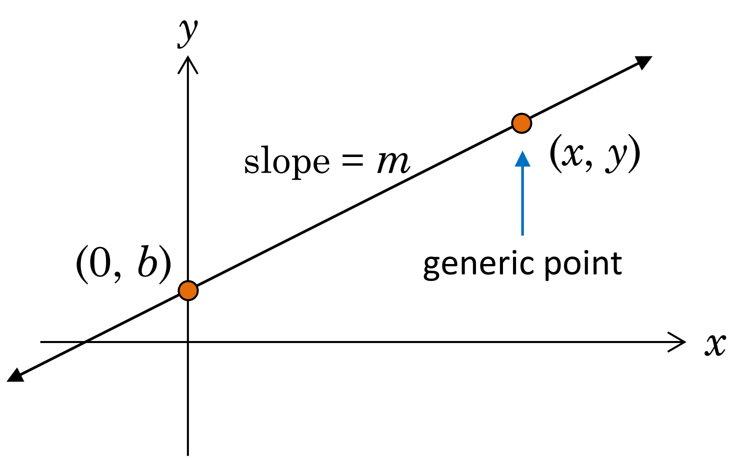
CH 6 – THE EQUATION OF A LINE

❑ INTRODUCTION

You might remember $y = mx + b$, the special form of a line where m is the slope and $(0, b)$ is the y-intercept. In previous Algebra courses you probably memorized it and learned how to apply it. In this course, we begin by deriving the equation from scratch. This chapter will also cover parallel and perpendicular lines, as well as an alternate formula for the equation of a line, well suited for future courses.

❑ DERIVING THE SLOPE-INTERCEPT FORM OF A LINE



We know that the slope of the line is m, and that its y-intercept is $(0, b)$. What is the *equation* of the line?

The "generic" point basically represents ALL the points on the line.

Our Goal: Given that a line has slope m and y-intercept $(0, b)$, find the equation of the line.

We've labeled the y-intercept and a generic point (x, y) on the line. [A *generic point* is a point that represents any point on the line.] On the one hand, the slope of the line is given to be m:

$$slope = m$$

On the other hand, we can calculate the slope of the line by applying the definition of slope to the two points labeled on the line:

$$slope = \frac{\Delta y}{\Delta x} = \frac{y-b}{x-0} = \frac{y-b}{x}$$

Since the slope of a line is the same everywhere on the line, the two slopes must be equal. That is,

$$\frac{y-b}{x} = m \qquad \text{(the two expressions for slope are equal)}$$

$$\Rightarrow \quad \left(\frac{y-b}{x}\right)x = mx \qquad \text{(multiply both sides of the equation by } x\text{)}$$

$$\Rightarrow \quad y - b = mx \qquad \text{(cancel the } x\text{'s on the left side of the equation)}$$

$$\Rightarrow \quad y = mx + b \qquad \text{(solve for } y\text{)}$$

In summary:

$$y = mx + b$$

SLOPE (m) y-INTERCEPT (b)

To be precise, b is not the y-intercept; b is the y-coordinate of the y-intercept. The y-intercept is properly written $(0, b)$. [Others disagree with me.]

EXAMPLE 1:

A. Find the slope and y-intercept of the line $y = -\frac{2}{3}x - 5$.
Answer: The slope is $-\frac{2}{3}$ and the y-intercept is (0, –5).

B. Find the equation of the line whose slope is –6 and whose y-intercept is $(0, \frac{4}{5})$. Answer: $y = -6x + \frac{4}{5}$.

EXAMPLE 2: **Find the slope and the y-intercept of the line $3x - 5y + 15 = 0$.**

Solution: The given equation, $3x - 5y + 15 = 0$, doesn't fit the slope-intercept form, $y = mx + b$, of a line. But we can make it fit; we solve the line equation $3x - 5y + 15 = 0$ for y.

$3x - 5y + 15 = 0$ (the original line)

$\Rightarrow 3x - 5y = -15$ (subtract 15 from each side)

$\Rightarrow -5y = -3x - 15$ (subtract $3x$ from each side)

$\Rightarrow \frac{-5y}{-5} = \frac{-3x-15}{-5}$ (divide each side by –5)

$\Rightarrow y = \frac{3}{5}x + 3$ (split the right-hand fraction; see the Prologue)

Now that the line is in the $y = mx + b$ form, we conclude that

The slope is $\frac{3}{5}$ and the y-intercept is (0, 3).

Homework

1. a. Find the slope and y-intercept of the line $y = -17x + 13$.

   b. Find the equation of the line whose slope is $\frac{2}{3}$ and whose y-intercept is $(0, -\pi)$.

   c. Find the slope and y-intercept of the line $y = -\frac{\pi}{2}x + \frac{\sqrt{2}}{\sqrt{3}}$.

   d. Find the equation of the line whose slope is $-\frac{w}{7}$ and whose y-intercept is $\left(0, \frac{a-b}{c}\right)$.

2. Find the slope and y-intercept of each line by converting the line to $y = mx + b$ form (if necessary):

   a. $y = 132x - 1000$

   b. $y = -\frac{8}{7}x - \frac{13}{9}$

   c. $7x - 9y = 10$

   d. $-3x - 5y + 1 = 0$

   e. $2x + 7y = 13$

   f. $-5x + 2y + 3 = 0$

   g. $y = \frac{9x - 5}{2}$

   h. $-17x - y = 4$

   i. $2x - 6y = 8$

   j. $-x + 4y - 2 = 0$

   k. $7y - 2x = 0$

   l. $7x + 4y + 5 = 0$

❑ FINDING THE LINE EQUATION FROM CLUES

EXAMPLE 3: **Find the equation of the line which has a slope of 7 and which passes through the point (–5, 3).**

Solution: The line equation we are using is $\boldsymbol{y = mx + b}$, where m is the slope and $(0, b)$ is the y-intercept. In this example, we are given the slope of 7. That's good.

And so the line equation $y = mx + b$

becomes $y = 7x + b$ (putting the 7 in for slope)

But we were not given the y-intercept. That's bad. Instead, we are given a point on the line, but it's certainly not the y-intercept (how can you tell?). So now our goal is to find the value of b.

Consider that the problem tells us that the point (–5, 3) is on the line. Therefore, it must work in the equation we just wrote: $y = 7x + b$. So we plug –5 in for x and plug 3 in for y:

$y = 7x + b$ (our line with the slope plugged in)

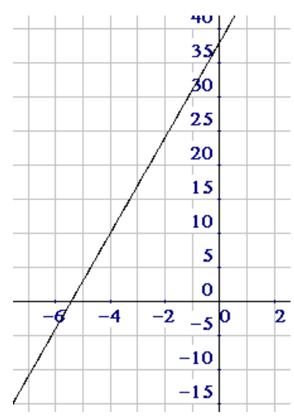
$\Rightarrow$ $\mathbf{3} = 7(\mathbf{-5}) + b$ (since (–5, 3) lies on the line)

$\Rightarrow$ $3 = -35 + b$ (multiply)

$\Rightarrow$ $b = 38$ (solve for b)

Now we put it all together. The value of m is 7 (given in the problem) and the value of b is 38 (we just calculated it). We get our final answer:

$$\boxed{y = 7x + 38}$$



Homework

3. Find the equation of the line with the given slope and passing through the given point:

a. $m = -3$	$(8, -16)$	b. $m = 4$	$(-1, -11)$
c. $m = 2$	$(0, -10)$	d. $m = -3$	$(3, -2)$
e. $m = 9$	$(2, 18)$	f. $m = -1$	$(-3, 11)$
g. $m = 1$	$(5, 5)$	h. $m = 7$	$(0, 10)$
i. $m = -8$	$(-2, 29)$	j. $m = 1$	$(10, -89)$

EXAMPLE 4: **Find the equation of the line passing through the points (–1, 3) and (8, –15).**

Solution: This problem will really test our deductive skills. Let's begin with the slope-intercept form of a line:

$$y = mx + b$$

Did the problem tell us what the slope is? It did not. Did the problem give us the y-intercept? Again, no. This is not good — how can we possibly do this problem? Well, even though the slope was not handed to us on a silver platter, we can use the two given points on the line to calculate the slope, using our $m = \frac{\Delta y}{\Delta x}$ formula. So, using the given points (–1, 3) and (8, –15), we find the slope:

$$m = \frac{\Delta y}{\Delta x} = \frac{3-(-15)}{-1-8} = \frac{3+15}{-1-8} = \frac{18}{-9} = \mathbf{-2}$$

We can now write our line as

$$y = -2x + b$$

How do we find b? The same way we did in the previous example: Plug the coordinates of one of the original points in for x and y — either point will do the job, since each of them lies on the line. Let's use the point (–1, 3).

$$y = -2x + b$$
$$\Rightarrow \quad \mathbf{3} = -2(\mathbf{-1}) + b$$
$$\Rightarrow \quad 3 = 2 + b$$
$$\Rightarrow \quad \boldsymbol{b} = \mathbf{1}$$

Putting the values of m and b into the formula $y = mx + b$ gives us the line which passes through the two points:

$$\boxed{y = -2x + 1}$$

Check: Let's check our final answer. If $y = -2x + 1$ is really the line passing through the two given points, then obviously each point should lie on the line. That is, each point should satisfy the equation of the line.

(–1, 3): $\mathbf{3} = -2(\mathbf{-1}) + 1 \Rightarrow 3 = 2 + 1 \Rightarrow 3 = 3$ ✓

(8, –15): $\mathbf{-15} = -2(\mathbf{8}) + 1 \Rightarrow -15 = -16 + 1 \Rightarrow -15 = -15$ ✓

EXAMPLE 5: **Find the equation of the line passing through the points (2, –1) and (5, –6).**

Solution: This problem will be solved in the same manner as the previous example. The equation of the line is

$$y = mx + b,$$

where m and b are to be determined.

$$m = \frac{\Delta y}{\Delta x} = \frac{-6-(-1)}{5-2} = \frac{-6+1}{5-2} = \frac{-5}{3} = -\frac{5}{3}$$

So our line equation is now

$$y = -\frac{5}{3}x + b$$

Placing the first point (either point will work) into this equation gives us

$$-1 = -\frac{5}{3}(2) + b \qquad \text{(since (2, –1) is on the line)}$$

$$\Rightarrow \quad -1 = -\frac{5}{3}\left(\frac{2}{1}\right) + b \qquad \text{(set up for multiply)}$$

$$\Rightarrow \quad -1 = -\frac{10}{3} + b \qquad \text{(multiply the fractions)}$$

$$\Rightarrow \quad -1 + \frac{10}{3} = b \qquad \left(\text{add } \frac{10}{3} \text{ to each side}\right)$$

$$\Rightarrow \quad -\frac{3}{3} + \frac{10}{3} = b \qquad \text{(common denominator)}$$

$$\Rightarrow \quad \frac{7}{3} = b \qquad \text{(combine the fractions)}$$

And now we have all we need to write the equation of the line:

$$\boxed{y = -\frac{5}{3}x + \frac{7}{3}}$$

Homework

4. Find the equation of the line passing through the two given points. Also be sure you know how to check your answer:

   a. (3, 13) and (–1, 5)
   b. (1, –9) and (–5, 39)
   c. (0, –1) and (2, 9)
   d. (1, –11) and (6, –1)
   e. (1, –8) and (–2, 31)
   f. (0, 0) and (–5, 35)

g. (7, 7) and (–3, –3) | h. (0, 17) and (–17, 0)

i. (–5, –4) and (2, 4) | j. (1, –3) and (–4, –5)

k. (2, –3) and (–1, –10) | l. (–4, 8) and (–1, 6)

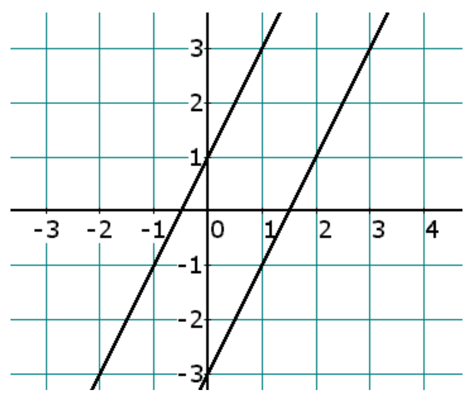
m. (0, –7) and (–2, –4) | n. (3, 7) and (5, –2)

❑ *PARALLEL LINES*

Let's begin by assuming that in this chapter we will never refer to horizontal or vertical lines. These special lines will be covered in detail in Chapter 8.

Let's graph the two lines $y = 2x + 1$ and $y = 2x - 3$ on the same grid:

Notice that the two lines are parallel (trust me — they are). Now, what do the equations of the two lines have in common? The formulas show that each line has a slope of 2. Since slope is a measure of steepness, does it seem reasonable that if two lines are parallel, then they are equally steep, and therefore they must have the same **slope**?



Parallel lines have the same slope.

For a simple example, suppose Line 1 has a slope of –9, and that Line 2 is parallel to Line 1. We can then deduce that the slope of Line 2 is also –9.

<u>EXAMPLE 6</u>: **Find the slope of <u>any</u> line which is *parallel* to the line $7x - 5y = 2$.**

<u>Solution</u>: Any line which is parallel to the line $7x - 5y = 2$ will have the *same* slope as the line $7x - 5y = 2$. So, if we can compute the slope of this line, we will have the slope of <u>any</u> line <u>parallel</u> to it. The easiest way to find the slope of the line is to convert it to $y = mx + b$ form:

$$7x - 5y = 2 \Rightarrow -5y = -7x + 2 \Rightarrow y = \frac{7}{5}x - \frac{2}{5}$$

The slope of the given line is $\frac{7}{5}$, and so we conclude that any line which is parallel to the line $7x - 5y = 2$ must have a slope of

$$\boxed{\frac{7}{5}}$$

<u>EXAMPLE 7</u>: **Find the <u>equation</u> of the line which is *parallel* to the line $3x + y = 5$, and which passes through the point (6, 2).**

<u>Solution</u>: We're looking for an unknown line

$$y = mx + b$$

The slope of our unknown line was not given to us, but we know that it's the same as the given line, since the two lines are parallel. Solving the given line for y yields the line $y = -3x + 5$, whose slope is clearly -3. So the slope of our unknown line is also -3. At this point in the problem we can write our line as

$$y = -3x + b$$

Plugging the given point (6, 2) into this equation allows us to find b:

$$\mathbf{2} = -3(\mathbf{6}) + b \quad \Rightarrow \quad 2 = -18 + b \quad \Rightarrow \quad 20 = b,$$

and we're done; our line is

$$\boxed{y = -3x + 20}$$

Homework

5. a. A given line has a slope of 7. What is the slope of any line that is parallel to the given line?

   b. A given line has a slope of $-\frac{2}{3}$. What is the slope of any line that is parallel to the given line?

6. a. What is the slope of any line that is parallel to the line $y = \frac{3}{4}x - 9$?

   b. What is the slope of any line that is parallel to the line $5x + 2y = 9$?

7. a. Prove that the lines $2x - 4y = 5$ and $3x - 6y = 1$ <u>are</u> parallel.

   b. Prove that the lines $3x - y = 4$ and $5x + 2y = 10$ are <u>not</u> parallel.

8. Find the equation of the line which is *parallel* to the given line, and which passes through the given point:

   a. $y = 3x + 4$; $(3, -5)$

   b. $y = \frac{1}{2}x - 5$; $(-1, 7)$

   c. $3x + y = 7$; $(1, 9)$

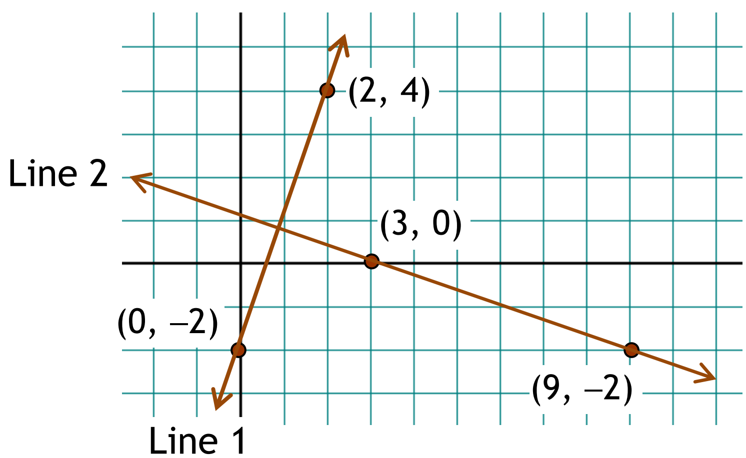
   d. $2x - y = 8$; $(-5, -4)$

   e. $2x - 3y = 1$; $(2, -5)$

   f. $3x + 4y = 10$; $(8, 0)$

❑ *Perpendicular Lines*

Parallel lines have the same slope; certainly *perpendicular* lines do not. But might there be some specific relationship between the slopes of perpendicular lines — some kind of connection that would allow us to deduce one of them if we knew the other? Let's see if we can discover one with an example. In the following grid, Line 1 is perpendicular to Line 2, and points have been labeled so we can easily calculate m_1 and m_2, the slopes of the two lines.



First we compute the slope of Line 1: $m_1 = \dfrac{\Delta y}{\Delta x} = \dfrac{4-(-2)}{2-0} = \dfrac{6}{2} = \mathbf{3}$

Next we compute the slope of Line 2: $m_2 = \dfrac{\Delta y}{\Delta x} = \dfrac{0-(-2)}{3-9} = \dfrac{2}{-6} = -\dfrac{\mathbf{1}}{\mathbf{3}}$

There are two things to note regarding the two slopes of the two perpendicular lines. First, one of the slopes is positive and the other is negative. This makes sense because Line 1 is "rising" (as you move from left to right) while Line 2 is "falling." Second, the slope of Line 1, m_1, is kind of a big number (the line's pretty steep), while the slope of Line 2, m_2, (ignoring the minus sign) is a relatively small number (the line's not very steep).

Specifically, the two slopes have opposite signs, and they are also (ignoring the minus sign) *reciprocals* of each other. In other words, when looking at the slopes of two **perpendicular lines**, each of the slopes is the **opposite reciprocal** of the other. [Some teachers call it the **negative reciprocal.**]

> Perpendicular lines have slopes that are *opposite reciprocals* of each other.

For example, if a line has a slope of $\frac{7}{4}$, then any perpendicular line must have a slope of $-\frac{4}{7}$.

Consider the line $y = -5x + 1$. Since its slope is -5, it follows that the slope of any perpendicular line must be $\frac{1}{5}$.

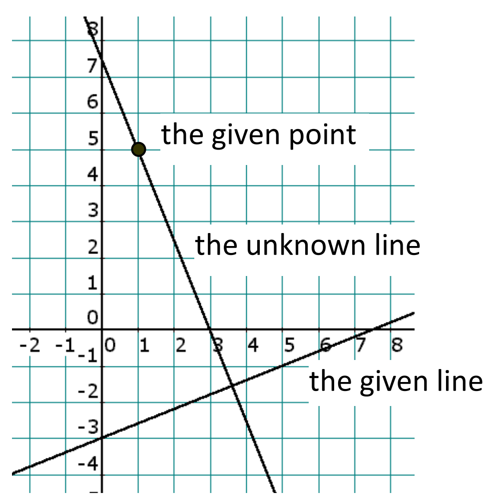
Now for the big example:

EXAMPLE 8: **Find the equation of the line which is *perpendicular* to the line $2x - 5y = 15$, and which passes through the point (1, 5).**

Solution: We're looking for an unknown line

$$y = mx + b$$

The slope of our unknown line was not given to us, but we know that it's the *opposite reciprocal* of the slope of the given line, since the two lines are perpendicular.



To determine the slope of the given line, we solve for y:

$$2x - 5y = 15$$

$$\Rightarrow -5y = -2x + 15$$

$$\Rightarrow \frac{-5y}{-5} = \frac{-2x+15}{-5}$$

$$\Rightarrow y = \frac{2}{5}x - 3$$

telling us that the slope of the given line is $\frac{2}{5}$. So the slope of our unknown line is the *opposite reciprocal* of that, which is $-\frac{5}{2}$. At this point in the problem we can write our line as

$$y = -\frac{5}{2}x + b$$

Plugging the given point (1, 5) into this equation allows us to find b:

$$\mathbf{5} = -\frac{5}{2}(\mathbf{1}) + b \quad \Rightarrow \quad 5 + \frac{5}{2} = b \quad \Rightarrow \quad b = \frac{15}{2}$$

and we're done; our line is

$$\boxed{y = -\frac{5}{2}x + \frac{15}{2}}$$

Final Note: We've learned that the slopes of two perpendicular lines are *opposite reciprocals* of each other. But some books say that two lines are perpendicular if *the product of their slopes* is <u>–1</u>. Do these statements mean the same thing? Yes — assume that the product of their slopes is –1:

$$m_1 m_2 = -1$$

Solving for m_1 gives us the equation

$$m_1 = -\frac{1}{m_2}$$

which says that one slope is the opposite reciprocal of the other. So both ways describe the slopes of perpendicular lines.

Homework

9. A given line has a slope of $-\frac{5}{3}$. What is the slope of any line that is *perpendicular* to the given line?

10. Prove that the lines $7x - 2y = 10$ and $4x + 14y = 23$ <u>are</u> perpendicular.

11. Prove that the lines $3x + 2y = 10$ and $2x + 3y = 9$ are <u>not</u> perpendicular.

12. Prove that the lines $5x - 3y = 10$ and $5x + 3y = 17$ are <u>not</u> perpendicular.

13. Find the <u>slope</u> of any line which is *perpendicular* to the given line:

    a. $y = -7x + 9$
    b. $y = \frac{5}{4}x + 10$
    c. $2x + 7y = 10$
    d. $3x - 2y = 0$

14. Find the <u>equation</u> of the line which is *perpendicular* to the given line, and which passes through the given point:

    a. $y = 3x + 4$; $(3, -5)$
    b. $y = \frac{1}{2}x - 5$; $(-1, 7)$
    c. $3x + y = 7$; $(1, 9)$
    d. $2x - y = 8$; $(-5, -4)$
    e. $5x - 3y = 1$; $(2, -5)$
    f. $3x + 4y = 10$; $(8, 0)$

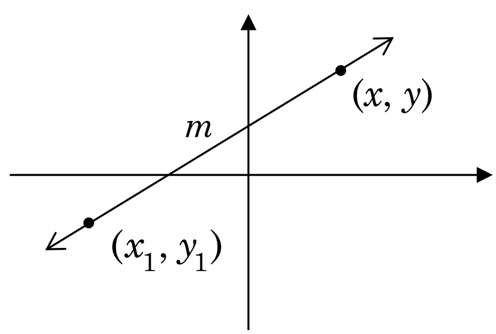
❑ THE POINT-SLOPE FORM OF A LINE

The slope-intercept form of a line, $y = mx + b$, that we've been using for the past 15 pages is perfect (especially in Statistics) when you have the slope and the y-intercept. It's just as likely (for example, in Business Calculus) that you'll be working with the slope and some point on the line other than the y-intercept.

<u>THEOREM:</u> The equation of the line with slope m and passing through the point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

<u>PROOF:</u> We begin by sketching the line and labeling the given point (x_1, y_1), the slope m, and a generic point (x, y):



On the one hand, the slope of the line is given by m. On the other hand, the slope of the line can be calculated using the definition of slope and the two points (x, y) and (x_1, y_1): $\frac{y - y_1}{x - x_1}$. And, of course, these two slopes must be the same, since they describe the same line:

$$\frac{y - y_1}{x - x_1} = m$$

$$\Rightarrow \quad y - y_1 = m(x - x_1) \qquad \text{And we're done.}$$

EXAMPLE 9: **Find the equation of the line whose slope is –3 and which passes through the point (8, –2).**

Solution: This is precisely the data we need to use the point-slope form: $y - y_1 = m(x - x_1)$. We're given the slope, so $m = -3$. We're also given a point on the line, so $(x_1, y_1) = (8, -2)$. Plugging these values into the point-slope form gives us

$$y - (-2) = -3(x - 8), \text{ or}$$

$$\boxed{y + 2 = -3(x - 8)}$$

EXAMPLE 10: **Find the equation of the line passing through the two points (3, –5) and (–2, –8).**

Solution: The point-slope form, $y - y_1 = m(x - x_1)$, requires a point (we have two of them), and the slope, which we'll have to calculate ourselves:

$$m = \frac{\Delta y}{\Delta x} = \frac{-5 - (-8)}{3 - (-2)} = \frac{-5 + 8}{3 + 2} = \frac{3}{5}$$

Now, using the point (3, –5) (although either point would work), we get our equation

$$y - (-5) = \frac{3}{5}(x - 3), \text{ or}$$

$$\boxed{y + 5 = \frac{3}{5}(x - 3)}$$

Homework

15. In Example 10, the claim was made that either point would work in finding the equation of the line through the two given points. Prove this claim.

16. Use the point-slope formula to find the equation of the line with slope 7 and passing through the point (6, –8).

17. Use the point-slope formula to find the equation of the line with slope 0 and passing through the point (–17, 9).

18. Use the point-slope formula to find the equation of the line with slope $-\frac{4}{7}$ and passing through the point $(\frac{1}{2}, \pi)$.

19. Use the point-slope formula to find the equation of the line which passes through the points (–2, 4) and (5, –5).

20. Use the point-slope formula to find the equation of the line which passes through the points $(\pi, \sqrt{2})$ and (–3, 1).

Practice Problems

21. The slopes of two parallel lines are ______.
22. The slopes of two perpendicular lines are ______.
23. The slope of a line is 5/7. What is the slope of any parallel line?
24. The slope of a line is –4/9. What is the slope of any perpendicular line?
25. T/F: The lines $y = 7x - 3$ and $14x - 2y = 22$ are parallel.

26. T/F: The lines $3x - 7y = 1$ and $7x + 3y = 0$ are perpendicular.

27. Find the equation of the line which is parallel to $3x - 7y = 9$ and passes through the point (–3, 10).

28. Find the equation of the line which is perpendicular to $4x - 9y = 11$ and passes through the point (3, –13).

29. Find the slope and y-intercept of the line $-7x - 3y = 10$.

30. Find the equation of the line with slope –8 and passing through the point (–1, –5).

31. Find the equation of the line passing through the points (–1, 9) and (9, –2).

32. Find the equation of the line which is parallel to the line $3x - 4y = 1$ and which passes through the point (–2, –7).

33. Find the equation of the line which is perpendicular to the line $3x - 4y = 1$ and which passes through the point (–2, –7).

34. Which one of the following lines is *parallel* to the line $5x - 3y = 7$?

a. $y = \frac{3}{5}x - 1$ b. $y = -\frac{3}{5}x + 4$ c. $y = \frac{5}{3}x - 3$

d. $y = -\frac{5}{3}x + 2$ e. $y = \frac{5}{3}$

35. Which one of the following lines is *perpendicular* to the line $5x - 3y = 7$?

a. $y = \frac{3}{5}x - 1$ b. $y = -\frac{3}{5}x + 4$ c. $y = \frac{5}{3}x - 3$

d. $y = -\frac{5}{3}x + 2$ e. $y = \frac{5}{3}$

Solutions

1. a. $m = -17$ y-int = (0, 13) b. $y = \frac{2}{3}x - \pi$

c. $m = -\frac{\pi}{2}$ y-int = $\left(0, \frac{\sqrt{2}}{\sqrt{3}}\right)$ d. $y = -\frac{w}{7}x + \frac{a-b}{c}$

2. a. $m = 132$ y-int = (0, –1000) b. $m = -\frac{8}{7}$ y-int = $(0, -\frac{13}{9})$

c. $m = \frac{7}{9}$ y-int = $(0, -\frac{10}{9})$ d. $m = -\frac{3}{5}$ y-int = $(0, \frac{1}{5})$

e. $m = -\frac{2}{7}$ y-int = $(0, \frac{13}{7})$ f. $m = \frac{5}{2}$ y-int = $(0, -\frac{3}{2})$

g. $m = \frac{9}{2}$ y-int = $(0, -\frac{5}{2})$ h. $m = -17$ y-int = (0, –4)

i. $m = \frac{1}{3}$ y-int = $(0, -\frac{4}{3})$ j. $m = \frac{1}{4}$ y-int = $(0, \frac{1}{2})$

k. $m = \frac{2}{7}$ y-int = (0, 0) l. $m = -\frac{7}{4}$ y-int = $(0, -\frac{5}{4})$

3. a. $y = -3x + 8$ b. $y = 4x - 7$ c. $y = 2x - 10$
d. $y = -3x + 7$ e. $y = 9x$ f. $y = -x + 8$
g. $y = x$ h. $y = 7x + 10$ i. $y = -8x + 13$
j. $y = x - 99$

4. a. $y = 2x + 7$ b. $y = -8x - 1$ c. $y = 5x - 1$
d. $y = 2x - 13$ e. $y = -13x + 5$ f. $y = -7x$
g. $y = x$ h. $y = x + 17$ i. $y = \frac{8}{7}x + \frac{12}{7}$
j. $y = \frac{2}{5}x - \frac{17}{5}$ k. $y = \frac{7}{3}x - \frac{23}{3}$ l. $y = -\frac{2}{3}x + \frac{16}{3}$
m. $y = -\frac{3}{2}x - 7$ n. $y = -\frac{9}{2}x + \frac{41}{2}$

5. a. 7 b. $-\frac{2}{3}$

6. a. $\frac{3}{4}$ b. $-\frac{5}{2}$

7. a. Each line has a slope of $\frac{1}{2}$. Same slope $\Rightarrow$ parallel lines.

b. The slopes are 3 and –5/2. Different slopes $\Rightarrow$ non-parallel lines.

8. a. $y = 3x - 14$ b. $y = \frac{1}{2}x + \frac{15}{2}$ c. $y = -3x + 12$

d. $y = 2x + 6$ e. $y = \frac{2}{3}x - \frac{19}{3}$ f. $y = -\frac{3}{4}x + 6$

9. $\frac{3}{5}$

10. The slopes are $\frac{7}{2}$ and $-\frac{2}{7}$, which are opposite reciprocals of each other.

11. The slopes are $-\frac{3}{2}$ and $-\frac{2}{3}$, which are not opposite reciprocals of each other. (They're reciprocals, but not opposites.)

12. The slopes are $\frac{5}{3}$ and $-\frac{5}{3}$, which are not opposite reciprocals of each other. (They're opposites, but not reciprocals.)

13. a. $\frac{1}{7}$ b. $-\frac{4}{5}$ c. $\frac{7}{2}$ d. $-\frac{2}{3}$

14. a. $y = -\frac{1}{3}x - 4$ b. $y = -2x + 5$ c. $y = \frac{1}{3}x + \frac{26}{3}$

d. $y = -\frac{1}{2}x - \frac{13}{2}$ e. $y = -\frac{3}{5}x - \frac{19}{5}$ f. $y = \frac{4}{3}x - \frac{32}{3}$

15. First rework the problem using the other point. Then take each line answer (the one in the example and the one you just got) and convert to $y = mx + b$ form. That should convince you.

16. $y + 8 = 7(x - 6)$

17. $y - 9 = 0$

18. $y - \pi = -\frac{4}{7}(x - \frac{1}{2})$

19. $y - 4 = -\frac{9}{7}(x + 2)$

20. $y - \sqrt{2} = \frac{1-\sqrt{2}}{-3-\pi}(x - \pi)$ Note: The slope can also be written: $\frac{\sqrt{2}-1}{\pi+3}$.

21. equal

22. opposite reciprocals

23. 5/7 **24**. 9/4 **25**. T **26**. T

27. $y = \frac{3}{7}x + \frac{79}{7}$

28. $y = -\frac{9}{4}x - \frac{25}{4}$

29. $m = -\frac{7}{3}$ $y\text{-int} = \left(0, -\frac{10}{3}\right)$

30. $y = -8x - 13$

31. $y = -\frac{11}{10}x + \frac{79}{10}$

32. $y = \frac{3}{4}x - \frac{11}{2}$

33. $y = -\frac{4}{3}x - \frac{29}{3}$

34. c.

35. b.

Ben Sweetland:

"We cannot hold a torch to light another's path without brightening our own."