MATH 182 QUIZ #9 KEY

SHOW ALL WORK NEATLY WHERE APPROPRIATE

1. PROVE that the function $y = 14x^2 - 10x + \pi$ has NO *point of diminishing returns*.

   $y'' = 28 \neq 0$ A point of diminishing returns occurs at an inflection point, which is where the 2nd derivative is 0, which it cannot be in this case.

2. Consider the following Cost function:

   $$C(x) = 4x^2 - 5x - 3$$

   a. Find the exact cost of producing the 5th item. **$31**

   $$C(5) - C(4) = 72 - 41 = 31$$

   b. Find the Marginal Cost function. MC = $C'(x) = 8x - 5$

   c. Use the Marginal Cost function to approximate the cost of producing the 5th item.

   $27

   $$C'(4) = 8(4) - 5 = 27$$

3. Given the Demand function

   $$p(x) = -3x + 7$$

   find the Marginal Revenue function. $\text{MR} = -6x + 7$

   $R(x) =$ Quantity $\times$ Price $= x(-3x + 7) = -3x^2 + 7x$

   Marginal Revenue $= R'(x) = -6x + 7$

4. Find A and B so that the graph of the function

$$y = \frac{Ax - 15}{Bx - 3}$$

will have a vertical asymptote of $x = 15$ and a horizontal asymptote of $y = -9$.

$$\underline{A = -\frac{9}{5} \quad B = \frac{1}{5}}$$

V.A. = 15 implies that $B(15) - 3 = 0 \Rightarrow B = 1/5$

H.A. = –9 implies that $\frac{A}{B} = -9 \Rightarrow \frac{A}{1/5} = -9 \Rightarrow A = -9\left(\frac{1}{5}\right) = -\frac{9}{5}$

5. Use Calculus to find two positive numbers whose sum is 10 and whose product is as large as possible.

The numbers are 5 and 5

Call the two numbers x and y, and define P to be their product. We need to maximize the objective function P, subject to the constraint that the sum of the two numbers is 10.

Maximize $P = xy$ subject to $x + y = 10$

$x + y = 10 \Rightarrow y = 10 - x$

So $P = xy = x(10 - x) = 10x - x^2$

To maximize P, we take its derivative and set it to 0:

$P'(x) = 10 - 2x = 0 \Rightarrow x = 5 \Rightarrow y = 5$ (since $x + y = 10$)

Extra Credit

1. The radius of a sphere is increasing at the rate of 5 m/s. How fast is the surface area of the sphere increasing at the moment when $r = 6$?

Surface Area $= 4\pi r^2$

Be sure to include the proper units in your answer. **240π m^2/sec**

2. Let MP = marginal profit, MR = marginal revenue, and MC = marginal cost. PROVE that MP = MR – MC.

$$P = R - C \Rightarrow P' = (R - C)' = R' - C'$$
$$\Rightarrow \text{MP} = \text{MR} - \text{MC}$$