# FUNCTION FORMULAS

The formula for the area of a square is

$$A = s^2$$

where $s$ is the length of a side. Since $A$ is based on $s$, we can say that $A$ is a function of $s$, insofar as $A$ depends on $s$. We can even use the terminology: $s$ is the input and $A$ is the output.

The volume of a cylinder is given by

$$V = \pi r^2 h$$

where $r$ is the radius and $h$ is the height. Here we say that $V$ is a function of both $r$ and $h$.

To express $X$ as a function of $b$ means to write an equation where $X$ is on the left side of the equation, while the right side consists of some formula containing only the variable $b$ (and perhaps some constants).

EXAMPLE 1: **Express the side of a square as a function of its area.**

Solution: The formula $A = s^2$ expresses $A$ as a function of $s$. To express the side $s$ as a function of the area $A$, all we need to do is solve the formula for $s$:

$$\boxed{s = \sqrt{A}}$$

Now $s$ is a function of $A$.

**NOTES:** This may seem like a trivial problem, but it's crucial when using computers. If you want to find the side of a square if you already know the area, then you must do the algebra to solve for $s$ -- the computer cannot do it for you.

The second thing is that technically, solving $A = s^2$ for $s$ should result in $s = \pm\sqrt{A}$. But the side of a square can never be negative, so in this context it's perfectly fine to drop the "±" sign and consider only the non-negative square root of $A$. In fact, you would type **`s = sqrt(A)`** in your computer program.

EXAMPLE 2: **Express the area of a circle as a function of its circumference.**

Solution: Normally, we think of the area of a circle as a function of its radius, namely $A = \pi r^2$, and we also consider the circumference as a function of the radius, as in $C = 2\pi r$.

To express the area, $A$, as a function of the circumference, $C$, we need to write a formula with $A$ on the left side and some formula with $C$ (but not $r$) on the right side. To accomplish this, we will remove the $r$ from the formula $A = \pi r^2$ and insert the $C$ from the formula $C = 2\pi r$. It's a substitution trick.

$$C = 2\pi r \qquad \text{(usual circumference formula)}$$

$$\Rightarrow \quad r = \frac{C}{2\pi} \qquad \text{(divide each side by } 2\pi\text{)}$$

Now substitute this version of $r$ into the area formula:

$$A = \pi r^2 \qquad \text{(usual area formula)}$$

$$\Rightarrow \quad A = \pi\left(\frac{C}{2\pi}\right)^2 \qquad \left(\text{since } r = \frac{C}{2\pi} \text{ from above}\right)$$

$$\Rightarrow \quad A = \pi\left(\frac{C^2}{4\pi^2}\right) \qquad \text{(square the fraction)}$$

$$\Rightarrow \quad \boxed{A = \frac{C^2}{4\pi}} \qquad \text{(cross-cancel \& multiply)}$$

**EXAMPLE 3:** **Express the surface area of a sphere as a function of its diameter.**

**Solution:** The standard formula for the surface area of a sphere is $S = 4\pi r^2$, a formula in which the surface area $S$ is written as a function of the radius $r$. To express the area as a function of the diameter, we need to get rid of the variable $r$ and replace it with $d$.

From the formula $d = 2r$, we solve for $r$: $r = \frac{d}{2}$. Thus,

$$S = \underbrace{4\pi r^2 = 4\pi\left(\frac{d}{2}\right)^2}_{\text{since } r = \frac{d}{2}} = 4\pi\left(\frac{d^2}{4}\right) = \pi d^2$$

Hence, $\boxed{S = \pi d^2}$

To help you absorb the concept of "something as a function of something else," let's summarize the formulas we've just seen.

| | |
|---|---|
| $V = \pi r^2 h$ | $V$ is a function of $r$ and $h$ |
| $s = \sqrt{A}$ | $s$ is a function of $A$ |
| $r = \frac{C}{2\pi}$ | $r$ is a function of $C$ |
| $A = \pi r^2$ | $A$ is a function of $r$ |
| $A = \frac{C^2}{4\pi}$ | $A$ is a function of $C$ |
| $S = 4\pi r^2$ | $S$ is a function of $r$ |
| $d = 2r$ | $d$ is a function of $r$ |
| $r = \frac{d}{2}$ | $r$ is a function of $d$ |
| $S = \pi d^2$ | $S$ is a function of $d$ |

# Homework

1. Express the area of a square as a function of its side.

2. Express the area of a square as a function of its perimeter.

3. Express the perimeter of a square as a function of its area.

4. Express the leg of a right triangle as a function of its other leg and hypotenuse.

5. Express the length of a rectangle as a function of its width and area.

6. Express the circumference of a circle as a function of its diameter.

7. Express the diameter of a circle as a function of its circumference.

8. Express the area of a circle as a function of its diameter.

9. Express the circumference of a circle as a function of its area.

10. Express the volume of a cylinder as a function of its height and radius.

11. Express the radius of a cylinder as a function of its volume and height.

12. Express the radius of a sphere as a function of its surface area.

13. Express the volume of a sphere as a function of its diameter. $(V = \frac{4}{3}\pi r^3)$

14. Express the radius of a sphere as a function of its volume.

15. Express the volume of a sphere as a function of its surface area.

16. Express the surface area of a sphere as a function of its volume.

17. Express the volume of a cube as a function of its surface area.

18. Express the surface area of a cube as a function of its volume.

19. In the statistics formula for the mean, $\bar{x} = \frac{x_1 + x_2}{2}$, express $x_1$ as a function of $x_2$ and $\bar{x}$.

20. In the statistics formula for the $z$-score, $z = \frac{x - \bar{x}}{s}$, express $x$ as a function of $z$, $s$ and $\bar{x}$.

# Solutions

**1.** $A = s^2$

**2.** $A = s^2$; but $P = 4s \Rightarrow s = \frac{P}{4}$; substituting into the area formula:

$$A = \left(\frac{P}{4}\right)^2 \Rightarrow A = \frac{P^2}{16}.$$

**3.** $P = 4s$; but $A = s^2 \Rightarrow s = \sqrt{A}$; substituting gives $P = 4\sqrt{A}$.

**4.** Since, $(\text{leg1})^2 + (\text{leg2})^2 = h^2$, it follows that $\text{leg1} = \sqrt{h^2 - (\text{leg2})^2}$.

**5.** $l = \frac{A}{w}$

**6.** $C = 2\pi r$; $d = 2r \Rightarrow r = \frac{d}{2}$. Therefore, $C = 2\pi\left(\frac{d}{2}\right) \Rightarrow C = \pi d$.

**7.** $d = \frac{C}{\pi}$

**8**. $A = \pi r^2$; $r = \frac{d}{2}$. Therefore, $A = \pi\left(\frac{d}{2}\right)^2 \Rightarrow A = \frac{\pi d^2}{4}$.

**9**. $C = 2\pi r$; $A = \pi r^2 \Rightarrow r = \sqrt{\frac{A}{\pi}}$. Thus, $C = 2\pi\sqrt{\frac{A}{\pi}}$.

**10**. $V = \pi r^2 h$ **11**. $r = \sqrt{\frac{V}{\pi h}}$ **12**. $r = \sqrt{\frac{S}{4\pi}}$

**13**. $V = \frac{4}{3}\pi r^3$; $r = \frac{d}{2}$. Therefore, $V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \Rightarrow V = \frac{\pi d^3}{6}$.

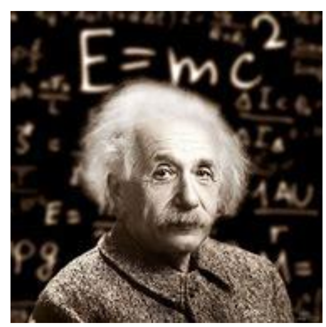
**14**. $r = \sqrt[3]{\frac{3V}{4\pi}}$ **15**. $V = \frac{4}{3}\pi\left(\frac{S}{4\pi}\right)^{3/2}$ **16**. $S = 4\pi\left(\frac{3V}{4\pi}\right)^{2/3}$

**17**. $V = \left(\frac{A}{6}\right)^{3/2}$ **18**. $A = 6V^{2/3}$

**19**. Solving the mean formula for $x_1$ gives $x_1 = 2\bar{x} - x_2$.

**20**. Multiply by $s$, then add $\bar{x}$, which results in $x = zs + \bar{x}$.

*"In the middle of difficulty lies opportunity."*




-- Albert Einstein (1879-1955)