CH 17 – PERCENT MIXTURE PROBLEMS

❑ INTRODUCTION

NaOCl
sodium hypochlorite

Suppose you have 12 quarts of a bleach solution which is at a 25% concentration. This means that 25% of the solution is bleach, while the other 75% is some neutral substance (usually water). How much of the 12 quarts is actually bleach? Since the solution is 25% bleach, we take 25% of 12, which is $12 \times 0.25 = 3$ quarts of bleach.

The 12 quarts of solution is called the total **quantity**.

The 25% is called the **concentration**.

The 3 quarts of bleach is called the actual **amount**.

The calculation above shows us the formula that ties all these ideas together, and is the basis for this chapter and the next one:

Quantity × % Concentration = Amount

Two Important Notes:

1. To convert 63% to a decimal, move the decimal point (it's after the 3) two places to the left and drop the percent sign, giving 0.63. To convert the decimal 0.08 to a percent, move the decimal point two places to the right and add a percent sign to get 8%.

2. Pure bleach would have a 100% bleach concentration, while pure water would have a 0% bleach concentration. Be sure this makes sense to you.

Homework

1. A 40-quart solution of sulfuric acid is at a 30% concentration. How many quarts of the solution are sulfuric acid? How many quarts are water?

2. A 25-kg solution of salt water is 20% NaCl (salt). How many kg of the solution consist of NaCl? How many kg are water?
3. What is the percent concentration of sodium in pure sodium?
4. What is the percent concentration of nitric acid in pure water?
5. Consider a gallon of pure antifreeze. What is the percent concentration of antifreeze? What is the percent concentration of water?
6. Consider a liter of pure water. What is the percent concentration of hydrochloric acid? What is the percent concentration of water?

❑ *SOLVING A SYSTEM OF EQUATIONS BY ELIMINATION*

To solve the applications in this chapter and future courses, we need to be able to solve two equations in two variables. A method that works very well in many cases is called the ***Elimination Method***. We multiply one or both equations by appropriate numbers (whatever that means), add the resulting equations to eliminate a variable, and then solve for the variable that survived. The *elimination method* is another term used to describe this procedure.

EXAMPLE 1: **Solve the system:** $\mathbf{5x - 2y = 20}$
$\mathbf{3x + 7y = -29}$

Solution: In the Elimination Method we may eliminate either variable. But there's a certain "orderliness" that comes in handy in future math courses if we always eliminate the first variable, so in this case we will eliminate the *x*. As mentioned above, we multiply one or both equations by some numbers, add the resulting equations to kill off one variable, and then solve for the variable that still lives. How do we find these numbers? Rather than some mystifying explanation, just watch — you'll catch on.

$$5x - 2y = 20 \xrightarrow{\text{times 3}} 15x - 6y = 60$$
$$3x + 7y = -29 \xrightarrow{\text{times -5}} -15x - 35y = 145$$

Add the equations: $0x - 41y = 205$

The *x*'s are gone!

Divide by –41: $\dfrac{-41y}{-41} = \dfrac{205}{-41}$

$\underline{y = -5}$

Now that we have the value of *y*, we can substitute its value of –5 into either of the two original equations to find the value of *x*. Using the first equation:

$$5x - 2(\mathbf{-5}) = 20$$
$$\Rightarrow 5x + 10 = 20$$
$$\Rightarrow 5x = 10$$
$$\Rightarrow \underline{x = 2}$$

Therefore, the final solution to the system of equations is

$$\boxed{x = 2 \ \& \ y = -5}$$

Homework

7. Solve each system using the Elimination Method, and be sure you practice checking your solution (your pair of numbers) in both of the original equations:

a. $$\begin{aligned} 2x + y &= 5 \\ -2x + 7y &= 19 \end{aligned}$$

b. $$\begin{aligned} 5a - 3b &= 5 \\ 10a + 4b &= -40 \end{aligned}$$

c. $$\begin{aligned} -2u - 3v &= -16 \\ -7u + 8v &= -56 \end{aligned}$$

d. $$\begin{aligned} 7x + 12y &= -24 \\ 6x - 7y &= 14 \end{aligned}$$

❑ *Mixing Chemicals*

EXAMPLE 2: **How many quarts each of a 62% poison solution and a 6% poison solution must a foreign spy need to get 14 quarts of a mixture that is 30% poison?**

Solution: Let x represent the quarts of the 62% poison. Let y represent the quarts of the 6% poison.

	Quantity x	Concentration =	Amount
62% poison	x	62%	$0.62x$
6% poison	y	6%	$0.06y$
final mixture	14	30%	14×0.30

Looking at the Quantity column, we're mixing x quarts with y quarts to get a total of 14 quarts in the final mixture. It makes sense to say that the sum of x and y must be 14:

$$x + y = 14 \qquad \textbf{[Equation \#1]}$$

Consider the Concentration column. Does adding the concentrations together make any sense? Of course not: $62\% + 6\% \neq 30\%$. In fact, our intuition tells us that the concentration of the final solution ought to be somewhere between the concentrations of the ingredients being mixed. But we need another equation; since we have two variables x and y, we'll need two equations in order to find x and y.

Now look at the Amount column. Each ingredient being mixed together contains poison and water. Does it make sense that the actual amount of poison in the final solution must be the sum of the actual amounts of poison in the ingredients? This leads to the second equation:

$$0.62x + 0.06y = 14 \times 0.30$$

or, $$0.62x + 0.06y = 4.2 \qquad \textbf{[Equation \#2]}$$

Equations 1 and 2 constitute a system of two equations in two variables, which we will solve using the Elimination Method.

$x + y = 14$ (times –0.62) $\Rightarrow$ $-0.62x - 0.62y = -8.68$

$0.62x + 0.06y = 4.2$ (leave alone) $\Rightarrow$ $0.62x + 0.06y = 4.2$

Adding $\Rightarrow$ $0 - 0.56y = -4.48$

$\Rightarrow$ $y = 8$

Since y stood for the number of quarts of the 6% solution, we see that the spy needs 8 quarts of the 6% solution. Moreover, since $x + y = 14$, we see that $x + 8 = 14 \Rightarrow x = 6$, which means that the spy also needs 6 quarts of the 62% solution.

6 quarts of the 62% poison, &
8 quarts of the 6% poison

EXAMPLE 3: **A druggist wants to create 4 liters of a 94% anti-malaria medicine. How many liters each of pure anti-malaria medicine and a 92% anti-malaria medicine must she mix together?**

Solution:

Let x represent the liters of pure anti-malaria medicine.
Let y represent the liters of the 92% anti-malaria medicine.

Remembering that pure anti-malaria medicine has an anti-malaria concentration of 100%, we put all our information in the chart:

	Quantity x	Concentration =	Amount
pure medicine	x	100%	$1.00x$
92% medicine	y	92%	$0.92y$
final mixture	4	94%	4×0.94

One of our equations comes from the fact that the quantities x and y must add up to 4:

$$x + y = 4$$

The second equation comes from the fact that the amount of anti-malaria medicine in the 100% medicine plus the amount of anti-malaria medicine in the 92% medicine must equal the amount of anti-malaria medicine in the final mixture:

$$1.00x + 0.92y = 0.94 \times 4,$$

or, $$x + 0.92y = 3.76$$

Let's solve this system of equations via the Elimination Method.

$$\begin{array}{lll} x + y = 4 & \text{(leave alone)} & \Rightarrow \quad x + y = 4 \\ x + 0.92y = 3.76 & \text{(times } -1\text{)} & \Rightarrow \quad -x - 0.92y = -3.76 \\ & \text{Adding} & \Rightarrow \quad 0 + 0.08y = 0.24 \\ & & \Rightarrow \quad y = 3 \end{array}$$

Since $y = 3$, and $x + y = 4$, it follows that $x = 1$. We now know how many liters of each medicine she must mix together.

3 liters of the 92% medicine, & 1 liter of the pure medicine

<u>EXAMPLE 4:</u> **How many fluid ounces each of pure water and a 10% albuterol inhalant must an allergist mix to get 5 fluid ounces of an inhalant that is 6% albuterol?**

<u>Solution:</u> This is the same scenario as the two previous examples, so we can get right to it; note, however, that pure water contains 0% albuterol (that is, there is <u>no</u> albuterol in pure water).

	Quantity	x Concentration	= Amount
pure water	x	0%	$0x$
10% albuterol	y	10%	$0.10y$
final mixture	5	6%	5×0.06

The two equations are $x + y = 5$ and $0x + 0.10y = 5 \times 0.06$, or $0.10y = 0.3$. Setting up the system of equations neatly gives

$$\begin{aligned} x + y &= 5 \\ 0.10y &= 0.3 \end{aligned}$$

The second equation can be easily solved for y, so we don't need the Elimination method. Dividing each side of the second equation by 0.10, we get $y = 3$. Using $x + y = 5$, we find that $x = 2$. Thus,

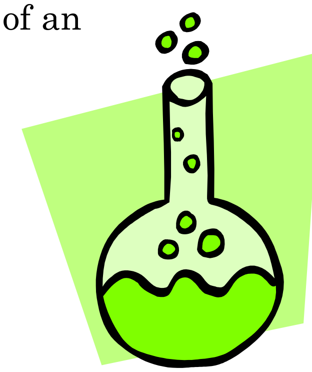
2 ounces of water, &
3 ounces of the 10% inhalant

Homework

8. How many pounds each of a 97% poison solution and a 67% poison solution must a spy mix to get 15 pounds of a solution that is 91% poison?

9. A druggist wants to create 24 ounces of a 92% alcohol medicine. How many ounces each of pure alcohol medicine and a 76% alcohol medicine must he mix together?

10. How many ounces each of pure water and an 80% albuterol inhalant must an allergist mix to get 20 ounces of an inhalant that is 64% albuterol?



11. A druggist wants to mix some 59% alcohol medicine with some 21% alcohol medicine. How many mL of each substance must she use to get a 19-mL mixture that is 57% alcohol?

12. A detective wants to mix some pure poison with some 20% poison solution. How many pounds of each substance must she use to get a 25-pound mixture that is 52% poison?

13. How many liters each of a 99% solution and an 88% solution must be mixed together to get 22 liters of a mixture whose concentration is 97%?

14. How many liters each of a pure solution and a 12% solution must be mixed together to get 88 liters of a mixture whose concentration is 42%?

15. How many liters each of a 24% solution and a 68% solution must be mixed together to get 22 liters of a mixture whose concentration is 44%?

16. How many liters each of pure water and a 57% solution must be mixed together to get 38 liters of a mixture whose concentration is 42%?

17. How many liters each of a 79% solution and a 70% solution must be mixed together to get 36 liters of a mixture whose concentration is 76%?

18. How many liters each of pure water and a 48% solution must be mixed together to get 72 liters of a mixture whose concentration is 14%?

19. How many liters each of a 9% solution and a 94% solution must be mixed together to get 68 liters of a mixture whose concentration is 69%?

20. How many liters each of pure water and a 20% solution must be mixed together to get 100 liters of a mixture whose concentration is 9%?

21. How many liters each of a 73% solution and a 97% solution must be mixed together to get 12 liters of a mixture whose concentration is 93%?

22. How many liters each of a pure solution and a 58% solution must be mixed together to get 48 liters of a mixture whose concentration is 93%?

❑ *THE RADIATOR PROBLEM*

Review of Main Formula: If a 12-liter radiator is filled with an antifreeze/water solution which is at a 25% concentration of antifreeze, then 25% of the 12 liters is antifreeze and the rest (75%) is water. Multiplying 12 liters by 25% gives us the fact that 3 of the liters of solution are antifreeze, and the other 9 liters (12 liters × 75%) are water. Thus, the key formula we need for the percent mixture problem in this section is

$$\text{Quantity} \times \%\ \text{Concentration} = \text{Amount}$$

<u>EXAMPLE 5:</u> **A 10-liter radiator is filled with 42% antifreeze solution. How much of the antifreeze solution must be drained and replaced with pure antifreeze to bring the concentration of the final solution up to 58%?**

<u>Solution:</u> We'll set up all the info in a chart, utilizing the above formula:

	Quantity (in liters)	× Concentration (of antifreeze)	= Actual Amount (of antifreeze)
Original	10	42%	4.2

Drain	x	42%	$0.42x$
Replace	x	100%	x
Final	10	58%	5.8

Notice that there are four rows in our chart, representing each of the four phases of the problem.

In the statement of the problem, we are given that it's a 10-liter radiator filled with 42% antifreeze. The product of these two numbers is 4.2, as per the formula given above.

The question asks us how much to remove. We don't know, so this is where a variable like x comes in. What is the concentration of the x liters we're to remove? It's the same as the concentration of the entire 10 liters: 42%. And, of course, the product of x with 42% is $0.42x$.

Now for the 3rd row of the chart. The number of liters of pure antifreeze needed to refill the radiator must be equal to the number of liters that were drained off, namely x. And the refill fluid is pure antifreeze, which has a concentration of 100%; hence, the last entry in the row is $x \times 100\%$, or simply x.

The bottom row: The radiator has been refilled, so it contains 10 liters of solution, just as in the original row. The concentration of this final mixture is stipulated to be 58%, and the product is clearly 5.8.

Thus, looking at the right-hand column of the chart (Actual Amount), we can say that we started with 4.2 liters of antifreeze, subtracted $0.42x$ liters, added x liters, and ended up with 5.8 liters of antifreeze. Now we translate these thoughts into an equation:

$$4.2 - 0.42x + x = 5.8$$

$$\Rightarrow \quad 0.58x = 1.6$$

$\Rightarrow \quad x = 2.76$, and thus

2.76 liters should be drained out.

Note: If the antifreeze solution needs to be diluted (that is, bring the concentration down), then we drain a certain amount of the solution and replace it with pure water, which has an antifreeze concentration of 0%.

Homework

23. A 9-liter radiator is filled with 67% antifreeze solution. How much of the antifreeze solution must be drained and replaced with pure antifreeze to bring the concentration of the final mixture up to 95% antifreeze?

24. A 5-liter radiator is filled with 58% antifreeze solution. How much of the antifreeze solution must be drained and replaced with pure water to bring the concentration of the final mixture down to 25% antifreeze?

25. A 4-liter radiator is filled with 32% antifreeze solution. How much of the antifreeze solution must be drained and replaced with pure antifreeze to bring the concentration of the final mixture up to 75% antifreeze?

26. A 20-liter radiator is filled with 75% antifreeze solution. How much of the antifreeze solution must be drained and replaced with pure water to bring the concentration of the final mixture down to 23% antifreeze?

Solutions

1. 12 qts acid; 28 qts H_2O

2. 5 kg NaCl; 20 kg H_2O

3. 100%

4. 0%

5. 100% antifreeze; 0% water

6. 0% hydrochloric acid; 100% water

7. a. $x = 1, y = 3$

Complete Check:

$2x + y = 5$
$2(\mathbf{1}) + \mathbf{3} = 5$
$2 + 3 = 5$
$5 = 5$ ✓

$-2x + 7y = 19$
$-2(\mathbf{1}) + 7(\mathbf{3}) = 19$
$-2 + 21 = 19$
$19 = 19$ ✓

b. $a = -2, b = -5$

c. $u = 8, v = 0$

d. $x = 0, y = -2$

8. 12 pounds of the 97% solution and 3 pounds of the 67% solution

9. 16 ounces of pure alcohol and 8 ounces of the 76% alcohol medicine

10. 4 ounces of water and 16 ounces of the 80% albuterol inhalant

11. 18 mL of the 59% alcohol medicine and 1 mL of the 21% alcohol medicine

12. 10 pounds of pure poison and 15 pounds of the 20% poison solution

13. 18 L of the 99% solution and 4 L of the 88% solution

14. 30 L of the pure solution and 58 L of the 12% solution

15. 12 L of the 24% solution and 10 L of the 68% solution

16. 10 L of pure water and 28 L of the 57% solution

17. 24 L of the 79% solution and 12 L of the 70% solution

18. 51 L of pure water and 21 L of the 48% solution

19. 20 L of the 9% solution and 48 L of the 94% solution

20. 55 L of pure water and 45 L of the 20% solution

21. 2 L of the 73% solution and 10 L of the 97% solution

22. 40 L of the pure solution and 8 L of the 58% solution

23. 7.64 liters

24. Note: The concentration of antifreeze in pure water is 0%.

$$5(.58) - .58x + 0x = 5(.25) \Rightarrow 2.84 \text{ liters}$$

25. 2.53 liters

26. 13.87 liters

“When you do the common things in life in an uncommon way, you will command the attention of the world.”

- George Washington Carver (1864-1943)