CH 9 – POLYNOMIALS

❑ INTRODUCTION

It's very difficult to define what a polynomial is at this point in your algebra studies, because we haven't come across many things that aren't polynomials. Suffice it to say that a typical polynomial looks like

$$3x^5 - \pi x^3 + x^2 - 9x + \frac{4}{5}$$

The main theme of a polynomial is that all of the exponents on the x (or whatever variable) must be one of the whole numbers 0, 1, 2, 3, ... The following are not polynomials: $8x^{-2}$ and $3x^{1/2}$, because the exponents –2 and 1/2 are not whole numbers.

❑ WORKING WITH POLYNOMIALS

A polynomial with one term is called a ***monomial***. The expressions $7n$ and $10x^2$ are monomials. The key to multiplying monomials is that each monomial is a single term whose final operation is multiplication.

For example, to find the product $(7x)(9x)$, we proceed the long way -- you don't ever have to do it this way -- but it's important to see.

	$(7x)(9x)$	(the original expression)
=	$7 \cdot 9 \times x \cdot x$	(it's all multiplication)
=	$(7 \cdot 9) \times (x \cdot x)$	(regroup the factors)
=	$63 \times x^2$	(something times itself is squaring)
=	$63x^2$	(no need for the multiplication sign)

Another example is $3(-10n) = (3 \cdot -10)n = -30n$.

But don't forget that adding and subtracting don't follow the same rules as multiplication. Two monomials can be added or subtracted only if they're like terms. See if the homework sorts all of this out.

Homework

1. Simplify each expression:

a. $3(7L)$	b. $-5(2x)$	c. $-6(-2T)$
d. $20(-3w)$	e. $3 + 7L$	f. $-5 + 2x$
g. $-6 - 2T$	h. $20 - 3w$	i. $(7y)5$
j. $(-2p)(-5)$	k. $(-3a)(10)$	l. $(5n)(-2)$
m. $7y + 5$	n. $(4x)(3x)$	o. $4x + 3x$
p. $(2n)(-3n)$	q. $2n - 3n$	r. $(-8x)(-7x)$
s. $(7u)(-u)$	t. $(-4c)(4c)$	u. $-4c + 4c$
v. $(7m)(6n)$	w. $7m - 6n$	x. $(13k)(-13k)$
y. $13k - 13k$	z. $-14x + 20x$	

Homework

2. Suppose a friend believed that $4n^2$ and $7n$ were like terms, and that their sum should be $11n^3$. Prove your friend wrong by letting $n = 2$, and then showing that

$$4n^2 + 7n \neq 11n^3$$

3. Simplify each expression by combining like terms:

a. $3x^2 - 7x + 5x^2 + 9$
b. $n^2 - 9 + 9 - n^2$
c. $1 - 3u - u^2 - 3u^2 + 7u - 1$
d. $7a^2 - 8a + 7 - 9a^2 + 7a - 7$
e. $x^2 - 3x - 1 + 7x^2 - 3x + 1$
f. $3y^2 - 2 + 3y^2 - 2$
g. $1 - 3x - x^2 + 5 - 7x + x^2$
h. $-5w^2 + 2 - 3w + 8w - 2 - w^2$

4. Simplify each expression by distributing and then combining like terms:

a. $(3c^2 - 2c - 1) + 2(c^2 + 5c - 7)$
b. $3(x^2 - 8x + 1) - 5(2x^2 + 7x - 1)$
c. $-(a^2 - a - 1) + 3(-a^2 + a)$
d. $7w^2 - 13w + 8 - (5w^2 - 3w - 2)$
e. $-(7u^2 - 7u - 6) - (-6u^2 + 3u + 5)$
f. $(3x^2 - x - 1) - (3x^2 - x - 1)$
g. $-2(x^2 - 3x + 7) - (3x^2 + 10x - 1)$
h. $-(3n^2 + 8n - 1) - 3(n^2 + 2n - 1)$

❑ *THE DOUBLE DISTRIBUTIVE LAW*

As stated before, a polynomial with one term is called a ***monomial***; a polynomial with two terms is called a ***binomial***. A problem where we must multiply a monomial by a binomial is the following:

$3x(2x + 10)$. (3*x* is the monomial and 2*x* + 10 is the binomial)

Finding the product of these two polynomials is pretty easy -- just distribute the 3*x* to the 2*x* and to the 10:

$(3x)(2x) + 3x(10)$

$= 6x^2 + 30x$, and it's done.

What we need now is a way to multiply <u>two binomials</u> together. For example, how do we simplify the product $(x + 7)(x + 5)$? The **double distributive law** says, in a nutshell,

Multiply each term in the first binomial

by each term in the second binomial.

<u>EXAMPLE 2:</u> **Multiply out (simplify): $(x + 7)(x + 5)$**

<u>Solution:</u> Multiply each term in the first binomial by each term in the second binomial:

i) Multiply the first x by the second x: x^2

ii) Multiply the first x by the 5: $5x$

iii) Multiply the 7 by the second x: $7x$

iv) Multiply the 7 by the 5: 35

Add the four terms together: $x^2 + 5x + 7x + 35$, and then combine like terms

$$\boxed{x^2 + 12x + 35}$$

<u>EXAMPLE 3:</u> **Simplify the given expression:**

A. $(2n + 1)(n - 8)$

$= 2n^2 - 16n + n - 8$ (double distribute)

$= \mathbf{2n^2 - 15n - 8}$ (combine like terms)

B. $(7a - 3)(4a - 5)$

$= 28a^2 - 35a - 12a + 15$ (double distribute)

$= \mathbf{28a^2 - 47a + 15}$ (combine like terms)

C. $(6k - 7)(6k + 7)$

$= 36k^2 + 42k - 42k - 49$ (double distribute)

$= \mathbf{36k^2 - 49}$ (combine like terms)

D. $(10 + y)(10 - y)$

$= 100 - 10y + 10y - y^2$ (double distribute)

$= \mathbf{100 - y^2}$ (combine like terms)

E. $(2x + 9)^2$ The square of a quantity is the product of the quantity with itself:

$(2x + 9)^2$

$= (2x + 9)(2x + 9)$ (since $N^2 = N \cdot N$)

$= 4x^2 + 18x + 18x + 81$ (double distribute)

$= \mathbf{4x^2 + 36x + 81}$ (combine like terms)

EXAMPLE 4: **Simplify: $(2x + 1)(x - 5) - (x - 4)^2$**

Solution: The Order of Operations tells us to square and multiply first, and subtract last:

$(2x + 1)(x - 5) - (x - 4)^2$

$= (2x^2 - 10x + x - 5) - \mathbf{(}x^2 - 4x - 4x + 16\mathbf{)}$ (multiply and square)

[Notice how parentheses still enclose the result of the squaring.]

$= (2x^2 - 9x - 5) - (x^2 - 8x + 16)$ (combine like terms)

$= 2x^2 - 9x - 5 - x^2 + 8x - 16$ (distribute the -1)

$= \boxed{x^2 - x - 21}$ (combine like terms)

Homework

5. Simplify each expression by double distributing:

a. $(x + y)(w + z)$ b. $(c + d)(a - b)$ c. $(x + 2)(y + 3)$

d. $(x + 3)(x + 4)$ e. $(n - 4)(n - 1)$ f. $(a + 3)(a - 7)$

g. $(y + 9)(y - 9)$ h. $(u - 3)(u + 3)$ i. $(t - 20)(t - 19)$

j. $(z + 3)(z + 3)$ k. $(v - 4)(v - 4)$ l. $(N + 1)(N - 1)$

6. Simplify each expression by double distributing:

a. $(3a + 7)(a - 9)$ b. $(2n - 3)(n + 4)$ c. $(3n - 8)(n - 1)$

d. $(5x + 7)(5x + 6)$ e. $(7w + 2)(7w - 2)$ f. $(x + 12)(x - 12)$

g. $(2y + 1)(2y + 1)$ h. $(7x + 3)(6x - 7)$ i. $(q + 7)(3q - 7)$

j. $(3n + 1)(3n + 1)$ k. $(3x - 7)(6x + 5)$ l. $(u - 7)(u - 7)$

7. Square and simplify each expression:

a. $(y + 4)^2$ b. $(z - 9)^2$ c. $(3x + 5)^2$

d. $(2a - 1)^2$ e. $(n + 12)^2$ f. $(6t - 7)^2$

g. $(q - 15)^2$ h. $(5b + 3)^2$ i. $(7u - 1)^2$

j. $(2x + 1)^2$ k. $(3h - 12)^2$ l. $(5y - 5)^2$

8. Simplify each expression:

a. $(a + b)(c - d)$ b. $(2x - 3)(2x + 3)$ c. $(3n - 1)^2$

d. $(3t + 1)(2t - 3)$ e. $(2x + 4)(3x - 6)$ f. $(n + 1)(n - 1)$

g. $(7a - 10)(6a - 10)$ h. $(10c + 7)^2$ i. $(L + 4)^2$

j. $(7x - 3)(3x + 7)$ k. $(13n - 7)(13n + 7)$ l. $(12d - 20)^2$

9. Simplify each expression:

a. $(2n + 1)(n + 1) + (n - 1)(n + 1)$

b. $(x + 1)^2 + (x + 2)^2$

c. $(3a + 2)(a - 1) - (a + 1)(a + 2)$

d. $(4w + 1)^2 - (w - 1)(w - 3)$

e. $(y + 2)(y - 3) - (2y - 1)^2$

f. $(2y + 1)^2 - (2y - 1)^2$

10. Prove that $(a + b)^2 \neq a^2 + b^2$ in two ways:

i) Plug in numbers.

ii) Simplify $(a + b)^2$ the correct way.

11. Use numbers to prove that $(x + y)^3 \neq x^3 + y^3$

❑ *TRINOMIALS*

A ***trinomial*** is a polynomial consisting of three terms. Here are a couple of problems where we subtract some trinomials and multiply with a trinomial.

<u>EXAMPLE 5:</u> **Simplify each expression:**

A. $(2x^2 - x + 1) - (x^2 - 7x + 2)$ (difference of 2 trinomials)

$= 2x^2 - x + 1 - x^2 + 7x - 2$ (distribute the minus sign)

$= 2x^2 - x^2 - x + 7x + 1 - 2$ (rearrange the terms)

$= \mathbf{x^2 + 6x - 1}$ (combine like terms)

B. $(a - 3)(a^2 + 2a - 5)$ (the product of a binomial and a trinomial)

The secret here is to multiply each of the terms in the binomial by each of the terms in the trinomial:

Multiply a by all three terms: $a^3 + 2a^2 - 5a$

Multiply -3 by all three terms: $-3a^2 - 6a + 15$

Now combine like terms: $\boldsymbol{a^3 - a^2 - 11a + 15}$

Homework

12. Simplify each expression:

a. $(3n^2 - 14n + 2) + (2n^2 + 2n - 1)$
b. $(4x^2 - x - 1) - (x^2 - 1)$
c. $(x + 2)(x^2 + 3x + 4)$
d. $(y - 1)(y^2 - 1)$
e. $(z + 3)(2z^2 - z - 1)$
f. $(2x - 5)(x^2 - 5x + 5)$
g. $(4w^2 - 3w - 1)(2w + 5)$
h. $(x + 3)(x^2 - 3x + 9)$
i. $(x^2 + 1)(x^2 + 2)$
j. $(2a + 1)(a^2 + 1)$
k. $(x - 3)(x^2 + 7x - 1)$
l. $(3t^2 - 5t + 3)(2t - 3)$

❑ *Cubing a Binomial*

EXAMPLE 6: **Cube the binomial $2x + 5$. That is, simplify the expression $(2x + 5)^3$.**

Solution: The cube of anything is found by multiplying three of those anythings together: $A^3 = A \times A \times A$. Therefore, the expression

$$(2x + 5)^3$$

can be expanded to get

$$(2x + 5)\ (2x + 5)\ (2x + 5)$$

We know that one way to multiply three things together is to multiply the first two of them together, and then multiply that result by the 3rd thing. Multiplying the first two factors together gives

$$(4x^2 + 10x + 10x + 25)(2x + 5) \qquad \text{(double distribute)}$$

$$= (4x^2 + 20x + 25)(2x + 5) \qquad \text{(combine like terms)}$$

We now have a trinomial times a binomial. What do we do? Most students find that reversing the trinomial and the binomial makes things a little easier to keep track of, so let's do it.

$$= (2x + 5)(4x^2 + 20x + 25) \qquad \text{(commutative property)}$$

We multiply each term in the binomial by each term in the trinomial:

$$= 2x(4x^2) + 2x(20x) + 2x(25) + 5(4x^2) + 5(20x) + 5(25)$$

$$= 8x^3 + 40x^2 + 50x + 20x^2 + 100x + 125$$

$$= \boxed{8x^3 + 60x^2 + 150x + 125}$$

Preview of a Future Chapter:

Consider simplifying (expanding) the expression $(a + b)^9$. You should realize that the answer is <u>not</u> $a^9 + b^9$.

First of all, previous examples have shown us that $(a + b)^2$ is not equal to $a^2 + b^2$. And the previous example showed us that $(2x + 5)^3$ is <u>not</u> equal to $(2x)^3 + 5^3$. It therefore seems reasonable that $(a + b)^9$ would <u>not</u> be equal to $a^9 + b^9$.

Second, watch what happens when we test the *conjecture* that $(a + b)^9 = a^9 + b^9$. Let a and b both take on the value 1. Then

$$(a + b)^9 = (1 + 1)^9 = 2^9 = 512;$$

but, $a^9 + b^9 = 1^9 + 1^9 = 1 + 1 = 2$ -- not even close!!

We therefore conclude that $(a + b)^9 \neq a^9 + b^9$. So how do we raise the sum of a and b to the 9th power? Here's the hard way:

$$(a + b)(a + b)(a + b)(a + b)(a + b)(a + b)(a + b)(a + b)(a + b)$$

Start with the first two binomials; multiply that result by the third binomial, and so on and so on. You'd be done in a few hours (most likely with errors), but there's a much quicker way that we'll learn about at the very end of this book.

Homework

13. Simplify each expression:

a. $(x + 3)^3$ b. $(y + 1)^3$ c. $(n - 5)^3$

d. $(2a + 4)^3$ e. $(3m - 2)^3$ f. $(5q + 3)^3$

14. Prove that $(x + y)^3 \neq x^3 + y^3$ by expanding the binomial.

❑ *Dividing a Polynomial by a Monomial*

In order to do the problem $\frac{a}{b} + \frac{c}{b}$ we just add the numerators, and place that sum over the common denominator b:

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$$

By reversing this reasoning we can take the fraction $\frac{a + c}{b}$ and, if we like, split it into the sum of two fractions:

$$\frac{a + c}{b} = \frac{a}{b} + \frac{c}{b}$$

This is the trick we need to divide a polynomial by a monomial.

EXAMPLE 7: **Divide:** $\dfrac{12x^2y^3 - 8x^3y^2 + 7xy^3}{2x^2y}$

Solution: Split the fraction into three separate fractions:

$$\frac{12x^2y^3}{2x^2y} - \frac{8x^3y^2}{2x^2y} + \frac{7xy^3}{2x^2y},$$

and then simplify each fraction:

$$\boxed{6y^2 - 4xy + \frac{7y^2}{2x}}$$

Homework

15. Perform each division problem, where the divisor is a monomial:

a. $\dfrac{x^3 - x^2 + x}{x}$ b. $\dfrac{14xy + 21x^2y - 28xy^2}{7xy}$ c. $\dfrac{x^2 + 3x + 1}{x}$

d. $\dfrac{a + b}{b}$ e. $\dfrac{x - y}{y}$ f. $\dfrac{ax + bx}{x}$

❑ *DIVIDING A POLYNOMIAL BY A POLYNOMIAL*

First we need the right terminology. When written as a fraction, a division problem has two parts:

$$\frac{\text{dividend}}{\text{divisor}}$$

When written in the standard "long division" format, we write

$$\text{divisor}\overline{\big)\text{dividend}}$$

The result of dividing is called the *quotient*, and the leftover is called the *remainder*. For example,

$$\begin{array}{r} 5 \\ 3\overline{)17} \\ \underline{15} \\ 2 \end{array}$$

dividend = 17
divisor = 3
quotient = 5
remainder = 2

We can then write the answer as $5+\frac{2}{3}$ $\left(\text{dividend} + \frac{\text{remainder}}{\text{divisor}}\right)$, which is written as the mixed number $5\frac{2}{3}$.

Think back when you were a kid and learned long division of numbers. Though I've seen different ways of doing this, the standard method boils down to a 4-step process, a process that is repeated until the problem is finished:

1. Divide the divisor into the first part of the dividend
2. Multiply the divisor by the part of the quotient calculated in step 1
3. Subtract
4. Bring down the next digit

And then repeat steps 1 – 4 as many times as necessary.

We use the same process for polynomial long division in algebra.

EXAMPLE 8: **Perform the long division:** $\dfrac{\mathbf{3x^3 - 5x - 2}}{\mathbf{x - 1}}$

Solution: The first step is to fill in the missing term in the dividend. Since there is no x^2 term, we put in the "place-holder" $0x^2$ between the cubic term and the linear term, giving us a dividend of $3x^3 + 0x^2 - 5x - 2$. So our long division problem is

$$x - 1 \overline{)\, 3x^3 + 0x^2 - 5x - 2}$$

1. Divide x into $3x^3$; it goes in $3x^2$ times (since $3x^2 \cdot x = 3x^3$):

$$\begin{array}{r} \mathbf{3x^2} \qquad\qquad \\ \mathbf{x} - 1 \overline{)\, \mathbf{3x^3} + 0x^2 - 5x - 2} \end{array}$$

2. Multiply $3x^2$ by the divisor, $x - 1$:

$$\begin{array}{r} \mathbf{3x^2} \qquad\qquad \\ \mathbf{x - 1} \overline{)\, 3x^3 + 0x^2 - 5x - 2} \\ \underline{\mathbf{3x^3 - 3x^2}} \qquad\qquad \end{array}$$

3. Subtract; $3x^3 - 3x^3 = 0$; $0x^2 - (-3x^2) = 3x^2$:

$$\begin{array}{r} 3x^2 \qquad\qquad \\ x - 1 \overline{)\, \mathbf{3x^3} + \mathbf{0x^2} - 5x - 2} \\ \underline{-\,(\mathbf{3x^3 - 3x^2})} \qquad\quad \\ \mathbf{0} \;\; + \mathbf{3x^2} \qquad\qquad \end{array}$$

4. Bring down the next term, $-5x$:

$$\begin{array}{r} 3x^2 \qquad\qquad \\ x - 1 \overline{)\, 3x^3 + 0x^2 - \mathbf{5x} - 2} \\ \underline{-\,(3x^3 - 3x^2)} \qquad\quad \\ 0 \;\; + 3x^2 - \mathbf{5x} \qquad \end{array}$$

1. And repeat: Divide x into $3x^2$:

$$\begin{array}{r|l} & 3x^2 \ + \mathbf{3x} \\ \mathbf{x} - 1 & 3x^3 + 0x^2 - 5x - 2 \\ & -(3x^3 - 3x^2) \\ & 0 \ + \mathbf{3x^2} - 5x \end{array}$$

2. Multiply $3x$ by $x - 1$, the divisor:

$$\begin{array}{r|l} & 3x^2 \ + \mathbf{3x} \\ \mathbf{x - 1} & 3x^3 + 0x^2 - 5x - 2 \\ & -(3x^3 - 3x^2) \\ & 0 \ + 3x^2 - 5x \\ & \mathbf{3x^2 - 3x} \end{array}$$

3. Subtract; $3x^2 - 3x^2 = 0$; $-5x - (-3x) = -2x$:

$$\begin{array}{r|l} & 3x^2 \ + 3x \\ x - 1 & 3x^3 + 0x^2 - 5x - 2 \\ & -(3x^3 - 3x^2) \\ & 0 \ + \mathbf{3x^2 - 5x} \\ & -(\mathbf{3x^2 - 3x}) \\ & \mathbf{0 \ - 2x} \end{array}$$

4. Bring down the next (and last) term, -2:

$$\begin{array}{r|l} & 3x^2 \ + 3x \\ x - 1 & 3x^3 + 0x^2 - 5x - \mathbf{2} \\ & -(3x^3 - 3x^2) \\ & 0 \ + 3x^2 - 5x \\ & -(3x^2 - 3x) \\ & 0 \ - 2x - \mathbf{2} \end{array}$$

1. Divide x into $-2x$:

$$\begin{array}{r} 3x^2 \ \ + 3x - \mathbf{2} \\ \boldsymbol{x} - 1 \overline{)\, 3x^3 + 0x^2 - 5x - 2} \\ \underline{-\,(3x^3 - 3x^2)} \qquad\qquad \\ 0 \ + 3x^2 - 5x \qquad \\ \underline{-\,(3x^2 - 3x)} \qquad \\ 0 \ \ - \mathbf{2}\boldsymbol{x} - 2 \end{array}$$

2. Multiply –2 by x – 1:

$$\begin{array}{r} 3x^2 \ \ + 3x - \mathbf{2} \\ \boldsymbol{x} - \mathbf{1} \overline{)\, 3x^3 + 0x^2 - 5x - 2} \\ \underline{-\,(3x^3 - 3x^2)} \qquad\qquad \\ 0 \ + 3x^2 - 5x \qquad \\ \underline{-\,(3x^2 - 3x)} \qquad \\ 0 \ \ - 2x - 2 \\ - \mathbf{2}\boldsymbol{x} + \mathbf{2} \end{array}$$

3. Subtract; –2x – (–2x) = 0; –2 – (+2) = –4

$$\begin{array}{r} 3x^2 \ \ + 3x - 2 \\ x - 1 \overline{)\, 3x^3 + 0x^2 - 5x - 2} \\ \underline{-\,(3x^3 - 3x^2)} \qquad\qquad \\ 0 \ + 3x^2 - 5x \qquad \\ \underline{-\,(3x^2 - 3x)} \qquad \\ 0 \ \ - \mathbf{2}\boldsymbol{x} - \mathbf{2} \\ \underline{-(-\mathbf{2}\boldsymbol{x} + \mathbf{2})} \\ \mathbf{0} \ \ - \mathbf{4} \end{array}$$

There are no terms left to bring down in the dividend, so we write the remainder (the –4) over the divisor and add it to the quotient. The final answer to the long division problem is

$$\boxed{3x^2 + 3x - 2 + \frac{-4}{x-1}}$$

Homework

16. Perform each polynomial long division problem, expressing any remainder as a fraction added to the quotient:

a. $\frac{x^2+5x+6}{x+3}$ b. $\frac{x^2-9}{x-3}$ c. $\frac{x^2+2x+1}{x+1}$

d. $\frac{n^2+n-4}{n+5}$ e. $\frac{2a^2-5a+2}{a+3}$ f. $\frac{3w^2+10}{w+5}$

g. $\frac{6b^2+b-15}{2b+3}$ h. $\frac{3y^2-9}{y+5}$ i. $\frac{10x^2+3x-7}{2x-1}$

j. $\frac{x^3+1}{x+1}$ Hint: $x^3 + 1 = x^3 + 0x^2 + 0x + 1$

k. $\frac{n^3-8}{n-2}$ l. $\frac{a^3+27}{a^2-3a+9}$

17. Perform each polynomial long division problem (hint: there is no remainder):

a. $\frac{40x^3+97x^2+60x+27}{5x+9}$ b. $\frac{8w^3+22w^2+13w+2}{2w^2+5w+2}$

c. $\frac{40r^3-4r^2-7r-3}{8r^2+4r+1}$ d. $\frac{63m^3+43m^2+13m+1}{7m^2+4m+1}$

Practice Problems

18. Simplify each expression:

a. $7x^2 - 3x + 7 - 7x^2 - 3x - 7$
b. $-8(3y^2 - 4y - 1)$
c. $2(a^2 - 8) - (a^2 - 2a - 1)$
d. $-(4n^2 - 4n) - (4n - 4n^2)$
e. $3(4g^2 - g + 3) - 2(6g^2 + g - 1)$
f. $(x + y)(w + z)$
g. $(3x)(-4x)$
h. $10(3y)$
i. $-3n + 4n$
j. $-2(x^2 - 3x - 1)$
k. $3(x^2 - x - 2) - (2x^2 + 7x + 8)$
l. $10x^2 + 29x$

19. Simplify each expression:

a. $(x + 9)(x + 8)$
b. $(y - 1)(y - 8)$
c. $(2z + 5)(2z - 5)$
d. $(N + 10)(N - 10)$
e. $(x - 9)^2$
f. $(a + 5)^2$
g. $(t + 9)(t - 5)$
h. $(a - 22)(a + 1)$
i. $(a - 11)(a + 2)$
j. $(2x + 1)(x - 5)$
k. $(3x + 8)(2x - 5)$
l. $(6x + 5)(x - 3)$
m. $(6a + 17)(a - 1)$
n. $(R + 12)(R - 12)$
o. $(5n - 3)^2$
p. $(1 - a)(2 - a)$
q. $(7w + 5)^2$
r. $(3a - 1)(3a - 2)$
s. $(9a - 1)(a - 2)$
t. $(x + 18)(x - 2)$
u. $(x + 36)(x + 1)$
v. $(5c - 1)(6c - 1)$
w. $(8a + 1)(2a - 1)$
x. $(6q + 5)^2$
y. $(3 + n)(3 - n)$
z. $(16n - 9)(2n - 3)$

20. Prove that $(u + w)^4 \neq u^4 + w^4$. Letting both u and w equal 1 will do the trick.

21. Simplify each expression:

a. $(2n - 5)(3n - 1)$
b. $(8x + 3)(8x - 3)$
c. $(7z - 5)^2$
d. $(8 - 7a)(8 + 7a)$
e. $(2x - 1)(3x + 4) - (4x - 1)^2$

22. Simplify each expression:

a. $(w - 5)(3w^2 - 2w - 1)$ b. $(2x - 5)^3$

23. True/False, and prove your answer:

a. $(a - b)^2 = a^2 + b^2$ b. $(x - y)^3 = x^3 - y^3$

24. Prove that $(a + b)^5 \neq a^5 + b^5$

25. Divide: $\dfrac{4x^3 - 8x^2 + 6x - 10}{4x^2}$

26. Divide: $\dfrac{x^2 + 9}{x - 5}$

27. Divide: $\dfrac{x^3 - 3x + 8}{x + 3}$

28. Divide: $\dfrac{x^4 - 1}{x + 1}$

29. Divide: $\dfrac{n^3 + 8}{n + 2}$

Solutions

1. a. $21L$ b. $-10x$ c. $12T$ d. $-60w$ e. As is f. As is
g. As is h. As is i. $35y$ j. $10p$ k. $-30a$ l. $-10n$
m. As is n. $12x^2$ o. $7x$ p. $-6n^2$ q. $-n$ r. $56x^2$
s. $-7u^2$ t. $-16c^2$ u. 0 v. $42mn$ w. As is x. $-169k^2$
y. 0 z. $6x$

2. $4n^2 + 7n = 4(\mathbf{2})^2 + 7(\mathbf{2}) = 4(4) + 7(2) = 16 + 14 = 30$,
whereas $11n^3 = 11(\mathbf{2})^3 = 11(8) = 88$
Therefore, $4n^2 + 7n \neq 11n^3$

3. a. $8x^2 - 7x + 9$ b. 0 c. $-4u^2 + 4u$ d. $-2a^2 - a$
e. $8x^2 - 6x$ f. $6y^2 - 4$ g. $-10x + 6$ h. $-6w^2 + 5w$

4. a. $5c^2 + 8c - 15$ b. $-7x^2 - 59x + 8$ c. $-4a^2 + 4a + 1$

d. $2w^2 - 10w + 10$ e. $-u^2 + 4u + 1$ f. 0
g. $-5x^2 - 4x - 13$ h. $-6n^2 - 14n + 4$

5. a. $xw + xz + wy + yz$ b. $ac - bc + ad - bd$ c. $xy + 3x + 2y + 6$
d. $x^2 + 7x + 12$ e. $n^2 - 5n + 4$ f. $a^2 - 4a - 21$
g. $y^2 - 81$ h. $u^2 - 9$ i. $t^2 - 39t + 380$
j. $z^2 + 6z + 9$ k. $v^2 - 8v + 16$ l . $N^2 - 1$

6. a. $3a^2 - 20a - 63$ b. $2n^2 + 5n - 12$ c. $3n^2 - 11n + 8$
d. $25x^2 + 65x + 42$ e. $49w^2 - 4$ f. $x^2 - 144$
g. $4y^2 + 4y + 1$ h. $42x^2 - 31x - 21$ i. $3q^2 + 14q - 49$
j. $9n^2 + 6n + 1$ k. $18x^2 - 27x - 35$ l. $u^2 - 14u + 49$

7. a. $y^2 + 8y + 16$ b. $z^2 - 18z + 81$ c. $9x^2 + 30x + 25$
d. $4a^2 - 4a + 1$ e. $n^2 + 24n + 144$ f. $36t^2 - 84t + 49$
g. $q^2 - 30q + 225$ h. $25b^2 + 30b + 9$ i. $49u^2 - 14u + 1$
j. $4x^2 + 4x + 1$ k. $9h^2 - 72h + 144$ l. $25y^2 - 50y + 25$

8. a. $ac - ad + bc - bd$ b. $4x^2 - 9$ c. $9n^2 - 6n + 1$
d. $6t^2 - 7t - 3$ e. $6x^2 - 24$ f. $n^2 - 1$
g. $42a^2 - 130a + 100$ h. $100c^2 + 140c + 49$ i. $L^2 + 8L + 16$
j. $21x^2 + 40x - 21$ k. $169n^2 - 49$ l. $144d^2 - 480d + 400$

9. a. $3n^2 + 3n$ b. $2x^2 + 6x + 5$ c. $2a^2 - 4a - 4$
d. $15w^2 + 12w - 2$ e. $-3y^2 + 3y - 7$ f. $8y$

10. i) By letting $a = 3$ and $b = 4$, for instance, we get:

$(a + b)^2 = (\mathbf{3} + \mathbf{4})^2 = 7^2 = 49$, whereas

$a^2 + b^2 = \mathbf{3}^2 + \mathbf{4}^2 = 9 + 16 = 25$.

Clearly, $(a + b)^2 \neq a^2 + b^2$

ii) $(a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$

11. Choosing, for example, $x = 1$ and $y = 2$, we would get the following results:

$$(x + y)^3 = (\mathbf{1} + \mathbf{2})^3 = 3^3 = 27;$$

on the other hand, $x^3 + y^3 = \mathbf{1}^3 + \mathbf{2}^3 = 1 + 8 = 9$.

12. a. $5n^2 - 12n + 1$ b. $3x^2 - x$
c. $x^3 + 5x^2 + 10x + 8$ d. $y^3 - y^2 - y + 1$
e. $2z^3 + 5z^2 - 4z - 3$ f. $2x^3 - 15x^2 + 35x - 25$
g. $8w^3 + 14w^2 - 17w - 5$ h. $x^3 + 27$
i. $x^4 + 3x^2 + 2$ j. $2a^3 + a^2 + 2a + 1$
k. $x^3 + 4x^2 - 22x + 3$ l. $6t^3 - 19t^2 + 21t - 9$

13. a. $x^3 + 9x^2 + 27x + 27$ b. $y^3 + 3y^2 + 3y + 1$
c. $n^3 - 15n^2 + 75n - 125$ d. $8a^3 + 48a^2 + 96a + 64$
e. $27m^3 - 54m^2 + 36m - 8$ f. $125q^3 + 225q^2 + 135q + 27$

14. $(x + y)^3 = (x + y)(x + y)(x + y) = (x + y)(x^2 + 2xy + y^2)$
$= x^3 + 3x^2y + 3xy^2 + y^3$,
which is most likely not equal to $x^3 + y^3$ for all values of x and y.

15. a. $x^2 - x + 1$ b. $2 + 3x - 4y$ c. $x + 3 + \frac{1}{x}$
d. $\frac{a}{b} + 1$ e. $\frac{x}{y} - 1$ f. $a + b$

16. a. $x + 2$ b. $x + 3$ c. $x + 1$
d. $n - 4 + \frac{16}{n+5}$ e. $2a - 11 + \frac{35}{a+3}$ f. $3w - 15 + \frac{85}{w+5}$
g. $3b - 4 + \frac{-3}{2b+3}$ h. $3y - 15 + \frac{66}{y+5}$ i. $5x + 4 + \frac{-3}{2x-1}$
j. $x^2 - x + 1$ k. $n^2 + 2n + 4$ l. $a + 3$

17. a. $8x^2 + 5x + 3$ b. $4w + 1$ c. $5r - 3$
d. $9m + 1$

18. a. $-6x$ b. $-24y^2 + 32y + 8$ c. $a^2 + 2a - 15$
d. 0 e. $-5g + 11$ f. $xw + xz + wy + yz$
g. $-12x^2$ h. $30y$ i. n
j. $-2x^2 + 6x + 2$ k. $x^2 - 10x - 14$ l. As is

19. a. $x^2 + 17x + 72$ b. $y^2 - 9y + 8$ c. $4z^2 - 25$
d. $N^2 - 100$ e. $x^2 - 18x + 81$ f. $a^2 + 10a + 25$
g. $t^2 + 4t - 45$ h. $a^2 - 21a - 22$ i. $a^2 - 9a - 22$
j. $2x^2 - 9x - 5$ k. $6x^2 + x - 40$ l. $6x^2 - 13x - 15$
m. $6a^2 + 11a - 17$ n. $R^2 - 144$ o. $25n^2 - 30n + 9$
p. $a^2 - 3a + 2$ q. $49w^2 + 70w + 25$ r. $9a^2 - 9a + 2$
s. $9a^2 - 19a + 2$ t. $x^2 + 16x - 36$ u. $x^2 + 37x + 36$
v. $30c^2 - 11c + 1$ w. $16a^2 - 6a - 1$ x. $36q^2 + 60q + 25$
y. $9 - n^2$, or $-n^2 + 9$ z. $32n^2 - 66n + 27$

20. $(1 + 1)^4 = 2^4 = 16$; whereas $1^4 + 1^4 = 1 + 1 = 2$.

21. a. $6n^2 - 17n + 5$ b. $64x^2 - 9$ c. $49z^2 - 70z + 25$
d. $64 - 49a^2$ e. $-10x^2 + 13x - 5$

22. a. $3w^3 - 17w^2 + 9w + 5$ b. $8x^3 - 60x^2 + 150x - 125$

23. a. False; let $a = 5$ and $b = 2$:
$(a - b)^2 = (5 - 2)^2 = 3^2 = 9$
$a^2 + b^2 = 5^2 + 2^2 = 25 + 4 = 29$

b. False; let $a = 2$ and $b = 3$

$(x - y)^3 = (5 - 4)^3 = 1^3 = 1$

$x^3 - y^3 = 5^3 - 4^3 = 125 - 64 = 61$

24. Let $a = 2$ and $b = 3$

$(2 + 3)^5 = 5^5 = 3125$

$2^5 + 3^5 = 32 + 243 = 275$

25. $x - 2 + \frac{3}{2x} - \frac{5}{2x^2}$

26. $x + 5 + \frac{34}{x - 5}$

27. $x^2 - 3x + 6 + \frac{-10}{x + 3}$

28. $x^3 - x^2 + x - 1$

29. $n^2 - 2n + 4$

"When one door closes, another opens; but we often look so long and so regretfully upon the closed door that we do not see the one which has opened for us."

- *Alexander Graham Bell*