of points:

a. (1, 0) and (2, 0)
b. (π, –π) and (–π, π)
c. (0, 0) and (–5, 10)
d. (3, –8) and (5, –10)
e. (–2, –4) and (–6, –8)
f. (100, 200) and (50, –2)

14. By plotting the two given points in the plane and using the Pythagorean Theorem, find the **distance** between the given pair of points:

a. (0, 0) and (5, 12)
b. (–1, –1) and (–2, –2)
c. (4, 4) and (4, 0)
d. (π, 3) and (π, 5)
e. (2, 3) and (5, 7)
f. (–1, 2) and (4, –3)

15. Find all the **intercepts** of each line:

a. $y = 7x - 3$	b. $y = -9x + 8$	c. $y = x + 1$
d. $y = -x - 1$	e. $y = \frac{2}{3}x + 5$	f. $y = -\frac{1}{2}x - \frac{4}{5}$
g. $-2x - 7y = 0$	h. $4x + 8y = 6$	i. $18x - 17y = 2$

16. Find the **slope** of the line connecting each pair of points:

a. $(-10, 7)$ and $(-12, -8)$	b. $(12, -10)$ and $(8, -5)$
c. $(12, 3)$ and $(-3, 10)$	d. $(1, 3)$ and $(10, 5)$
e. $(-8, 10)$ and $(12, 8)$	f. $(-9, 1)$ and $(-10, 11)$
g. $(-2, -1)$ and $(1, 5)$	h. $(6, -1)$ and $(-12, -1)$
i. $(4, 6)$ and $(9, -5)$	j. $(3, -3)$ and $(12, 6)$
k. $(3, -12)$ and $(-2, -7)$	l. $(-7, -6)$ and $(-8, 12)$

17. Find the midpoint of the line segment connecting the points $(7, -3)$ and $(14, -9)$.

18. Find the midpoint of the line segment connecting the points (n, w) and $(-2n, 3w)$.

19. Find the distance between the points $(-2, 5)$ and $(1, -4)$.

20. Find the distance between the origin and the point $(7, 24)$.

21. Find all the intercepts of the line $3x - 17y = 102$.

22. Find the slope of the line connecting the points $(2, -7)$ and $(-8, 5)$.

23. Find the slope of the line connecting the points $(2a, b)$ and $(a, -b)$.

Solutions

1.

a.	15	b.	17.5	c.	–17	d.	2	e.	–0.5	f.	0
g.	–60	h.	21.5	i.	–25	j.	–25	k.	11	l.	☺

2.

a.	(0, 6)	b.	(0, 7/2)	c.	(–5, 0)	d.	(3/2, 2)
e.	(–11, –19)	f.	(–3, –9/2)	g.	(5, 4)	h.	(7, –9/2)
i.	(–11/2, –11/2)	j.	(0, 0)	k.	(0, 0)	l.	$\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$
m.	(a, 0)	n.	(0, 2b)				

3. a. $|7-2| = |5| = 5$

b. $|-2-9| = |-11| = 11$

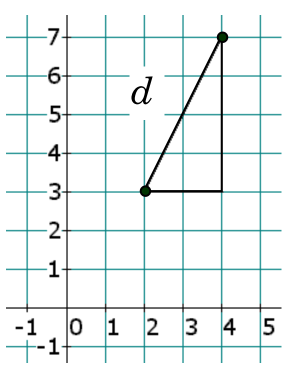
c. $|-3-(-3)| = |-3+3| = |0| = 0$

d. $|99-(-99)| = |99+99| = |198| = 198$

e. 8

f. 16

4. a.



The two points have been plotted in the plane. By counting squares we see the leg at the bottom is 2 while the other leg is 4. The distance between the two given points is simply the length of the hypotenuse. The Pythagorean Theorem gives us

$d^2 = 2^2 + 4^2 \Rightarrow d^2 = 20 \Rightarrow d = 4.472$ (approximately).

(Actually, the equation $d^2 = 20$ has <u>two</u> solutions: ±4.472, but the negative solution is silly since we're looking for distance.)

b.	1.414	c.	7.071	d.	2.236	e.	3.606	f.	4
g.	4	h.	4	i.	7.211	j.	0		

5. a. y-intercept b. x-intercept c. both intercepts
d. x-intercept e. y-intercept f. neither

6. a. (6, 0) (0, 12) b. (8, 0) (0, –6) c. (–7, 0) (0, 4)
d. (7, 0) (0, –1) e. (–4, 0) (0, 12) f. (10, 0) (0, 12)
g. $(\frac{1}{2}, 0)$ $(0, -\frac{2}{3})$ h. $(\frac{1}{3}, 0)$ $(0, \frac{1}{8})$ i. (0, 0)
j. $(\frac{3}{7}, 0)$ (0, –3) k. $(\frac{2}{9}, 0)$ (0, 2) l. (–5, 0) (0, –5)

7. (–3, 0) (–1, 0) (0, 0) (2, 0) (3, 0) (4, 0) (0, –5) (0, 5)

8. The x-intercepts are (5, 0) and (–5, 0). The y-intercepts are (0, 5) and (0, –5).

9. Hint: There are two x-intercepts (and one y-intercept).

10. a. 2 b. 2 c. $-\frac{8}{3}$ d. 1 e. 1 f. $-\frac{3}{4}$
g. $-\frac{2}{3}$ h. 4 i. $-\frac{1}{4}$ j. –3 k. $-\frac{5}{4}$ l. $-\frac{1}{5}$

11. a. 1 b. 2 c. –2 d. 3 e. –3 f. –1
g. $-\frac{1}{2}$ h. $\frac{2}{3}$ i. 3 j. $\frac{3}{2}$ k. $-\frac{2}{5}$ l. $\frac{3}{4}$

12. a. 2 b. 2 c. $-\frac{8}{3}$ d. 1 e. 1 f. $-\frac{3}{4}$
g. $-\frac{2}{3}$ h. 4 i. $-\frac{1}{4}$ j. –3 k. $-\frac{5}{4}$ l. $-\frac{1}{5}$

13. a. (1.5, 0) b. (0, 0) c. (–2.5, 5)
d. (4, –9) e. (–4, –6) f. (75, 99)

14. a. 13 b. 1.414 c. 4 d. 2 e. 5 f. 7.071

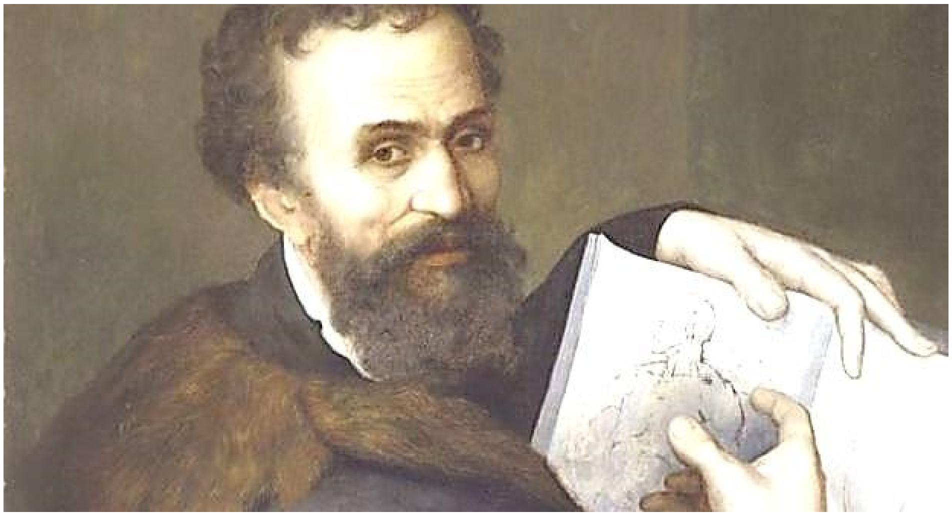
15. a. (0, –3) $(\frac{3}{7}, 0)$ b. (0, 8) $(\frac{8}{9}, 0)$ c. (0, 1) (–1, 0)
d. (0, –1) (–1, 0) e. (0, 5) $(-\frac{15}{2}, 0)$ f. $(0, -\frac{4}{5})$ $(-\frac{8}{5}, 0)$
g. (0, 0) h. $(0, \frac{3}{4})$ $(\frac{3}{2}, 0)$ i. $(0, -\frac{2}{17})$ $(\frac{1}{9}, 0)$

16. a. $\frac{15}{2}$ b. $-\frac{5}{4}$ c. $-\frac{7}{15}$ d. $\frac{2}{9}$ e. $-\frac{1}{10}$ f. -10

g. 2 h. 0 i. $-\frac{11}{5}$ j. 1 k. -1 l. -18

17. $(10.5, -6)$ **18**. $\left(-\frac{n}{2}, 2w\right)$ **19**. 9.49

20. 25 **21**. $(34, 0)$ $(0, -6)$ **22**. $-\frac{6}{5}$ **23**. $\frac{2b}{a}$



"I am still learning."

– **Michelangelo, at age 87**