CH 3 – FROM EQUATION TO GRAPH

❑ *INTRODUCTION*

In this chapter we make pictures out of equations. Specifically, we take an equation containing the variables x and y, find some solutions to that equation, plot those solutions in a Cartesian Coordinate System (the x-y plane), and then "connect the dots" to create a final graph of the equation.

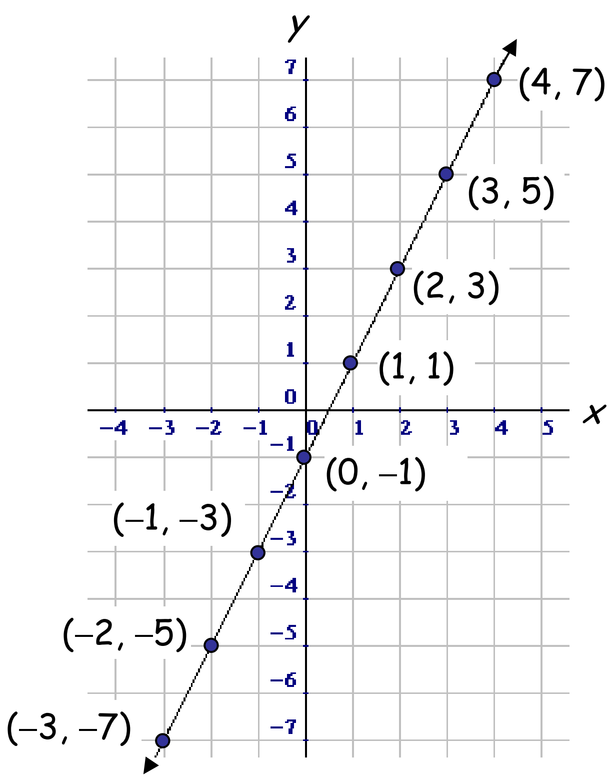
❑ *GRAPHING LINES*

EXAMPLE 1: **Graph:** $y = 2x - 1$

Solution: The most important aspect of graphing is to learn where the x-values come from. They basically come from your head — you get to make them up. Later in this course this process will become more sophisticated; but for now, just conjure them up from your imagination. I'm going to choose x-values of $-3, -2, -1, 0, 1, 2, 3$, and 4. For each of these x-values, I will calculate the corresponding y-value using the given formula, $y = 2x - 1$. Organizing all this information in a table is useful:

x	$2x - 1$	(x, y)
–3	2(**–3**) – 1 = –6 – 1 = –7	(–3, –7)
–2	2(**–2**) – 1 = –4 – 1 = –5	(–2, –5)
–1	2(**–1**) – 1 = –2 – 1 = –3	(–1, –3)
0	2(**0**) – 1 = 0 – 1 = –1	(0, –1)
1	2(**1**) – 1 = 2 – 1 = 1	(1, 1)
2	2(**2**) – 1 = 4 – 1 = 3	(2, 3)
3	2(**3**) – 1 = 6 – 1 = 5	(3, 5)
4	2(**4**) – 1 = 8 – 1 = 7	(4, 7)

Now I will plot the eight points just calculated (each of which is a solution of the equation) on an *x*-*y* coordinate system.



Then the points will be connected with the most reasonable graph, in this case a straight line. Notice that the graph passes through every quadrant except the second; also notice that as we move from left to right (as the *x*'s grow larger) the graph rises.

Final comment: We could let $x = 1{,}000{,}000$ for this equation, in which case y would equal 2(**1,000,000**) – 1 = 1,999,999. That is, the point (1000000, 1999999) is on the line. Can we graph it? Not with the scale we've selected for our graph. But if we traveled along the line, up and up and up, we would eventually run into the point (1000000, 1999999).

We could have used rational numbers like 2/7 for x. This would have given us the point (2/7, −3/7), which is in Quadrant IV. This point, too, is definitely on the line.

And last, we could have used a number like π for x, in which case we would obtain the point $(\pi, 2\pi - 1)$. This point is impossible to plot precisely, but I guarantee that it's in the first quadrant, and it's on the line we've drawn.

In short, every solution of the equation $y = 2x - 1$ is a point on the line, and every point on the line is a solution of that equation.

Note: It may be true that it takes only two points to completely determine a line. So some students plot exactly two points, connect them with a straight line, and they're done. One warning: You're taking a big gamble when you plot just two points — if you make an error with either one, you'll get the wrong line, and there may be no way for you to know you've goofed. Even worse, what if the equation isn't even a line in the first place? Plotting just two points (even if they're both correct) will not suffice to plot a curve that is not a simple straight line.

The more points you plot, the better!

1. Graph each equation:

a. $y = 3x - 1$　　b. $y = -2x + 3$　　c. $y = -x$

d. $y = x$　　e. $y = -x + 3$　　f. $y = -2x - 1$

❑ GRAPHING ABSOLUTE VALUES

EXAMPLE 2: **Graph:** $y = |x - 3| + 2$

Solution: Let's plot lots of points and see what we get. I'll do a couple for you, but be sure you confirm each of the y-values that appears in the table. Recall from Chapter 0 – Prologue that the vertical bars represent **absolute value**. Some examples of absolute value are

$$|17| = 17 \qquad |-44| = 44 \qquad |0| = 0$$

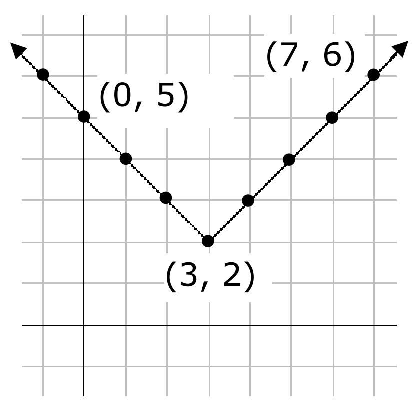
We'll start by letting $x = 6$. Then

$$y = |x-3|+2 = |6-3|+2 = |3|+2 = 3+2 = 5$$

Now choose x to be –1:

$$y = |x-3|+2 = |-1-3|+2 = |-4|+2 = 4+2 = 6$$

x	y
–1	6
0	5
1	4
2	3
3	2
4	3
5	4
6	5
7	6



$y = |x - 3| + 2$

Notice that the graph is in the shape of a "V" with a sharp corner at its bottom point (3, 2).

Homework

2. Graph each equation:

a. $y = |x|$ b. $y = |x+2|$ c. $y = |x|-4$

d. $y = -|x|$ e. $y = |-x|$ f. $y = |x-1|$

g. $y = |x|+3$ h. $y = |x+1|+2$ i. $y = |x+2|-1$

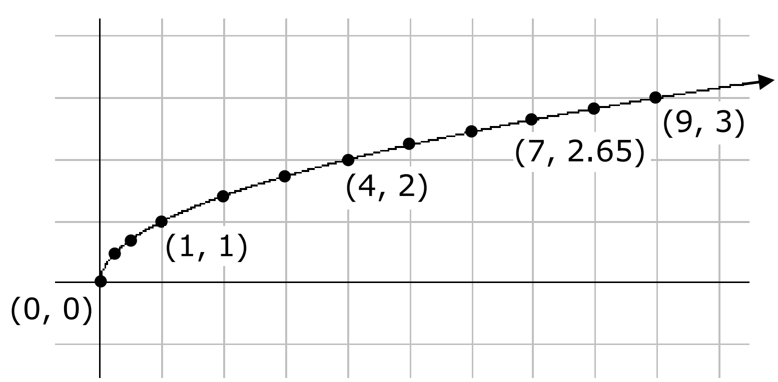
j. $y = -|x+1|$ k. $y = -|x|+3$ l. $y = -|x|-2$

❑ *Graphing Square Roots*

EXAMPLE 3: **Graph:** $y = \sqrt{x}$

x	y
0	0
0.25	0.5
0.5	0.71
1	1
2	1.41
3	1.73
4	2
5	2.24
6	2.45
7	2.65
8	2.83
9	3

Solution: In order to take a square root without killing the problem, we recall (from the Prologue) that x must be 0 or bigger (which we write as $x \geq 0$). This requirement exists so that $\sqrt{x}$ will be a real number. So we can use 0 or any positive number for x, but we can't use any negative values for x. We'll use a calculator to help us estimate the square roots of numbers that don't have nice square roots.



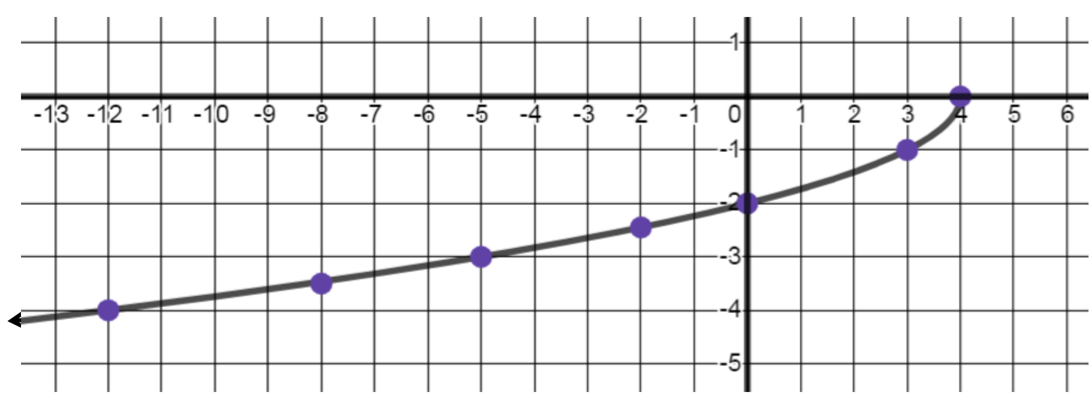
$y = \sqrt{x}$

EXAMPLE 4: **Graph:** $y = -\sqrt{4-x}$

Solution: We know something's up with that square root in the formula; after all, we know we can't take the square root of a negative number and stay within the world of ℝ, the real numbers. But it's not obvious exactly which values of *x* we're allowed to use, so let's not worry about it now; we'll cover this issue in Chapter 36 – Domain. Let's just make up our favorite values of *x* and check them one at a time.

x	y	*Calculation*
0	–2	$y = -\sqrt{4-0} = -\sqrt{4} = -2$
3	–1	$y = -\sqrt{4-3} = -\sqrt{1} = -1$
4	0	$y = -\sqrt{4-4} = -\sqrt{0} = 0$
5	Undefined	$y = -\sqrt{4-5} = -\sqrt{-1} =$ Not a real #
6	Undefined	$y = -\sqrt{4-6} = -\sqrt{-2} =$ Not a real #
–2	–2.45	$y = -\sqrt{4-(-2)} = -\sqrt{6} \approx -2.45$
–5	–3	$y = -\sqrt{4-(-5)} = -\sqrt{9} = -3$
–8	–3.5	$y = -\sqrt{4-(-8)} = -\sqrt{12} \approx -3.5$
–12	–4	$y = -\sqrt{4-(-12)} = -\sqrt{16} = -4$

Plotting the seven points gives us the following graph:



Homework

3. Graph each equation:

a. $y = \sqrt{x+4}$ b. $y = \sqrt{-x}$ c. $y = -\sqrt{x}$

d. $y = \sqrt{x+9}$ e. $y = \sqrt{x} + 3$ f. $y = \sqrt{x} - 2$

g. $y = -\sqrt{-x}$ h. $y = -\sqrt{x+4}$ i. $y = -\sqrt{x-1}$

Practice Problems

4. Graph: $y = 3x - 4$

5. Graph: $y = |x+3| - 2$

6. Graph: $y = \sqrt{x+9} - 5$

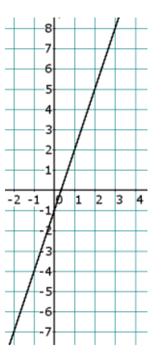
7. Graph: $y = \sqrt{x^2}$

8. Graph: $y = -|x-2| + 4$

9. Graph: $y = -\sqrt{x-3} + 2$

Solutions

1. a.



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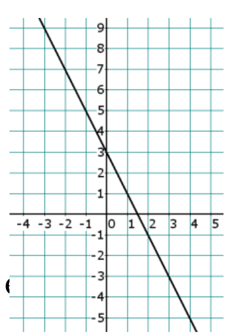

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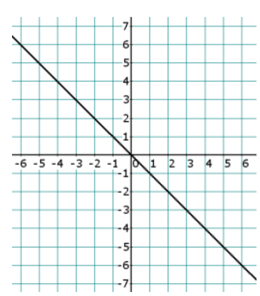

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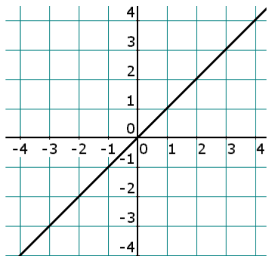

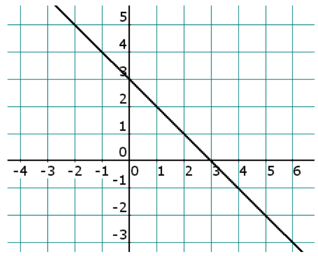

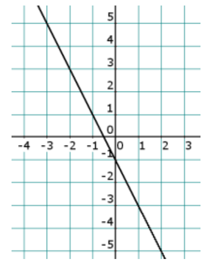

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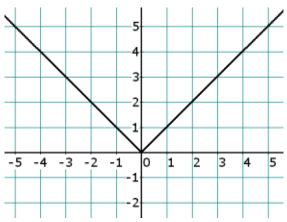

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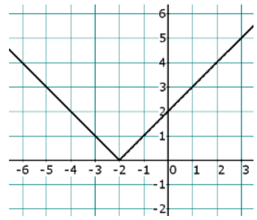

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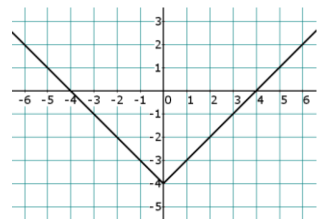

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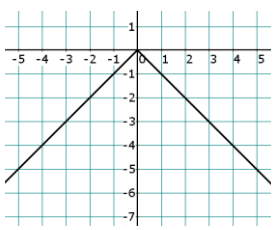

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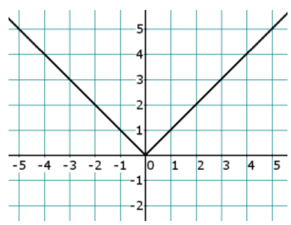

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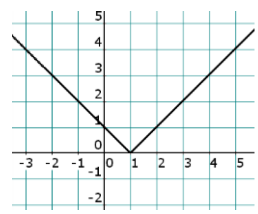

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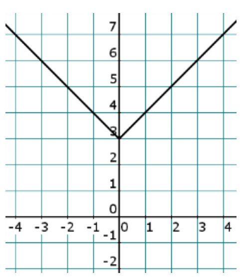

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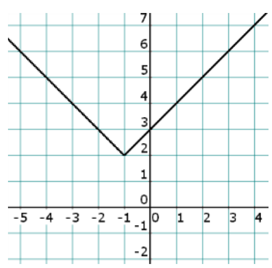

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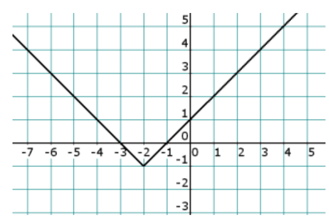

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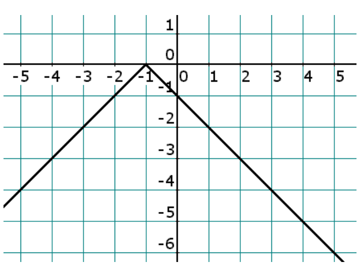

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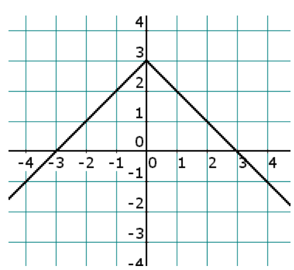

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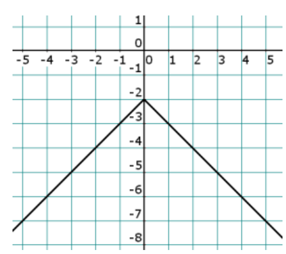

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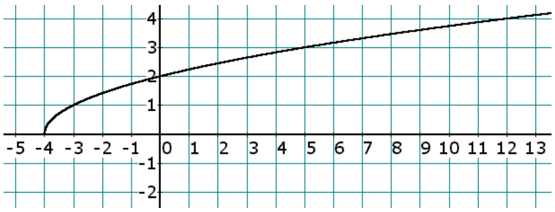
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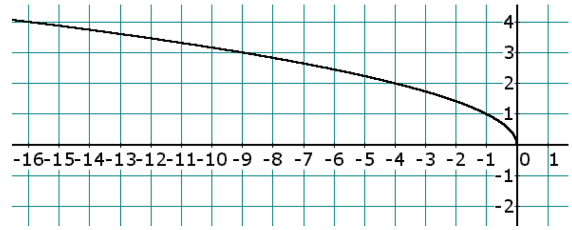
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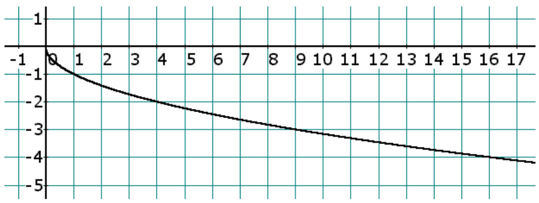
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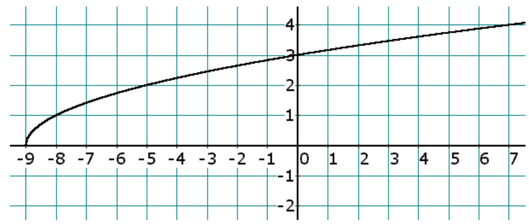
d.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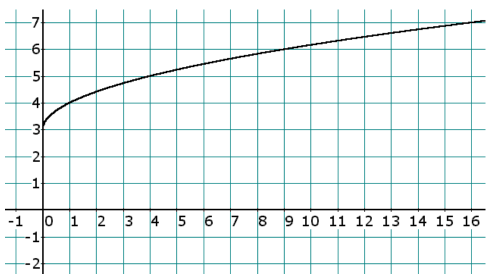
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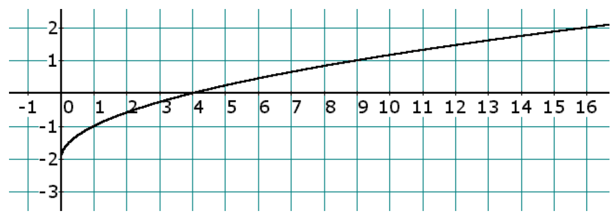
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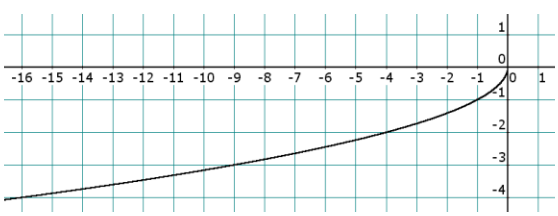
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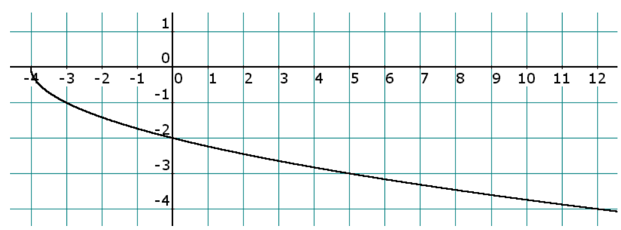
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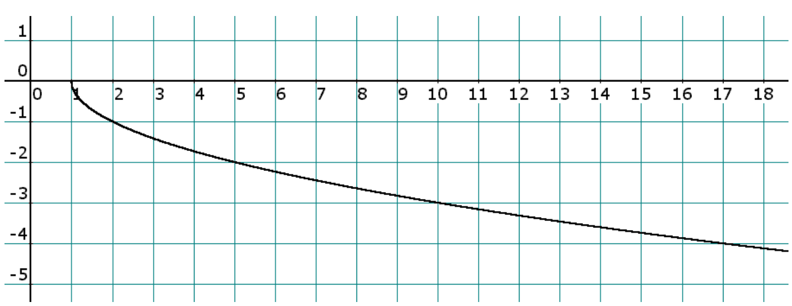
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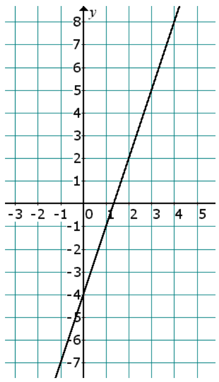
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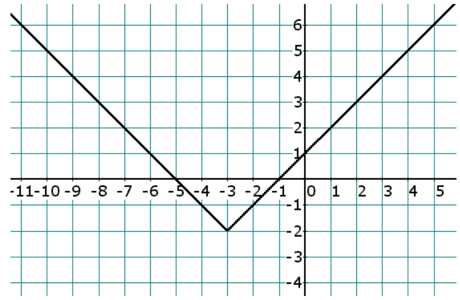
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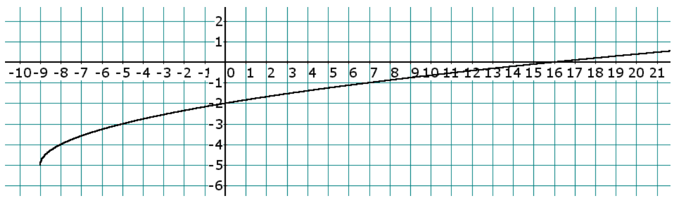
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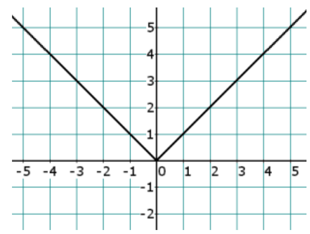
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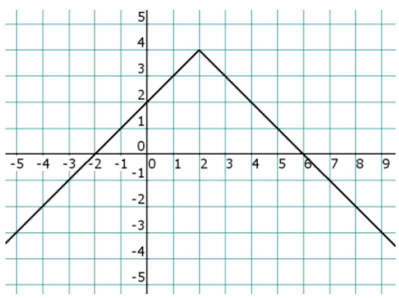
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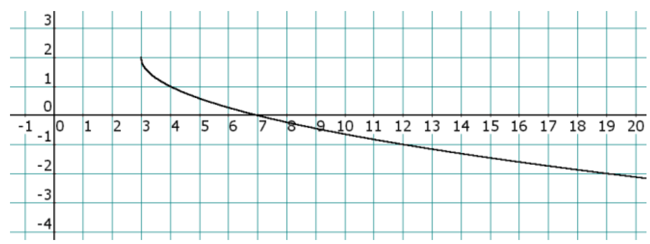
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-1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

"A good head and good heart are always a formidable combination. But when you add to that a literate tongue or pen, then you have something very special."

Nelson Mandela
(1918 – 2013)