-Addendum-

0	1	2	3	4
5	6	7	8	9
10	11	12	13	14
15	16	17	18	19
20	21	22	23	24
25	26	27	28	29
30	31	32	33	34
35	36	37	38	39
40	41	42	43	44

Ch 0 – Prologue

IN A NUTSHELL

A. Repeating (and thus terminating) decimals are Rational Numbers.
Non-repeating decimals are Irrational Numbers.
The Real Numbers are the Rationals and Irrationals combined.
$\sqrt{-9}$ is not a real number. Equivalently, $\sqrt{N}$ is a real number only if $N \geq 0$.

B. The opposite of N is $-N$.
Every real number has an opposite.
The sum of a number and its opposite is 0.

C. The reciprocal of N is $\frac{1}{N}$.
Every real number except 0 has a reciprocal.
The product of a number and its reciprocal is 1.

D. The absolute value of a non-negative number is itself. The absolute value of a negative number is its opposite.

E. $\frac{ax+b}{c} = \frac{a}{c}x + \frac{b}{c}$

F. The even power of a negative number is positive.
The odd power of a negative number is negative.

G. Order of Operations: Parentheses, Exponents, Multiply/Divide (left to right), Add/Subtract (left to right)

H. Division <u>by</u> zero is UNDEFINED: $\frac{0}{9} = 0$ $\quad \frac{5}{0} =$ Undefined $\quad \frac{0}{0} =$ Undefined

I. If *a* and *b* are the legs of a right triangle and *c* is its hypotenuse, then

$$a^2 + b^2 = c^2$$

J. Inequalities and Intervals

Inequality	Interval
$x > 4$	$(4, \infty)$
$x \geq -2$	$[-2, \infty)$
$x < 0$	$(-\infty, 0)$
$x \leq 13$	$(-\infty, 13]$
$-5 < x \leq 2$ which means $x > -5$ AND $x \leq 2$, which means that x is *between* -5 and 2, excluding the -5, but including the 2.	$(-5, 2]$
$x \leq -1$ OR $x > 3$	$(-\infty, -1] \cup (3, \infty)$

Frequently Asked Questions

Q: Regarding the issue of dividing by 0, my teacher told me that a fraction like $\frac{5}{0}$ is actually equal to *infinity*. So, is $\frac{5}{0}$ *undefined*, or is it *infinity*?

A: You be the judge. Let's see happens when we divide 5 by smaller and smaller (positive) numbers:

$\frac{5}{2}$	$\frac{5}{1}$	$\frac{5}{0.2}$	$\frac{5}{0.01}$	$\frac{5}{0.003}$	$\frac{5}{0.00005}$
2.5	5	25	500	1,667.7	1,000,000

First note that the denominators (the numbers we are dividing into 5) are getting smaller and smaller (kind of "approaching 0"). What's happening to the quotient (the result of the division)? Can you see that it's getting bigger and bigger? So, if we were to let the denominator get smaller and smaller (teeny-tiny), you might see that the quotient is growing without bound; that is, it's approaching infinity. In short, the smaller the bottom, the larger the fraction.

In summary, if we look at $\frac{5}{0}$ as asking "What number times 0 would make 5?", there's no answer (anything times 0 is 0, never 5): It's UNDEFINED. But if we can appreciate the table above — letting the bottom get infinitely small, becoming essentially 0 — we can see that the fraction is approaching INFINITY. So, who knows?

Later in the course, we'll summarize this whole thing like this:

> Consider the function $f(x) = \frac{5}{x}$.
>
> As $x \to 0$ (from the right), $f(x) \to \infty$.

PROGRAMMING

A. Consider the problem: *The legs of a right triangle are 7 and 10. Find the hypotenuse.* Can we program a solution to solve this problem? Yes, we can, and it takes only two lines of code.

```
c = SQRT(7^2 + 10^2)
Print c
```

The term SQRT in this program is called a built-in ***<u>function</u>*** of the programming language.

Try editing the program to solve the following problem: *The user inputs the two legs of a right triangle, and the program outputs the hypotenuse.*

[Hint: Place a new line of code at the top allowing the user to input the two legs; then replace the 7 and the 10 in the calculation of **c** with the variables you chose for the Input statement.]

B. What two values of *N* will make the following program die?

```
a = 1/N + 7
b = 1/(N + 7)
```

The answers to these two questions are located below.

To ∞ and Beyond

A ***Pythagorean Triple*** is a triple of numbers, written (a, b, c), such that

$$a^2 + b^2 = c^2$$

a. Prove that (3, 4, 5) is a Pythagorean Triple. Is (4, 3, 5) a Pythagorean Triple? How about (5, 4, 3)?

b. Consider the triple of numbers (6, 8, 10), obtained from (3, 4, 5) by multiplying each of the numbers by 2. Prove that (6, 8, 10) is also a Pythagorean Triple.

c. Suppose (a, b, c) is a Pythagorean Triple. Prove that (ka, kb, kc) is also a Pythagorean Triple (where k represents any whole number ≥ 2). You must keep k a variable to make the proof hold for any number.

d. Find a Pythagorean Triple which is not a multiple of (3, 4, 5).

Solutions to Programming Problems –

A.
```
Input a, b
 c = SQRT(a^2 + b^2)
Print c
```

B. N = 0 and –7

Ch 1 – Intro to Graphing

DESMOS GRAPHING

Use DESMOS to graph each of the following on the same grid. Be sure to scale the axes (just drag your mouse to move around; move your mouse roller up and down to zoom in and out) so that whatever you want me to see actually shows up.

1) Line

2) Parabola opening up or down

3) Parabola opening left or right

4) Circle

Note: You may have to do a little research (books, friends, internet) to obtain these graphs. For this activity, that's not cheating because I haven't yet taught you about these graphs.

When you print out your graphs (click the arrow to the right of the Sign Up button), two pages should appear: one with the four graphs and the other with the four formulas. Scan the two pages into a single PDF (Adobe Scan for phones), and email me with your PDF attached.

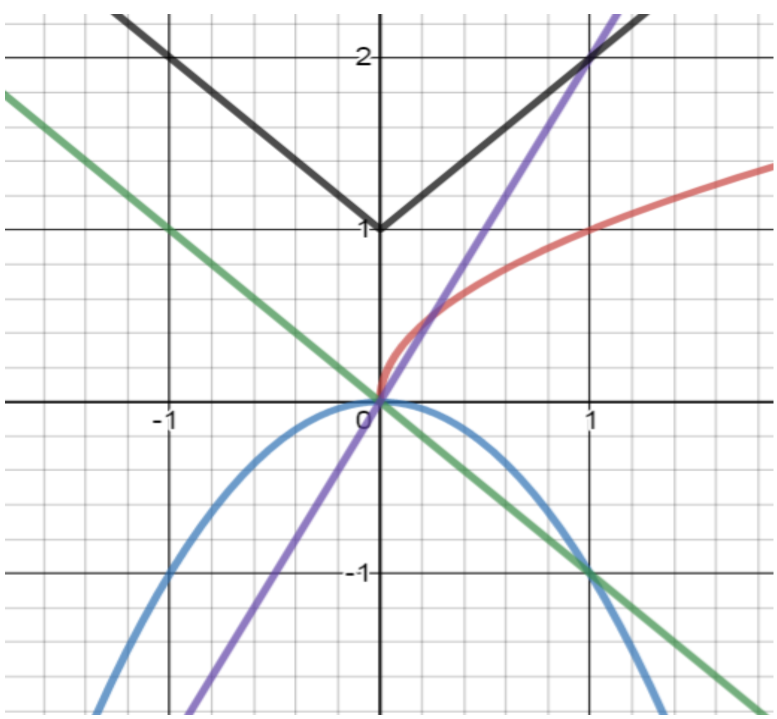
Ch 2 – From Graph to Equation

FREQUENTLY ASKED QUESTIONS

Q: In other classes, my teacher always gave us an equation and asked us to graph it. Why does this chapter do it the other way around?

A: Because in the real world (business, science, computers, etc.), the usual sequence of event is *a*) perform an experiment, *b*) collect data, *c*) plot (graph) the data, *d*) come up with an equation (formula) that best describes that data, and then *e*) use that equation to predict values that were never in your data.

MISCELLANEOUS



BLUE: ______________________________

GREEN: ______________________________

PURPLE: ______________________________

BLACK: ______________________________

RED: ______________________________

Ch 3 – From Equation to Graph

DESMOS GRAPHING

A. Graph each of the following on the same grid, using any graphing program you like:

$y = x^2$ $\quad y = x^2 + 3$ $\quad y = x^2 - 5$

$y = (x + 2)^2$ $\quad y = (x - 4)^2$

B. WITHOUT graphing, answer the following question (within the brackets, circle your choice):

The graph of $y = (x + 3)^2 - 7$ is the graph of $y = x^2$ shifted [left / right] by ____ units and shifted [up / down] by ____ units.

C. WITHOUT graphing, answer the following question:

The graph of $y = \sqrt{x-5} + \pi$ is the graph of $y = \sqrt{x}$ shifted [left / right] by ____ units and shifted [up / down] by ____ units.

D. WITHOUT graphing, answer the following question:

The graph of $y = |x + 2| + 100$ is the graph of $y = |x|$ shifted [left / right] by ____ units and shifted [up / down] by ____ units.

E. 1. Graph $y = \sqrt{x}$.

2. On the same grid, graph $y = -\sqrt{x}$.

3. Find a single formula whose graph will be the union of the two graphs (that is, the same as the two graphs combined).

PROGRAMMING

Write a program that will graph the function $y = x^2 + 7$. Let x take on the values from –10 to +10. Let's assume that our programming language has a built-in function called PLOT, which takes two inputs, the x-coordinate and the y-coordinate, and then plots a dot at the point (x, y) in the x-y plane. For example, the statement PLOT(4, –5) will plot a dot in the plane at the point (4, –5). So, consider this:

```
1   For x = -5 to 5
2   y = x^2 + 7
3       PLOT(x,y)
4   Next x
```

<u>Line 1:</u> The values of x go from –5 to 5; the default increment is 1;
i.e., x = –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5

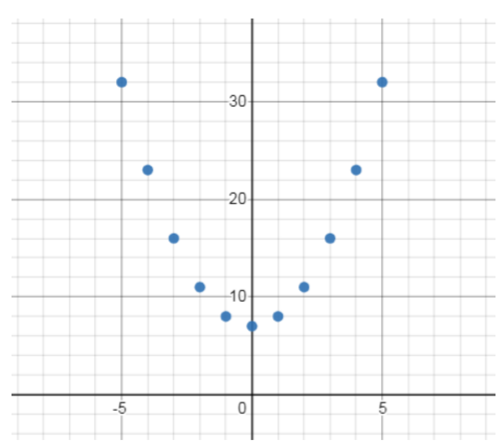
<u>Line 2:</u> y is calculated for the current x-value.

<u>Line 3:</u> The point is plotted.

<u>Line 4:</u> On to the next value of x. Once x exceeds 5, the For/Next loop is done, and the program ends.

The output of this program will be dots plotted at the points

(–5, 32), (–4, 23), . . . , up to and including (5, 32).



Notice that our computer program doesn't really graph a continuous curve, just 11 discrete points. BTW, do you see how going from –5 to 5 (counting by 1's) will yield 11 points?

Ch 4 – Equations and Inequalities: a Graphical Approach

DESMOS GRAPHING

Solve each equation:

A. $\sin x = x$

B. $\frac{1}{x} = x$

C. $\cos x = 2$

D. $x^3 = -12x + 16$
[Round to 2 digits]

Ch 5 – Midpoint, Distance, Intercepts, and Slope

IN A NUTSHELL

A. Midpoint on the Line

The midpoint of the segment connecting the points a and b on a line is $\dfrac{a+b}{2}$.

B. Midpoint in the Plane

The midpoint of the segment connecting the points (a, b) and (c, d) in the plane is $\left(\dfrac{a+b}{2}, \dfrac{c+d}{2}\right)$.

C. Distance on the Line

The distance between the points a and b on a line is $|a-b|$.

D. Distance in the Plane

Create a right triangle, and then calculate the hypotenuse.

E. Intercepts

1) x-int: Set $y = 0$

2) y-int: Set $x = 0$

F. Slope: $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{\text{change in } y}{\text{change in } x} = \dfrac{\Delta y}{\Delta x}$

PROGRAMMING 

First, evaluate the following expression, remembering the Order of Operations: **`10 + 6 / 2`**

If your answer was 8, you made a classic error; do it again until you understand that the answer is 13.

How could you rewrite the problem so that the addition <u>would</u> be done first? Answer: Put parentheses around the sum: **`(10 + 6) / 2 = 8`**

Programming problem **#1**: *Write a program that will ask the user for two points on the x-axis, and then output the <u>midpoint</u> of the line segment connecting those two points.*

```
Input x1, x2
m = (x1 + x2)/2
Print m
```

Notice the critical need for the parentheses.

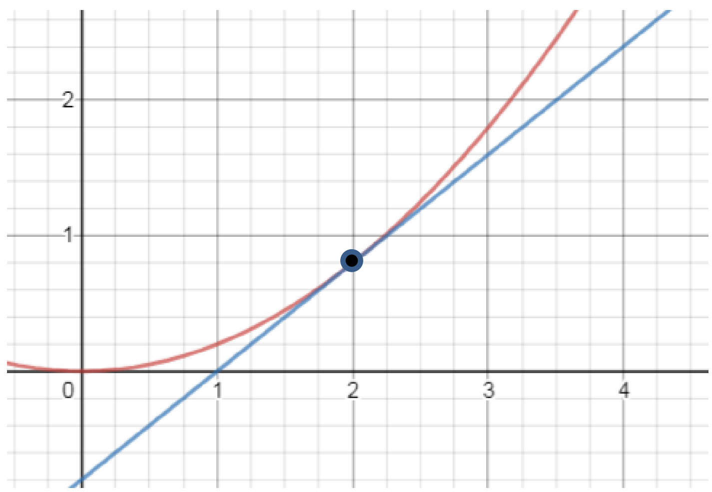
Programming problem **#2**: *Write a program that will ask the user for two points in the plane, and then output the <u>midpoint</u> of the line segment connecting those two points.* Your job is to fill in the three blanks.

The variables we'll use: **`(x1,y1)`** for the first point, **`(x2,y2)`** for the second point, **`midx`** for the *x*-coordinate of the midpoint, and **`midy`** for the*y*-coordinate of the midpoint.

`Input x1,y1`	Input the first point.
____________________	Input the second point.
`midx = (x1 + x2)/2`	Calculate the *x*-coordinate of the midpoint.
____________________	
`Print` ________________	

THE CALCULUS CORNER

I hope it's clear that the *slope* of a line is the same at every point on the line. But what about the slope of a nonlinear *curve* at some point? What does slope at a point even mean? We agree it means the slope of the ***tangent line*** at that point. But what's a tangent line at a point? The following definition is loose, but it's a line that touches the curve at that point, *but stays on the same side of the curve*, at least near that point. Here's an example:



The blue line is *tangent* to the red curve at the point (2, 0.8). Notice that the tangent line touches the curve at (2, 0.8), but always stays on the same side of the curve (in this case, below the curve).

Click *DESMOS* and then do the following:

First, click the Play Button on Equation #3. Is it clear what is meant by "the tangent line to a curve at a point," and also that the slope of that tangent line changes from point to point? Click the Stop Button .

Second, drag the point (the blue dot on the curve) to the point (1, 1). Now click the tangent line at any other point on it and write down the coordinates of that point. Using that point, the point (1, 1), and $m = \dfrac{\Delta y}{\Delta x}$, estimate the slope of the tangent line.

Third, drag the blue dot on the curve to the point (3, 9). Now click the tangent line at any other point on it and write down the coordinates of that point. Using that point and the point (3, 9), estimate the slope of the tangent line.

Fourth, do the same thing at the point (–4, 16).

Fifth, organize your results in the following table:

Point of Tangency	Another Point on the Tangent Line	Slope of the Tangent Line
(1, 1)		
(3, 9)		
(–4, 16)		

Sixth, use the data in the table to *predict* the slope of the tangent line at the point (100, 10000).

Ch 6 – The Equation of a Line

In a Nutshell

A. The equation of the line with slope m and y-intercept $(0, b)$ is given by

$$y = mx + b$$

B. If two lines are <u>parallel</u>, then their slopes are *equal*.

C. If two lines are <u>perpendicular</u>, then their slopes are *opposite reciprocals* of each other.

D. The equation of the line with slope m and passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = m(x - x_1)$$

Ch 7 – Break-Even Point, Linear Functions

In a Nutshell

Profit = Revenue – Cost

Break-Even Point:

either Revenue = Cost

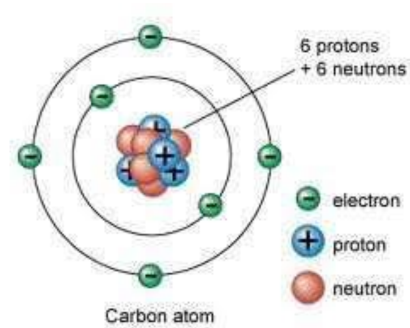
or, Profit = 0

Ch 9 – Linear Modeling

To ∞ and Beyond

Linear modeling works well for things that are … well … linear. The following are a couple of notions from the fields of chemistry and physics which lead to non-linear models:

A. The electrons which revolve around the nucleus of an atom do so in well-defined energy levels. Any given electron must be in one energy level or another; it cannot be in between two of them. [In fact, when electrons change their energy levels, they may absorb or emit energy.] The following table shows the <u>maximum</u> number of electrons that can be in any given energy level:



Energy Level	Maximum # of Electrons
1	2
2	8
3	18
4	
5	
6	
n	

Fill in all the blank cells of the table.

Note that the last entry must be a formula with an n in it.

Use Desmos to graph the formula you got, and confirm that it's NOT linear.

Last, use your formula in the last cell to predict the maximum number of electrons that could be in energy level 100. [Even though there's no known element in the universe that could have that many energy levels].

B. When you drop an object in a gravitational field, the object does not fall at the same distance in equal time periods. The following table shows the distance that the object has fallen from the moment it's been dropped (assuming no resistance):

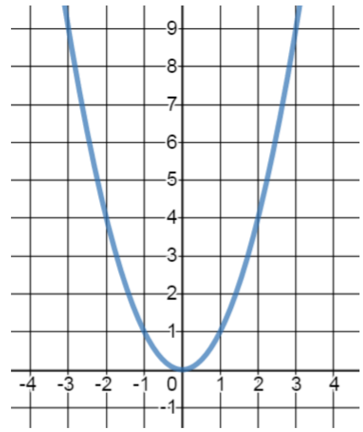
Time (in seconds)	Distance (in meters)
0	0
1	5
2	20
3	45
4	80
5	
6	
t	

Fill in all the blank cells of the table.

Use your formula in the last cell to predict the distance that the object has fallen after 100 seconds (assuming, of course, that it doesn't hit the ground).

Ch 10 – Maximizing Company Profit

THE CALCULUS CORNER



If you look back at The Calculus Corner in Chapter 5 of this Addenda, you may have noticed that the function we worked with in Desmos was

$$y = x^2$$

And the conclusion that you might have drawn from that problem was that the <u>slope of the tangent line</u> at any point (x, y) on the curve was given by

$m = 2x$ [The slope is always <u>twice</u> the x-coordinate.]

So, for example, the slope of the tangent line at the point (15, 225) is given by 2(15) = <u>30</u>.

What about the slope at the origin? Looking at the graph, would you agree that the tangent line is horizontal? [In fact, it's just the x-axis.] And we learned in Chapter 8 – Special Lines that the slope of any horizontal line is **0**. Does our slope formula, $m = 2x$, work at the origin?

$m = 2(0) = \mathbf{0}$ It does!

Now we'll skip a couple of steps (which you'll learn if you take Business Calculus) and state the formula that will be relevant to Chapter 10 of the book:

The function

$$y = ax^2 + bx + c$$

has the following *slope* formula:

$$m = 2ax + b$$

Here's a question for you: For the graph $y = 7x^2 - 20x + 92$,

a. What is the slope function?

b. What is the slope of the tangent line at the point (10, 592)?

Ch 11 – Polynomials

FREQUENTLY ASKED QUESTIONS

Q: It's stated in the Introduction to Ch 11 that the following is a polynomial because "all the exponents on the *x* are whole numbers":

$$3x^5 - \pi x^3 + x^2 - 9x + \frac{4}{5}$$

I see that the exponents 5, 3, and 2 on the *x* are whole numbers, but I don't understand how those last two terms have that same property. In fact, the fifth term, the fraction, doesn't even have an *x* in it. So how can their exponents be whole numbers?

A: For the 4th term, consider that $-9x$ can be written $-9x^1$, and 1 is a whole number. For the 5th term, $\frac{4}{5}$, you might remember from a previous algebra course that anything to the *zero* power is 1. Thus, we can write $\frac{4}{5}$ as $\frac{4}{5}x^0$. When written that way, it's indeed the case that the *x* is raised to a whole-number exponent.

DESMOS GRAPHING

As tempting as it might be to figure that $(x + 3)^2$ and $x^2 + 9$ are the same thing, prove that they're NOT.

Ch 12 – Factoring, Part 1

Programming

When we factor — using trial-and-error — we take educated guesses until we land on the right combination of numbers. So, for example, to factor $x^2 + 10x + 24$, we try to find two numbers whose <u>sum</u> is 10 and whose <u>product</u> is 24. We eventually find that the numbers 6 and 4 do the trick:

$$x^2 + 10x + 24 = (x + 6)(x + 4)$$

So I guess if we just use the same logic, we shouldn't have much trouble factoring $n^2 + 535n + 56{,}056$. Yeah, right!

Is it possible to program a solution? If we assume that the numbers we need are whole numbers, the answer is Yes. We therefore need two numbers, call them a and b, such that

$$a + b = 535 \text{ and } ab = 56{,}056$$

Here's one way to let the computer do all the work, while we just sit back and wait for the final numbers:

```
For a = 1 to 56056
    b = 535 - a
    If a*b = 56056 Then Exit Loop
Next a
Print a, b
```

The output to this program will be 143,392. You should verify that their sum is 535 and their product is 56,056.

Therefore,

$$n^2 + 535n + 56{,}056 = (n + 143)(n + 392)$$

Notice that our program could factor similar trinomials by merely changing the numbers in the code.

Ch 13 – Break-Even Point, Quadratic Functions

To ∞ and Beyond

1. Show that $\dfrac{-5+\sqrt{21}}{2}$ is a solution of the quadratic equation

$$x^2 + 5x + 1 = 0$$

using **direct substitution**. Do NOT solve the equation; do not use the Quadratic Formula. Just put $\dfrac{-5+\sqrt{21}}{2}$ (not a decimal approximation) in for x, simplify everything, and confirm that it comes out to be 0.

2. Find the three x-intercepts of $y = x^3 - 39x + 70$.

Hint: One factor of $x^3 - 39x + 70$ is $x - 2$, so if you use long division, you can find the other factor.

]

3. Use the same technique as #2 to find the three x-intercepts of

$$y = x^3 + 3x^2 - x - 3.$$

<u>Note:</u> Unlike #2, I'm not giving you one of the factors, so you need to experiment until you find one that leaves a 0 remainder.

Ch 14 – Inequalities and Absolute Value Equations

TO ∞ AND BEYOND

A. Marty was trying to solve the inequality $ax + b > c$ for x, and wrote

$$ax + b > c$$

$$\Rightarrow \quad ax > c - b$$

$$\Rightarrow \quad x > \frac{c-b}{a}$$

Explain the fallacy (logical error) in Marty's reasoning.

B. Solve for x: $\left|3x - 5\right| \geq 7$

C. Solve for n: $\left|-5n + 1\right| < 8$

Ch 15 – Variation

To ∞ and Beyond

1. Suppose that the area, *A*, varies directly as the square of the radius, *r*, and further suppose that the radius varies directly as the time, *t*.

   Prove that *A* varies directly as the square of *t*.

2. Assume that *Y* is directly proportional to the cube of *X*, and also that *X* is directly proportional to the square of *T*.

   Prove that *Y* is directly proportional to the sixth power of *T*.

3. The force of gravity, *F*, between two objects varies directly as the product of their masses, m_1 and m_2, and inversely as the square of the distance, *r*, between the objects.

   a) Using *G* as the constant of variation, write a formula which describes this situation.

   b)

Ch 16 – Motion Problems

GETTING READY

#1: A bike leaves the theatre at 1 pm traveling at a rate of 10 mph. At 2 pm a scooter traveling at a rate of 12 mph leaves the theatre in pursuit of the bike. At what time will the scooter catch up with the bike?

Time	Bike Distance (mi)	Scooter Distance (mi)
1 pm	0	0
2 pm	10	0
3 pm		12
4 pm		
5 pm		
6 pm		
7 pm		
8 pm		
9 pm		
10 pm		

Conclusion: The scooter catches up with the bike at _____ pm. The bike traveled for a total of _____ hours and the scooter _____ hours. Each of them traveled a distance of _____ miles.

<u>#2</u>: A bike leaves the theatre at 12 pm traveling at a rate of 10 km/h. At 2 pm a scooter traveling at a rate of 12 km/h leaves the theatre in pursuit of the bike. At what time will the scooter catch up with the bike?

Time	Bike Distance (km)	Scooter Distance (km)
12 pm		
1 pm		
2 pm		
3 pm		
4 pm		
5 pm		
6 pm		
7 pm		
8 pm		
9 pm		
10 pm		
11 pm		
12 am		
1 am		
2 am		

<u>Conclusion</u>: The scooter catches up with the bike at _____ (am/pm). The bike traveled for a total of _____ hours and the scooter _____ hours. Each of them traveled a distance of _____ km.

TO ∞ AND BEYOND

Janie drove from her home to her college at an average speed of 40 mph, and returned home (same distance) at an average speed of 60 mph. What was Janie's *average speed* for the entire trip?

Ch 17 – Percent Mixture Problems

To ∞ and Beyond

A 10-liter radiator is filled with 42% antifreeze solution. How much of the antifreeze solution must be drained and replaced with pure antifreeze to bring the concentration of the final solution up to 58%?

Ch 18 – Exponents

DESMOS GRAPHING

Even though $x^2 \cdot x^3$ is a multiplication problem, and even though $2 \cdot 3 = 6$, prove that $x^2 \cdot x^3 \neq x^6$.

Ch 19 – Factoring, Part II

TO ∞ AND BEYOND

A. Factor completely: $x^6 - 1$

[You must have **four** factors in your answer.]

Ch 20 – Fractions, Part I

To ∞ and Beyond

Simplify to a single fraction: $\dfrac{\dfrac{A}{B} + \pi}{C + \dfrac{D}{Rx}}$

Ch 21 – Negative Exponents

THE CALCULUS CORNER

Recall our formula that calculates the ***slope*** of the curve

$$y = x^n$$

at any point (x, y):

$$m = nx^{n-1}$$

So suppose we want the slope formula for the function $y = \dfrac{1}{x^7}$. The form in which that formula is written does not lend itself to using our slope formula. But . . . if we first express the function as

$$y = x^{-7}$$

then we have the function written in a perfect form for the slope formula, and that slope is therefore

$$m = -7x^{-7-1} = -7x^{-8} = \boxed{\dfrac{-7}{x^8}}$$

Use our slope formula to show that the slope of the curve $y = \dfrac{1}{x^7}$ at the point (2, 128) is $m = -\dfrac{7}{256}$.

Ch 22 – Fractions, Part II

To ∞ and Beyond

Add and write your answer in reduced form: $\dfrac{x^2}{x^3-8}+\dfrac{x}{x-2}$

Ch 23 – Fractional Equations

To ∞ and Beyond

By direct substitution, prove that $x = \dfrac{-2z}{y - wz}$ is a solution of the fractional equation $\dfrac{2}{x} + \dfrac{y}{z} = w$.

Ch 24 – Radicals, Part I

FREQUENTLY ASKED QUESTIONS

Q: The book keeps talking about the *real numbers*, and that a number like $\sqrt{-9}$ does not belong to that set of numbers. I'm getting the feeling that it might be some other kind of number.

A: You're right — we do consider $\sqrt{-9}$ to be a number. Recall from the Prologue that we called $\sqrt{-9}$ an imaginary number. We start with the simplest imaginary number, $\sqrt{-1}$, and call it $\boldsymbol{i}$. By definition,

$$i = \sqrt{-1}$$

First notice that from this definition we conclude that

$$i^2 = -1$$

[Some books define $\boldsymbol{i}$ to be the number such that $i^2 = -1$.]

From the definition, $i = \sqrt{-1}$, we can evaluate $\sqrt{-9}$:

$$\sqrt{-9} \;=\; \sqrt{9 \cdot -1} \;=\; \sqrt{9}\sqrt{-1} \;=\; 3i$$

And we can evaluate $\sqrt{-50}$ thusly:

$$\sqrt{-50} \;=\; \sqrt{50 \cdot -1} \;=\; \sqrt{50}\sqrt{-1} \;=\; 5\sqrt{2}\,i$$

Notice that we simplified $\sqrt{50}$ as we've learned in this chapter. It's also imperative that you realize that the $\boldsymbol{i}$ is <u>outside</u> the radical sign. In fact, to make the answer more explicit, some books would write the answer as $5i\sqrt{2}$.

Ch 25 – More Equations

DESMOS GRAPHING

Find approximate solutions to each equation:

A. $x^3 - 3x^2 + 4x + 7 = 0$

B. $x^3 - 3x^2 + x = 0$

C. $0.5x^4 - 5x^3 - 5x^2 + 3x + 1 = 0$

D. $5x^4 + 2x^2 + 1 = 0$

E. $\sqrt{x^2 + 2x + 5} - 3 = 0$

F. $|3x + 1| + |x| - 5 = 0$

G. $\sin x + \ln x - 4 = 0$ [Give the 3 smallest solutions]

Ch 26 – Fractional Exponents

THE CALCULUS CORNER

Recall our formula that calculates the ***slope*** of the curve

$$y = x^n$$

at any point (x, y):

$$m = nx^{n-1}$$

So suppose we want the slope formula for the function $y = \sqrt[3]{x^2}$. The form in which that function is written does not lend itself to using our slope formula. But . . . if we first express the function as

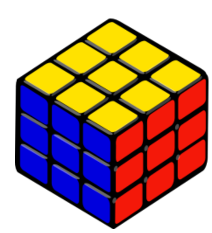
$$y = x^{2/3}$$

then we have the function written in a perfect form for calculating the slope formula, and that slope is thus

$$m = \frac{2}{3}x^{\frac{2}{3}-1} = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}} = \boxed{\frac{2}{3\sqrt[3]{x}}}$$

PROGRAMMING

Write a program that will calculate the side of a cube given its volume.



First, the geometry: If each side of a cube has length s, then the volume of the cube is given by the formula

$$V = s^3 \qquad\qquad [\text{In any box, } V = l \times w \times h]$$

The problem is, this gives us the volume if we know the length of each side (the reverse of what we're trying to program). So now we do the algebra and solve for *s* by taking the cube root of each side of the formula:

$$s = \sqrt[3]{V}$$

But we're still not ready — there's no built-in *cube root* function built into our programming language. Here's where this chapter comes to the rescue: We know that a cube root can be represented by an exponent of 1/3, and we can write the following code:

```
Input V
s = V^(1/3)
Print s
```

A final question: Explain why the statement

```
s = V^1/3
```

will produce an answer (we say that there were no syntax errors), but it would be the <u>wrong</u> answer.

Ch 27 – Radicals, Part II

TO ∞ AND BEYOND

A. Simplify to a single radical: $\sqrt[5]{\sqrt[3]{\sqrt{x}}}$

B. Simplify: $\sqrt[51]{a^{17}}$

Ch 28 – Preparing for the Quadratic Formula

To ∞ and Beyond

The following is a quadratic equation in the variable x:

$$wx^2 - (y^2 + z)x + (1 + x)x - \sqrt{2} = \pi$$

Convert it to standard form ($ax^2 + bx + c = 0$) and determine the values of a, b, and c.

Ch 29 – Completing the Square

TO ∞ AND BEYOND

Solve for *x* by Completing the Square:

$$x^4 - 5x^2 - 36 = 0$$

You should discover 4 answers. You may need to refer back to the Frequently Asked Questions from Chapter 24 of this document.

Ch 30 – The Quadratic Formula

PROGRAMMING

Write the code that would display the two solutions of the quadratic equation $ax^2 + bx + c = 0$ after the user enters the numerical values of a, b, and c.

```
Input a, b, c
x1 = (-b + SQRT(b^2-4*a*c))/(2*a)
x2 = (-b - SQRT(b^2-4*a*c))/(2*a)
Print x1, x2
```

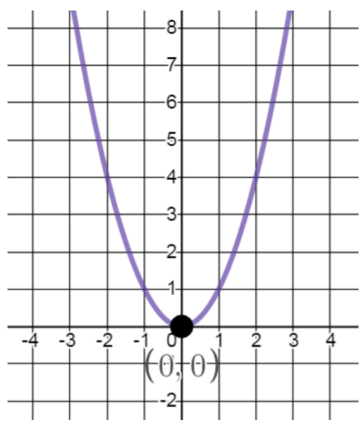
TO ∞ AND BEYOND

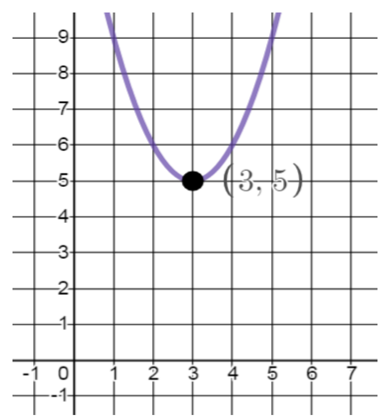
Solve for x: $\sqrt{2}x^4 + \sqrt{3}x^2 + \sqrt{6} = 0$

Ch 31 – The Parabola

TO ∞ AND BEYOND

Recall that the graph of $y = (x-3)^2 + 5$ can be created by shifting the graph of $y = x^2$ (whose vertex is the origin) three units to the right and five units up, which implies that the ***vertex*** is the point (3, 5).





In general, if a parabola is written in the form

$$y = a(x-h)^2 + k \quad \text{[called } \textbf{\textit{standard form}}\text{]}$$

its vertex is the point (h, k). For example, the vertex of the parabola $y = -3(x+\pi)^2 - 17$ is the point $(-\pi, -17)$.

By Completing the Square, convert each parabola to standard form to determine its vertex:

A. $y = x^2 + 6x + 19$ B. $y = -3x^2 + 30x - 79$

Ch 32 – Quadratic Modeling

PROGRAMMING

The goal of this program is to calculate the vertex of the parabola

$$y = ax^2 + bx + c$$

```
Input a, b, c
xcoord = -b/(2*a)
ycoord = a*xcoord^2 + b*xcoord + c
Print xcoord, ycoord
```

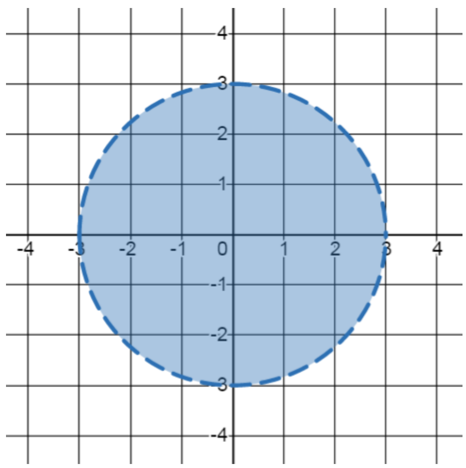
Follow-up Question: If line 2 was

```
xcoord = -b/2*a
```

Would the program have worked? Explain.

Ch 33 – The Circle

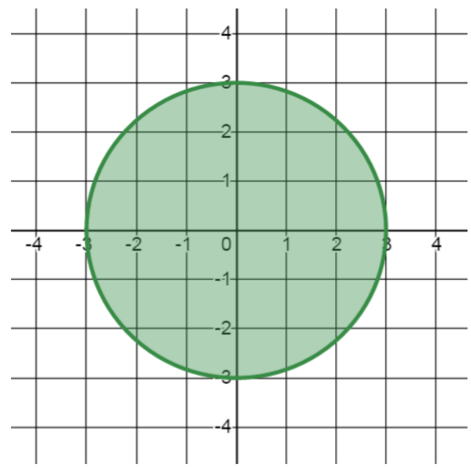
MISCELLANEOUS 



The graph of

$$x^2 + y^2 < 9$$

is called an ***open disk***. It consists of all the points <u>inside</u> the circle $x^2 + y^2 = 9$ but not the points <u>on</u> the circle.



The graph of

$$x^2 + y^2 \leq 9$$

is called a ***closed disk***. It consists of all the points <u>inside</u> the circle $x^2 + y^2 = 9$ as well as all the points <u>on</u> the circle.

A. Describe the graph of $x^2 + y^2 < 100$.

B. Describe the graph of $x^2 + y^2 \leq 144$.

C. Write the inequality whose graph is the interior (<u>only</u>) of a circle whose center is at the origin and whose radius is 5.

D. Write the inequality whose graph is the interior of a circle whose center is at the origin and whose radius is 17, and which <u>also</u> includes the circle itself.

Ch 34 – The Ellipse

to ∞ and beyond:
center off the origin
    standard form
    general form

Ch 35 – Functions

To ∞ and Beyond

Assume that a graphing program can graph only functions. How would you graph the following circles?

A. $x^2 + y^2 = 25$

[Hint: Solve for *y*.]

B. $(x - 2)^2 + (y + 1)^2 = 9$

C. $x^2 + y^2 + 6x - 10y = 8$

[Solve for *y* using the Quadratic Formula.]

Ch 36 – Domain

DESMOS GRAPHING

Graph each of the following, and then use that graph to determine the **domain**:

a. $y = \dfrac{10}{x^3 + x^2 - 12x}$

b. $y = \dfrac{10x + 4}{x^2 + 9}$

c. $y = \sqrt{100 - x^2}$

d. $x = |y - 5| + 3$

[Notice where the *x* is.]

PROGRAMMING

Print out all the ordered pairs of the function $y = \dfrac{1}{x - 3}$, letting x take on integer values from –6 to +6. Here goes:

```
For x = –6 to 6
    y = 1/(x-3)
    Print x, y
Next x
```

Code looking good? It may look good, but there's a lethal **bug** in this program. Your job is to find it.

Now for two follow-up questions: 1) How many ordered pairs will be printed? 2) Explain why the formula **`y = 1/x - 3`** will FAIL to produce the correct ordered pairs.

Ch 37 – Rational Functions

DESMOS GRAPHING

There's a bug in Desmos . . .

Graph the following function:

$$y = \frac{x^2 - 9}{x - 3}$$

and describe what you see.

According to the graph, what is the **domain** of the function?

According to what we've learned about division in this chapter, what is the domain of the function? Do you see Desmos' problem?

Now here's the irony: Click any point on the graph where $x \neq 3$, and notice that it shows a *solid* dot — that makes sense. Now click the point on the graph where $x = 3$, and this time notice that it displays an *open* dot. This means that although Desmos knows that x cannot be 3 in this function (the open dot), it ignores that fact, and shows a continuous line.

Ch 38 – Intro to Exponential Equations

FREQUENTLY ASKED QUESTIONS

Q: I know that $\boldsymbol{e}$ is an irrational number (infinite, non-repeating decimal), and its *approximate* value is 2.718. But where the heck did it come from? Also, how can I find more digits of $\boldsymbol{e}$ if my calculator doesn't have an $\boxed{e^x}$ button?

A: First, a way to think about the origin of $\boldsymbol{e}$. If you put one dollar in the bank for one year with interest compounded over n periods (for example, $n = 12$ would mean monthly), at 100% interest per year, the amount of money you'd have in the bank at the end of the year would be

$$\left(1+\frac{1}{n}\right)^n \text{ dollars}$$

Now, to achieve *continuous compounding*, you'd let the number of compounding periods get larger and larger (for example, for daily compounding, you'd let $n = 365$). In other words, you'd let n approach infinity. When you do that, you'd find that

$$\text{As } n \to \infty,\ \left(1+\frac{1}{n}\right)^n \to 2.71828\ldots \text{, which we call } \boldsymbol{e}.$$

So, if you want lots of digits of $\boldsymbol{e}$, just let n get really, really big. For example, if we let $n = 1{,}000{,}000$, we can calculate

$$\left(1+\frac{1}{n}\right)^n = \left(1+\frac{1}{1{,}000{,}000}\right)^{1{,}000{,}000} = 1.000001^{1{,}000{,}000} \approx 2.71828138$$

Need more digits of $\boldsymbol{e}$? Just let n grow even larger.

Ch 39 – Logarithms

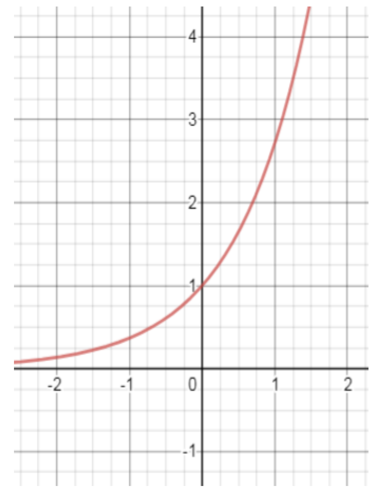
Ch 40 – Log and Exponential Equations

Ch 41 – Exponential Functions

THE CALCULUS CORNER

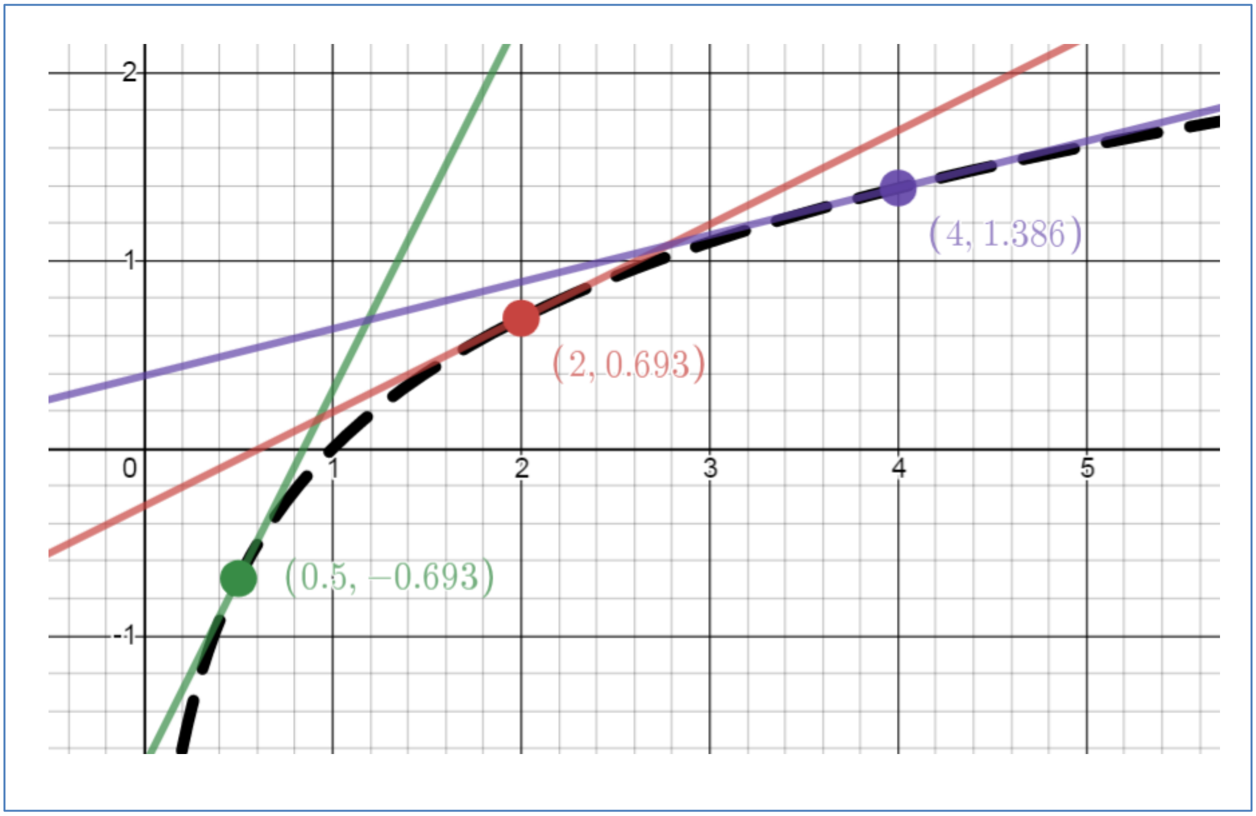
Our goal is to calculate the *slope* of the *tangent line* at various points on the curve $y = e^x$, hopefully ending up with a slope formula for any point (x, e^x) on the curve. But this time I'm going to do all the grunt work for you — I've given you the approximate slopes of the tangent lines at five points on the curve. Your job is to figure out the pattern, and turn that into a general slope formula.

x	y	Slope of Tangent Line
-1	$\frac{1}{e}$	0.3679
0	1	1
1	e	2.7183
2	e^2	7.3891
3	e^3	20.0855
x	e^x	???



Ch 42 – Log Functions

THE CALCULUS CORNER



The black, dashed, graph you see above is the graph of $y = \ln x$. Three *tangent lines* have been graphed, at the points where $x = 0.5$, 2, and 4. Your job is to approximate the *slopes* of those tangent lines, and then take an educated as to what the slopes would be at other points on the *ln* graph, and indeed at a general point $(x, \ln x)$.

For example, let's work through the calculations for the tangent line at the point (2, 0.693). We'll go up the tangent line to where $x = 3$, and approximate the y-value there to be 1.2. We have two points on the tangent line: (2, 0.693) and (3, 1.2). So we're ready to calculate the slope of that tangent line:

$$m = \frac{1.2 - 0.693}{3 - 2} = \frac{0.507}{1} = 0.507$$

Note: If you estimate the y-coordinate for $x = 3$ to be a little different from the value 1.2 that I saw, you'll get a somewhat different slope. That's OK.

x	y	Slope of Tangent Line
$0.5 = \frac{1}{2}$		
2	0.693	0.507
4		
20		
x	$\ln x$	

Ch 43 – Sequences and Series

Frequently Asked Questions

Q: Will I ever see the sigma notation, Σ, again?

A: Quite possibly. If you take statistics, you'll see the following: Given a set of n numbers: $x_1, x_2, x_3, \ldots, x_n$, the ***mean*** might be given as

$$\mu = \frac{\sum_{i=1}^{n} x_i}{n}$$

which simply means **add** up all n of the numbers, and then divide that sum by n. [Not only is Σ a Greek letter, but the μ in the formula is the lower-case Greek letter 'mu.' (pronounced myoo).]

Programming

#1 Can I write a computer program that will generate the terms of a sequence; for example, the one in Example 3?

Of course, assuming we know the formula that generated the sequence: nth term $= 3n + 1$. The following is one way to accomplish this goal:

```
NumberOfTerms = 1000
For n = 1 to NumberOfTerms
    Print 3*n + 1
Next n
```

This will print the first 1000 terms of the sequence.

#2 Follow-up Question: What if I want to give the user the ability to specify the number of terms of the sequence to be printed?

```
Input NumberOfTerms
For n = 1 to NumberOfTerms
    Print 3*n + 1
Next n
```

Changing just the first line now allows the user to specify the number of terms they want.

Ch 44 – The Binomial Theorem

To ∞ and Beyond

I understand how to raise *a* + *b* to some power using the Binomial Theorem, but what if I have to raise 2*x* + 5 to some power?

OK, suppose that we want to expand $(2x + 5)^7$. First, to lay down the foundation, we use what we've learned to expand $(a + b)^7$:

$$(a+b)^7 = a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$$

Now we substitute $2x$ for a and 5 for b, since we're expanding $(2x + 5)^7$:

$$(2x+5)^7 = (2x)^7 + 7(2x)^6 \cdot 5 + 21(2x)^5 \cdot 5^2 + 35(2x)^4 \cdot 5^3 + 35(2x)^3 \cdot 5^4 + 21(2x)^2 \cdot 5^5 + 7(2x) \cdot 5^6 + 5^7$$

$$= 128x^7 + 7 \cdot 64x^6 \cdot 5 + 21 \cdot 32x^5 \cdot 25 + 35 \cdot 16x^4 \cdot 125 + 35 \cdot 8x^3 \cdot 625 + 21 \cdot 4x^2 \cdot 3125 + 7 \cdot 2x \cdot 15{,}625 + 78{,}125$$

$$= \mathbf{128x^7 + 2240x^6 + 16{,}800x^5 + 70{,}000x^4 + 175{,}000x^3 + 262{,}500x^2 + 218{,}750x + 78{,}125}$$