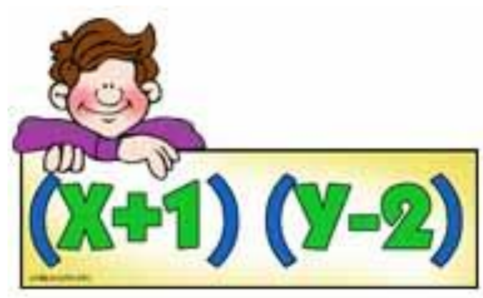
CH 12 – FACTORING, PART I

❑ INTRODUCTION



In the next chapter we will continue our discussion break-even discussion from Chapter 7 by finding *break-even points* with *quadratic equations* (where the variable is squared). But instead of using a table or a graph, we will focus on solving those *quadratic equations* to find the break-even points.

One method of solving a quadratic equation is called the **factoring method**. Therefore, in this chapter, we have to become very skilled at factoring quadratic expressions.

❑ A DIFFERENT VIEW OF THE DISTRIBUTIVE PROPERTY

We've generally viewed the distributive property in a form like

$$A(B + C) = AB + AC \qquad \textbf{DISTRIBUTING}$$

and saw the power of such a property in simplifying expressions and solving complicated equations. But the distributive property is a statement of equality. We might find it useful to flip it around the equals sign and write it as

$$AB + AC = A(B + C) \qquad \textbf{FACTORING}$$

This provides a whole new perspective. It allows us to take a pair of terms, the sum $AB + AC$, find the **common factor** A (it's in both terms), and "pull" the A out in front, and write the sum $AB + AC$ as the product $A(B + C)$.

This use of the distributive property in reverse is called ***factoring***. Notice that using the distributive property in reverse converts two (or more than two) terms into one term.

For example, suppose we want to factor $9a + 9b$; that is, we want to convert $9a + 9b$ from a sum to a product. First we notice that 9 is a common factor of both terms. We pull the 9 away from both terms, and put it out in front to get $9(a + b)$, and we're done factoring:

$$9a + 9b \text{ factors into } 9(a + b)$$

To check this answer, distribute $9(a + b)$ and you'll get the original $9a + 9b$.

RECAP:

$9(a + b)$	Distribute →	$9a + 9b$	**One term to two terms**
$9a + 9b$	Factor →	$9(a + b)$	**Two terms to one term**

❑ *FACTORING OUT THE GCF*

EXAMPLE 1: **Factor each expression:**

A. $7x + 7y = 7(x + y)$

B. $3x + 12 = 3(x + 4)$

C. $ax + bx = x(a + b)$

D. $Rw - Ew = w(R - E)$

E. $9z + 9 = 9(z + 1)$

F. $mn - m = m(n - 1)$

G. $-6R + 8 = -2(3R - 4)$

Alternatively, we could pull out a positive 2, yielding $2(-3R + 4)$, but it's customary to pull out the leading negative sign.

H. $-ax - at = -a(x + t)$

I. $-x + 5 = -(x - 5)$

J. $-n - 9 = -(n + 9)$

K. $6r + 8s - 10t = 2(3r + 4s - 5t)$

Note that every problem in the preceding example (and the next example) can be checked by distributing the answer.

<u>EXAMPLE 2:</u> **Factor each expression:**

A. $x^2 + 3x = x(x + 3)$

B. $n^2 - 7n = n(n - 7)$

C. $t^2 + t = t(t + 1)$

D. $y^2 - y = y(y - 1)$

E. $m^2 - 10m = m(m - 10)$

F. $a^2 + 40a = a(a + 40)$

<u>EXAMPLE 3:</u> **Factor: $2a^2 - 8a$**

<u>Solution:</u> What common factor can be pulled out in front? Since 2 is a factor of both terms, it can be pulled out. But a is also a common factor, so it needs to come out in front, also. In other words, the quantity $2a$ is common to both terms (and it's the <u>largest</u> quantity that is common to both terms). So we factor it out and leave in the parentheses what must be left.

$2a^2 - 8a = 2a(a - 4)$ (check by distributing)

New Terminology: Look at the previous example. We decided that 2 was a common factor of the two given terms. We also realized that a was another common factor of the two terms. But the quantity $2a$, which is, of course, common to both terms, is called the <u>greatest</u> common factor. This is because $2a$ is certainly a factor of both terms, and it is the biggest such factor — nothing bigger than $2a$ is common to both terms. We call $2a$ the ***greatest common factor***, or **GCF**, of the expression $2a^2 - 8a$.

Homework

1. How would you convince your buddy that factoring $20x + 30y$ produces a result of $10(2x + 3y)$?

2. Your friend adamantly believes that $6w + 9z$ factors to $6(w + 3z)$. Prove her wrong.

3. Finish the factorization of each expression:

a. $wx + wz = w(\quad)$ b. $4P - 4Q = 4(\quad)$

c. $9x - 36 = 9(\quad)$ d. $8y - 12t = 4(\quad)$

e. $7u + 7 = 7(\quad)$ f. $-2n + 8 = -2(\quad)$

g. $-a + b = -(\quad)$ h. $-c - d = -(\quad)$

i. $2x + 4y - 8z = 2(\quad)$ j. $aw - au + az = a(\quad)$

k. $14x^2 - 21x = 7x(\quad)$ l. $20a^2 + 30a - 40 = 10(\quad)$

4. Factor each expression:

a. $3P + 3Q$ b. $9n - 27$ c. $cn + dn$

d. $wx - xy$ e. $7t - 7$ f. $x + xy$

g. $-8L + 10$ h. $-ab - bc$ i. $-u - 5$

j. $-z - x + 10$ k. $2x + 2y + 2z$ l. $5a - 10b + 15c$

5. Finish the factorization of each expression:

a. $4a + 8b = 4(\quad)$ b. $9u^2 - 3u = 3u(\quad)$

c. $15Q - 45R = 15(\quad)$ d. $18x^2 + 12x = 6x(\quad)$

e. $10y^2 - 20y = 10y(\quad)$ f. $50a + 75b = 25(\quad)$

g. $7t^2 + 28t = 7t(\quad)$ h. $48w - 64z = 16(\quad)$

i. $100a^2 - 80a = 20a(\quad)$ j. $47y^2 + 47y = 47y(\quad)$

❑ *USING THE GCF TO SOLVE FORMULAS*

Do you remember how, in the Prologue, we solved for x in the formula (literal equation)

$$wx + y = A \ ?$$

We subtracted y from each side of the equation:

$$wx = A - y$$

and then we divided each side of the equation by w:

$$x = \frac{A-y}{w}$$ and we've isolated the x.

This is all fine and dandy when the unknown, in this case the x, occurs only once in the formula. But how do we isolate something that occurs more than once in an equation? For example, how do we solve for x in the formula

$$ax - c = bx \ ?$$

There's an x-term on each side of the equation. This x is going to be tough to isolate. What would you do if, instead of the symbols a, b, and c in the equation, they had been numbers?

> For example, suppose the equation had been
>
> $$7x - 10 = 4x$$
>
> We subtract $4x$ from each side: $7x - 4x - 10 = 0$
>
> Then combine like terms: $3x - 10 = 0$
>
> Now add 10 to each side: $3x = 10$
>
> And lastly, divide each side by 3: $\boldsymbol{x = \frac{10}{3}}$

We follow the same procedure for solving the formula $ax - c = bx$ for x.

The original formula: $ax - c = bx$

Subtract bx from each side: $ax - bx - c = 0$

Add c to each side: $ax - bx = c$

How do we "combine the like terms" ax and $-bx$? Here's where factoring comes to the rescue; by factoring x out of $ax - bx$, we get $x(a - b)$: $x(a - b) = c$ (Factoring out the GCF)

Divide each side by $a - b$, and we're done: $\boldsymbol{x = \dfrac{c}{a-b}}$

Note: There are no x's on the right side of the answer. Can you explain why this fact is so important?

EXAMPLE 4: **Solve for n: $Qn - n + P = R$**

Solution:

$Qn - n + P = R$ (the original formula)

$\Rightarrow Qn - n = R - P$ (subtract P from each side)

$\Rightarrow n(Q - 1) = R - P$ (factor out the n)

$\Rightarrow \dfrac{n(Q-1)}{Q-1} = \dfrac{R-P}{Q-1}$ (divide each side by $Q - 1$)

$\Rightarrow \boxed{n = \dfrac{R-P}{Q-1}}$ (simplify)

EXAMPLE 5: **Solve for a: $c(a - d) + 3 = 5(e - a)$**

Solution:

$c(a - d) + 3 = 5(e - a)$ (the original formula)

$\Rightarrow ac - cd + 3 = 5e - 5a$ (distribute)

$\Rightarrow ac + 5a - cd + 3 = 5e$ (add $5a$ to each side)

$\Rightarrow \quad ac + 5a - cd = 5e - 3$ (subtract 3 from each side)

$\Rightarrow \quad ac + 5a = 5e - 3 + cd$ (add *cd* to each side)

Note: These steps were designed to get the variable *a* on one side of the equation, and the rest of the things on the other side.

$\Rightarrow \quad a(c + 5) = 5e - 3 + cd$ (factor out the *a*)

$\Rightarrow \quad \dfrac{a(c+5)}{\mathbf{c+5}} = \dfrac{5e-3+cd}{\mathbf{c+5}}$ (divide each side by *c* + 5)

$\Rightarrow \quad \boxed{a = \dfrac{5e-3+cd}{c+5}}$ (simplify)

Be sure you understand thoroughly why there mustn't be any *a*'s on the right side of the answer. And can you see why *c* cannot be equal to –5 in this answer?

Homework

6. Solve each formula for *n*:

a. $cn + dn = 3$

b. $an - cn = d$

c. $Ln + n = M$

d. $tn = c - sn$

e. $rn = 3 + tn$

f. $m(n + 1) - Qn - R = 0$

g. $a(n + 3) + b(n + c) = R$

h. $an + bn + cn = d$

i. $an - n - a = 0$

j. $2(n + 1) + an = c$

7. Solve each formula for *x*:

a. $cx + 7x = 14$

b. $rx - ux = w + v$

c. $wx - x = w$

d. $ax - b = c - dx$

e. $u(x - a) + x = w$

f. $a(x + 1) + b(x - 1) = 0$

g. $mx - 3(x - w) = z + u$ h. $p(x - 3) = q(x + 2)$

i. $c(x + a) - a(x - 1) = a - b$ j. $a(b - x) + c(2 - x) = R - Q$

❑ *A Quick Review of Double Distributing*

To set the stage for factoring, we recall from the previous chapter the concept we called "double distributing" to multiply two binomials:

$$(2x + 3)(x + 5)$$

i) Multiply the FIRST terms in each set of parentheses: $2x$ and x. The product is $\underline{2x^2}$.

ii) Multiply the OUTER terms: $2x$ and 5. The product is $\underline{10x}$.

iii) Multiply the INNER terms: 3 and x. The product is $\underline{3x}$.

iv) Multiply the LAST terms in each set of parentheses: 3 and 5. The product is $\underline{15}$.

v) Writing out these four products,

=	$2x^2$	+	$10x$	+	$3x$	+	15
	↑		↑		↑		↑
	product of First terms		product of Outer terms		product of Inner terms		product of Last terms

$$= \mathbf{2x^2 + 13x + 15}$$

The key idea to absorb here is that the $2x^2$ in the answer is the product of the first terms, while the 15 is the product of the last terms. Also, the middle term in the answer, $13x$, is the sum of the outer and inner products.

Homework

8. Find the following products — do all the work in your head:

a. $(2x - 1)(x + 4)$ b. $(3n - 3)(2n - 5)$

c. $(a + 9)(3a - 1)$ d. $(6y - 1)(2y + 7)$

e. $(2m - 7)(2m + 7)$ f. $(4w + 5)(4w + 5)$

g. $(5x + 1)(5x - 1)$ h. $(2n + 3)(7n - 10)$

i. $(7u - 3)(7u - 3)$ j. $(12a + 13)(12a - 13)$

❑ *REVERSE DOUBLE DISTRIBUTING*

We know that

$$(2x + 3)(x + 5) = 2x^2 + 13x + 15,$$

and we call this **double distributing**.

But as described at the beginning of this chapter, we can turn the equality around,

$$2x^2 + 13x + 15 = (2x + 3)(x + 5)$$

and call it **factoring**.

One more example before this section's homework. Suppose I ask you whether the following statement is true or false:

$$9n^2 - 25 \text{ factors into } (3n + 5)(3n - 5)$$

You can determine the answer to this question by multiplying out the expression on the right:

$$(3n + 5)(3n - 5) = 9n^2 - 15n + 15n - 25 = 9n^2 - 25 \checkmark$$

We conclude that it's a true statement.

Homework

9. True/False:

a. $x^2 + 5x + 6$ factors into $(x + 3)(x + 2)$.
b. $y^2 - 16$ factors into $(y + 4)(y - 4)$.
c. $x^2 + 9x + 15$ factors into $(x + 3)(x + 5)$.
d. $n^2 - 6n + 9$ factors into $(n - 3)(n - 3)$.
e. The factorization of $2a^2 - 11a - 6$ is $(2a + 1)(a - 6)$.
f. The factorization of $u^2 + 25$ is $(u + 5)(u + 5)$.
g. The factorization of $a^2 + 10a + 25$ is $(a + 5)^2$.
h. The factorization of $y^2 - 2y + 4$ is $(y - 2)^2$.

10. Matching:

a. ___	$x^2 + 4x + 3$	1. $(2x + 1)(x - 6)$
b. ___	$x^2 - 9$	2. $(x + 5)(x - 5)$
c. ___	$x^2 + 10x + 25$	3. $(x + 3)(x + 1)$
d. ___	$x^2 + 16$	4. $(x + 3)(x - 3)$
e. ___	$x^2 + 4x + 4$	5. $(2x - 3)(x + 2)$
f. ___	$2x^2 - 11x - 6$	6. $(x + 5)(x + 5)$
g. ___	$x^2 - 25$	7. $(x + 2)(x + 2)$
h. ___	$2x^2 + x - 6$	8. Not factorable

11. Finish the factorization of each expression:

a. $x^2 - 10x + 16 = (x - 8)(\qquad)$

b. $n^2 + 5n - 14 = (n + 7)(\qquad)$

c. $a^2 - 17a + 72 = (a - 9)(\qquad)$

d. $q^2 - 49 = (q + 7)(\qquad)$

e. $c^2 + 6c + 9 = (c + 3)(\qquad)$

12. Finish the factorization of each expression:

a. $12n^2 + 8n - 15 = (6n - \quad)(2n + \quad)$

b. $16x^2 - 9 = (4x + \quad)(4x - \quad)$

c. $21z^2 - 4z - 1 = (7z + \quad)(3z - \quad)$

d. $9a^2 + 24a + 16 = (3a + \quad)(3a + \quad)$

e. $6x^2 - 23x + 7 = (2x - \quad)(3x - \quad)$

f. $25t^2 - 49 = (5t + \quad)(5t - \quad)$

g. $9w^2 - 225 = (3w + \quad)(3w - \quad)$

h. $16c^2 - 24c + 9 = (4c - \quad)(4c - \quad)$

i. $14x^2 - 58x + 8 = (7x - \quad)(2x - \quad)$

❑ *THE PROCESS OF FACTORING*

Hopefully, we now understand the concept of factoring. We now present a series of examples that try to turn the random method into something a little more methodical; but no matter what method is used, it's essentially a matter of *trial-and-error*.

EXAMPLE 6: **Factor: $6x^2 - 7x - 5$**

Solution: Factoring a trinomial like $6x^2 - 7x - 5$ can be viewed as a 3-step process:

1) Split the $6x^2$ into two factors (by guessing)
2) Split the -5 into two factors (by guessing)
3) See if we guessed right by double distributing

Step 1: $6x^2 - 7x - 5$

$(3x \quad)(2x \quad)$

Split $6x^2$ into 2 factors and place these factors in the front of each binomial.

Step 2: $6x^2 - 7x - 5$

$(3x - 5)(2x + 1)$

Split –5 into 2 factors and place these factors in the back of each binomial.

Step 3: See if we guessed right:

$$(3x - 5)(2x + 1) = 6x^3 + 3x - 10x - 5 = 6x^2 - 7x - 5 \checkmark$$

We got lucky on the first try. The factorization of $6x^2 - 7x - 5$ is

$$\boxed{(3x - 5)(2x + 1)}$$

EXAMPLE 7: **Factor: $n^2 - 5n + 6$**

Solution: As before, we focus on the first and last terms.

Step 1: Split the n^2; there's only one way to do this:

$(n \quad)(n \quad)$

Step 2: Split the 6; let's try 6 and 1:

$(n + 6)(n + 1)$

Step 3: See if we guessed right:

$$(n + 6)(n + 1) = n^2 + n + 6n + 6 = n^2 + 7n + 6 \quad ☹$$

Let's redo Step 2: Split the 6 into 3 and 2:

$$(n + 3)(n + 2) = n^2 + 2n + 3n + 6 = n^2 + 5n + 6 \quad ☹$$

Notice that the $n^2 + 5n + 6$ we obtained is almost the original expression that we're trying to factor; it differs only in the sign of the middle term. Let's redo Step 2 again: Split the 6 into –3 and –2:

$$(n-3)(n-2) = n^2 - 2n - 3n + 6 = n^2 - 5n + 6 \quad ☺$$

and thus our final factorization is

$$\boxed{(n-3)(n-2)}$$

<u>EXAMPLE 8:</u> **Factor: $y^2 + 7y + 14$**

<u>Solution:</u> Looking at the y^2 first, we see that the only way to split it up is $y \cdot y$. Since all the terms of the trinomial are positive, a couple of ways to split up the 14 is $14 \cdot 1$ and $7 \cdot 2$. Let's give it a try:

$$(y+14)(y+1) = y^2 + y + 14y + 14 = y^2 + 15y + 14 \quad ☹$$

$$(y+7)(y+2) = y^2 + 2y + 7y + 14 = y^2 + 9y + 14 \quad ☹$$

No luck yet; let's reverse the order of the factors:

$$(y+1)(y+14) = y^2 + 14y + y + 14 = y^2 + 15y + 14 \quad ☹$$

$$(y+2)(y+7) = y^2 + 7y + 2y + 14 = y^2 + 9y + 14 \quad ☹$$

Nothing but sad faces, and we've tried every possible arrangement. This can mean only one thing — that there's no way to factor the expression $y^2 + 7y + 14$ — the expression is

$$\boxed{\text{Not factorable}}$$

EXAMPLE 9: **Factor: $w^2 - 25$**

Solution: This quadratic expression has only two terms, but which one is missing? When a quadratic starts with w^2, we expect the next term to contain a w (which is w^1). So it's the middle term that is missing. But the middle term is not the one we focus on anyway, so let's begin the usual process. Notice that the fact that the last term is negative means that it must split into one positive factor and one negative factor.

$(w + 25)(w - 1) = w^2 - w + 25w - 25 = w^2 + 24w - 25$ ☹

$(w - 25)(w + 1) = w^2 + w - 25w - 25 = w^2 - 24w - 25$ ☹

$(w + 1)(w - 25) = w^2 - 25w + w - 25 = w^2 - 24w - 25$ ☹

$(w - 1)(w + 25) = w^2 + 25w - w - 25 = w^2 + 24w - 25$ ☹

$(w + 5)(w - 5) = w^2 - 5w + 5w - 25 = w^2 - 25$ ☺

Therefore, the expression $w^2 - 25$ factors into

$$(w + 5)(w - 5)$$

EXAMPLE 10: **Factor: $9u^2 + 12u + 4$**

Solution: This is quite a problem. We need to notice that $9u^2$ can be split in two ways: $(9u)(u)$ and $(3u)(3u)$. And worse, the 4 can also be split in two ways: (4)(1) and (2)(2). And even worse, the order in which we arrange the factors of 4 matters, too. The only good news is that all the terms of the trinomial are positive.

Let's start the multiplications, the first three using $(9u)(u)$:

$(9u + 4)(u + 1) = 9u^2 + 9u + 4u + 4 = 9u^2 + 13u + 4$ ☹

$(9u + 1)(u + 4) = 9u^2 + 36u + u + 4 = 9u^2 + 37u + 4$ ☹

$(9u + 2)(u + 2) = 9u^2 + 18u + 2u + 4 = 9u^2 + 20u + 4$ ☹

Now we'll use $(3u)(3u)$:

$$(3u + 4)(3u + 1) = 9u^2 + 3u + 12u + 4 = 9u^2 + 15u + 4$$ ☹

$$(3u + 1)(3u + 4) = 9u^2 + 12u + 3u + 4 = 9u^2 + 15u + 4$$ ☹

$$(3u + 2)(3u + 2) = 9u^2 + 6u + 6u + 4 = 9u^2 + 12u + 4$$ ☺

Eureka! The factorization of $9u^2 + 12u + 4$ is $(3u + 2)(3u + 2)$, which we can write more succinctly as

$$\boxed{(3u+2)^2}$$

EXAMPLE 11: **Factor: $a^2 + 49$**

Solution: This looks simple enough. Let's start with the most obvious choice: $49 = 7 \times 7$

$$(a + 7)(a + 7) = a^2 + 7a + 7a + 49 = a^2 + 14a + 49$$ ☹

Hey, that didn't work. Let's try 49×1:

$$(a + 49)(a + 1) = a^2 + a + 49a + 49 = a^2 + 50a + 49$$ ☹

How can we arrange the factors of 49 so that there's no middle term when the factors are double distributed? How about one of each sign:

$$(a + 7)(a - 7) = a^2 - 7a + 7a - 49 = a^2 - 49$$

The middle term's gone, but the 49 is the wrong sign.

None of our attempts panned out. As we saw before, not every expression can be factored. So we say that $a^2 + 49$ is

$$\boxed{\text{Not factorable}}$$

First Notice: Even though $a^2 + 49$ is not factorable, $a^2 - 49$ is, since $a^2 - 49 = (a + 7)(a - 7)$. The difference between a plus sign and a minus sign makes all the difference in the world, so be careful!

Second Notice: Some students jump to the conclusion that when two terms are separated by a plus sign, the expression is not factorable. Consider $4x^2 + 16$. It may not factor with two sets of parentheses, but it does have a common factor of 4, which can be factored out to produce $4(x^2 + 4)$. So $4x^2 + 16$ is factorable.

Homework

13. Factor each expression:

a. $2x^2 + 3x + 1$ b. $3n^2 - 7n + 2$ c. $5a^2 + 3a - 2$

d. $3m^2 - 11m - 20$ e. $4x^2 - 3x - 1$ f. $6u^2 + 7u - 10$

g. $4z^2 - 4z - 3$ h. $6y^2 - 5y - 6$ i. $7n^2 - 45n + 18$

14. Factor each expression:

a. $x^2 + 5x + 6$ b. $x^2 - 5x + 6$ c. $x^2 - 5x - 6$

d. $x^2 + 5x - 6$ e. $n^2 + 10n + 9$ f. $z^2 - 4z - 5$

g. $t^2 - 20t + 96$ h. $u^2 - 6u - 16$ i. $Q^2 + 34Q - 72$

15. Factor each expression:

a. $x^2 + 8x + 16$ b. $y^2 - 10y + 25$ c. $a^2 + 18a + 81$

d. $b^2 - 20b + 100$ e. $4z^2 + 4z + 1$ f. $9n^2 - 24n + 16$

g. $25x^2 - 30x + 9$ h. $x^2 + 6x + 36$ i. $2t^2 + 33t + 100$

16. Factor each expression:

a. $p^2 - 1$ b. $c^2 - 4$ c. $R^2 - 16$ d. $z^2 - 36$

e. $x^2 - 25$ f. $y^2 - 81$ g. $n^2 - 10$ h. $w^2 + 16$

i. $a^2 - 144$ j. $e^2 - 72$ k. $m^2 + 100$ l. $W^2 - 1$

17. Factor each expression:

a. $4x^2 - 9$ b. $9y^2 - 49$ c. $u^2 - 2$

d. $v^2 + 1$ e. $16z^2 - 49$ f. $49w^2 - 16$

g. $49a^2 - 144$ h. $121b^2 - 64$ i. $9x^2 + 25$

j. $1 - x^2$ k. $16 - n^2$ l. $25 - 4g^2$

m. $9 + t^2$ n. $144N^2 - 169$ o. $225a^2 - 1$

18. Factor each expression:

a. $3x^2 + 10x - 8$ b. $t^2 - 121$ c. $y^2 + 10y + 25$

d. $16a^2 - 121$ e. $b^2 - 20$ f. $n^2 + 121$

g. $x^2 + 3x + 1$ h. $12q^2 - 23q + 5$ i. $6a^2 - 13a + 6$

j. $x^2 + 14x + 13$ k. $4y^2 - 49$ l. $9Q^2 + 12Q + 4$

m. $25z^2 - 10z + 1$ n. $16x^2 + 34x - 15$ o. $16x^2 + 118x - 15$

p. $16x^2 - 77x - 15$ q. $16x^2 - 72x + 45$ r. $16a^2 - 8a + 1$

s. $x^2 + 7x + 5$ t. $8c^2 + 2c - 21$ u. $8c^2 - 13c - 21$

Practice Problems

19. Factor each expression:

a. $4x + 12$ b. $9x - 9$ c. $7y^2 + 13y$

d. $2n^2 + 8n$ e. $10w^2 - 25w$ f. $8x + 11$

g. $-x + 3$ h. $14x^2 + 21x + 28$ i. $10n^2 + 10n$

20. Factor each expression:

a. $3x - 12$ b. $9x + 9$ c. $7y^2 - 14y$

d. $2n^2 - 10n$ e. $10w^2 + 45w$ f. $8x + 13$

g. $-x - 4$ h. $14n^2 - 21n + 35$ i. $20n^2 - 20n$

21. Factor each expression:

a. $x^2 + 17x + 72$ b. $y^2 - 9y + 8$ c. $N^2 + 100$

d. $N^2 - 100$ e. $x^2 - 18x + 81$ f. $a^2 + 10a + 25$

g. $t^2 + 4t - 45$ h. $a^2 - 21a - 22$ i. $a^2 - 9a - 22$

j. $2x^2 - 9x - 5$ k. $6x^2 + x - 40$ l. $6x^2 - 13x - 15$

m. $6x^2 + 11x - 17$ n. $R^2 - 144$ o. $25n^2 - 30n + 9$

p. $T^2 + 144$ q. $49w^2 + 70w + 25$ r. $9a^2 - 9a + 2$

s. $9a^2 - 19a + 2$ t. $x^2 + 16x - 36$ u. $x^2 + 37x + 36$

v. $30c^2 - 11c + 1$ w. $16a^2 - 6a - 1$ x. $36q^2 + 60q + 25$

y. $x^2 + 8x + 18$ z. $32n^2 - 66n + 27$

22. Solve for x: $a(x - 2) = b(c - x)$

23. Factor: $25y^2 - 49$

24. Factor: $4x^2 + 25$

25. Factor: $9n^2 + 6n + 1$

26. Factor: $14x^2 - 43x + 3$

Solutions

1. Here's what I would do. First, the given expression, $20x + 30y$, consists of two terms, and the result, $10(2x + 3y)$, consists of one term. Since factoring is the process of converting two or more terms into a single term, so far so good. Moreover, if I take my answer, $10(2x + 3y)$, and distribute to remove the parentheses, I will get $20x + 30y$, the original problem. I hope your buddy is now convinced.

2. While it may be true that the original expression consists of two terms, and her answer consists of one term, there's still one big problem. Ask her to take her answer, $6(w + 3z)$, and distribute it to remove the parentheses. She will get $6w + 18z$, which is <u>not</u> equal to the original problem. Therefore, her factorization can't possibly be right.

3. a. $x + z$ b. $P - Q$ c. $x - 4$ d. $2y - 3t$
e. $u + 1$ f. $n - 4$ g. $a - b$ h. $c + d$
i. $x + 2y - 4z$ j. $w - u + z$ k. $2x - 3$ l. $2a^2 + 3a - 4$

4. a. $3(P + Q)$ b. $9(n - 3)$ c. $n(c + d)$
d. $x(w - y)$ e. $7(t - 1)$ f. $x(1 + y)$
g. $-2(4L - 5)$ h. $-b(a + c)$ i. $-(u + 5)$
j. $-(z + x - 10)$ k. $2(x + y + z)$ l. $5(a - 2b + 3c)$

5. a. $a + 2b$ b. $3u - 1$ c. $Q - 3R$ d. $3x + 2$
e. $y - 2$ f. $2a + 3b$ g. $t + 4$ h. $3w - 4z$
i. $5a - 4$ j. $y + 1$

6. a. $cn + dn = 3 \Rightarrow n(c+d) = 3 \Rightarrow \frac{n\cancel{(c+d)}}{\cancel{c+d}} = \frac{3}{c+d} \Rightarrow n = \frac{3}{c+d}$

b. $n = \frac{d}{a-c}$

c. $n = \frac{M}{L+1}$

d. $tn = c - sn \Rightarrow tn + sn = c \Rightarrow n(t+s) = c \Rightarrow n = \frac{c}{t+s}$

e. $n = \dfrac{3}{r-t}$

f. $m(n+1) - Qn - R = 0 \Rightarrow mn + m - Qn - R = 0$
$\Rightarrow mn - Qn + m - R = 0 \Rightarrow mn - Qn = R - m$
$\Rightarrow n(m-Q) = R - m \Rightarrow n = \dfrac{R-m}{m-Q}$

g. $a(n+3) + b(n+c) = R \Rightarrow an + 3a + bn + bc = R$
$\Rightarrow an + bn = R - 3a - bc \Rightarrow n(a+b) = R - 3a - bc$
$\Rightarrow n = \dfrac{R-3a-bc}{a+b}$

h. $n(a+b+c) = d \Rightarrow n = \dfrac{d}{a+b+c}$

i. $n(a-1) = a \Rightarrow n = \dfrac{a}{a-1}$

j. $2n + 2 + an = c \Rightarrow n(2+a) = c - 2 \Rightarrow n = \dfrac{c-2}{a+2}$

7. a. $x = \dfrac{14}{c+7}$ b. $x = \dfrac{w+v}{r-u}$ c. $x = \dfrac{w}{w-1}$
d. $x = \dfrac{c+b}{a+d}$ e. $x = \dfrac{w+au}{u+1}$ f. $x = \dfrac{b-a}{a+b}$
g. $x = \dfrac{z+u-3w}{m-3}$ h. $x = \dfrac{2q+3p}{p-q}$

i. $x = \dfrac{-b-ac}{c-a}$, or, upon multiplying top and bottom by –1, $\dfrac{b+ac}{a-c}$

j. $x = \dfrac{R-Q-2c-ab}{-a-c}$, or, upon multiplying top and bottom by –1,
$\dfrac{Q+2c+ab-R}{a+c}$

8. a. $2x^2 + 7x - 4$ b. $6n^2 - 21n + 15$ c. $3a^2 + 26a - 9$
d. $12y^2 + 40y - 7$ e. $4m^2 - 49$ f. $16w^2 + 40w + 25$
g. $25x^2 - 1$ h. $14n^2 + n - 30$ i. $49u^2 - 42u + 9$
j. $144a^2 - 169$

9. a. T b. T c. F d. T e. T f. F g. T h. F

10. a. 3 b. 4 c. 6 d. 8 e. 7 f. 1 g. 2 h. 5

Ch 12 – Factoring, Part I

11. a. $x - 2$ b. $n - 2$ c. $a - 8$ d. $q - 7$ e. $c + 3$

12. a. 5, 3 b. 3, 3 c. 1, 1 d. 4, 4 e. 7, 1
f. 7, 7 g. 15, 15 h. 3, 3 i. 1, 8

13. a. $(2x + 1)(x + 1)$ b. $(3n - 1)(n - 2)$ c. $(5a - 2)(a + 1)$
d. $(3m + 4)(m - 5)$ e. $(4x + 1)(x - 1)$ f. $(6u - 5)(u + 2)$
g. $(2z + 1)(2z - 3)$ h. $(2y - 3)(3y + 2)$ i. $(7n - 3)(n - 6)$

14. a. $(x + 3)(x + 2)$ b. $(x - 2)(x - 3)$ c. $(x - 6)(x + 1)$
d. $(x + 6)(x - 1)$ e. $(n + 9)(n + 1)$ f. $(z - 5)(z + 1)$
g. $(t - 12)(t - 8)$ h. $(u - 8)(u + 2)$ i. $(Q + 36)(Q - 2)$

15. a. $(x + 4)^2$ b. $(y - 5)^2$ c. $(a + 9)^2$
d. $(b - 10)^2$ e. $(2z + 1)^2$ f. $(3n - 4)^2$
g. $(5x - 3)^2$ h. Not factorable i. $(2t + 25)(t + 4)$

16. a. $(p + 1)(p - 1)$ b. $(c + 2)(c - 2)$ c. $(R + 4)(R - 4)$
d. $(z + 6)(z - 6)$ e. $(x + 5)(x - 5)$ f. $(y + 9)(y - 9)$
g. Not factorable h. Not factorable i. $(a + 12)(a - 12)$
j. Not factorable k. Not factorable l. $(W + 1)(W - 1)$

17. a. $(2x + 3)(2x - 3)$ b. $(3y + 7)(3y - 7)$ c. Not factorable
d. Not factorable e. $(4z + 7)(4z - 7)$ f. $(7w + 4)(7w - 4)$
g. $(7a + 12)(7a - 12)$ h. $(11b + 8)(11b - 8)$ i. Not factorable
j. $(1 + x)(1 - x)$ k. $(4 + n)(4 - n)$ l. $(5 + 2g)(5 - 2g)$
m. Not factorable n. $(12N + 13)(12N - 13)$ o. $(15a + 1)(15a - 1)$

18. a. $(3x - 2)(x + 4)$ b. $(t + 11)(t - 11)$ c. $(y + 5)^2$
d. $(4a + 11)(4a - 11)$ e. Not factorable f. Not factorable
g. Not factorable h. $(3q - 5)(4q - 1)$ i. $(2a - 3)(3a - 2)$
j. $(x + 1)(x + 13)$ k. $(2y + 7)(2y - 7)$ l. $(3Q + 2)^2$
m. $(5z - 1)^2$ n. $(8x - 3)(2x + 5)$ o. $(8x - 1)(2x + 15)$
p. $(16x + 3)(x - 5)$ q. $(4x - 3)(4x - 15)$ r. $(4a - 1)^2$
s. Not factorable t. $(4c + 7)(2c - 3)$ u. $(8c - 21)(c + 1)$

19. a. $4(x + 3)$ b. $9(x - 1)$ c. $y(7y + 13)$
d. $2n(n + 4)$ e. $5w(2w - 5)$ f. Not factorable
g. $-(x - 3)$ h. $7(2x^2 + 3x + 4)$ i. $10n(n + 1)$

20. a. $3(x-4)$ b. $9(x+1)$ c. $7y(y-2)$
d. $2n(n-5)$ e. $5w(2w+9)$ f. Not factorable
g. $-(x+4)$ h. $7(2n^2-3n+5)$ i. $20n(n-1)$

21. a. $(x+9)(x+8)$ b. $(y-1)(y-8)$ c. Not factorable
d. $(N+10)(N-10)$ e. $(x-9)^2$ f. $(a+5)^2$
g. $(t+9)(t-5)$ h. $(a-22)(a+1)$ i. $(a-11)(a+2)$
j. $(2x+1)(x-5)$ k. $(3x+8)(2x-5)$ l. $(6x+5)(x-3)$
m. $(6x+17)(x-1)$ n. $(R+12)(R-12)$ o. $(5n-3)^2$
p. Not factorable q. $(7w+5)^2$ r. $(3a-1)(3a-2)$
s. $(9a-1)(a-2)$ t. $(x+18)(x-2)$ u. $(x+36)(x+1)$
v. $(5c-1)(6c-1)$ w. $(8a+1)(2a-1)$ x. $(6q+5)^2$
y. Not factorable z. $(16n-9)(2n-3)$

22. $x = \dfrac{bc+2a}{a+b}$

23. $(5y+7)(5y-7)$

24. Not factorable

25. $(3n+1)^2$

26. $(14x-1)(x-3)$

"Ninety-nine percent of the failures come from people who have the habit of making excuses."

– George Washington