Ch 18 – Exponents

❑ Introduction

We already know that an exponent usually means repeated multiplication, and sometimes we find expressions with two or more exponents involved. We need to know how to simplify such expressions so that they're easier to understand. This chapter also introduces us to using zero as an exponent. Later chapters will deal with negative and fractional exponents.

❑ The Meaning of Exponents

$$x^5 = \underbrace{\boldsymbol{x \cdot x \cdot x \cdot x \cdot x}}_{\text{5 factors of } x}$$

$$3^4 = \underbrace{3 \cdot 3 \cdot 3 \cdot 3}_{\text{4 factors of 3}} = \mathbf{81}$$

$$2^5 = \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{\text{5 factors of 2}} = \mathbf{32}$$

$$1^{10} = \underbrace{1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1}_{\text{10 factors of 1}} = \mathbf{1}$$

$$0^8 = \underbrace{0 \times 0 \times 0 \times 0 \times 0 \times 0 \times 0 \times 0}_{\text{8 factors of 0}} = \mathbf{0}$$

$$(-4)^6 = \underbrace{(-4)(-4)(-4)(-4)(-4)(-4)}_{\text{6 factors of } -4} = \mathbf{4096}$$

$$(-2)^7 = \underbrace{(-2)(-2)(-2)(-2)(-2)(-2)(-2)}_{\text{7 factors of } -2} = \mathbf{-128}$$

Homework

1. Rewrite each expression using exponents:

a. $nnnn$ b. $x \cdot x \cdot x$ c. $a \times a \times a \times a \times a$

d. $(yyy)(zzz)$ e. $ababa$ f. $10qqq$

2. Evaluate each expression:

a. 3^3 b. 2^{10} c. 1^{321}

d. 0^{4231} e. 4^4 f. 5^3

g. $(-2)^3$ h. $(-3)^4$ i. $(-3)^2$

j. $(-1)^{123}$ k. $(-1)^{234}$ l. $(-2)^8$

m. -5^2 n. -10^3 o. -2^4

❑ *EXPONENTS WITH NUMBERS*

This section will introduce the Five Laws of Exponents using numbers only. For each calculation we use the Order of Operations, since these rules are the only ones we know for sure. Also remember that *product* means multiply, *quotient* means divide, and *power* means exponent.

I. Let's calculate a **product of powers**. For example,

$$2^3 \times 2^5 = 8 \times 32 = 256$$

On the other hand, 256 can also be written as 2^8. So it should be clear that we can legally write

$$2^3 \times 2^5 = 2^8$$

Is it possible that we can *multiply powers of the same base* simply by adding the exponents?

II. Now let's raise a **power to a power**. For instance,

$$(5^2)^3 = 25^3 = 15{,}625 \qquad \text{(Order of Operations)}$$

But you can also calculate that $5^6 = 15{,}625$. Hence,

$$(5^2)^3 = 5^6$$

Can calculating *a power of a power* be as simple as multiplying the exponents?

III. It's time for a **power of a product**. We can try this:

$$(2 \times 3)^4 = 6^4 = 1{,}296 \qquad \text{(Order of Operations)}$$

But here's another way to get the same result:

$$2^4 \times 3^4 = 16 \times 81 = 1{,}296$$

Do we raise *a product to a power* merely by raising each factor to the power?

IV. Let's try a **quotient of powers**. Here's an example:

$$\frac{10^6}{10^2} = \frac{1{,}000{,}000}{100} = 10{,}000 \qquad \text{(Order of Operations)}$$

Now watch this: If we subtract the exponents and keep the base of 10, we get 10^4, which is also equal to 10,000.

Let's do a second example where we put the bigger exponent on the bottom:

$$\frac{2^3}{2^7} = \frac{8}{128} = \frac{1}{16}$$

But $\frac{1}{16}$ can also be written as $\frac{1}{2^4}$, where the 4 results from subtracting the exponents. There's something going on here.

V. Our fifth example in this section will look at a **power of a quotient**.

$$\left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{16}{81} \qquad \text{(the meaning of exponent)}$$

But if we simply raise 2 to the 4th power, and then raise 3 to the 4th power, and write one over the other, we get the same result:

$$\frac{2^4}{3^4} = \frac{16}{81}$$

Homework

3. Use the Order of Operations to evaluate each expression:

a. $2^2 \times 2^3$ b. $3^1 \times 3^4$ c. $4^2 \cdot 4^2$

d. $(3^2)^3$ e. $(2^3)^3$ f. $(5^2)^1$

g. $(2 \times 3)^3$ h. $(3 \times 4)^2$ i. $(1 \cdot 10)^6$

j. $\frac{2^7}{2^5}$ k. $\frac{10^6}{10^3}$ l. $\frac{3^3}{3^5}$

m. $\left(\frac{2}{5}\right)^3$ n. $\left(\frac{1}{8}\right)^2$ o. $\left(\frac{3}{4}\right)^5$

❑ *EXPONENTS WITH VARIABLES*

It's now time for a change in tactics, in order to give us a deeper understanding of the laws of exponents. For each of the following five examples, we will "stretch and squish," and then we'll generalize what we observe to an official law of exponents.

I. We start by finding the product of x^3 and x^4:

$$x^3x^4 = (xxx)(xxxx) = xxxxxxx = x^7$$

Notice that the bases (the x's) are the same, and it's a multiplication problem. As long as the bases are the same, and it's a multiplication problem, it appears that we merely need to write down the base, and then add the exponents together to get the exponent of the answer. That is, $\boldsymbol{x^ax^b = x^{a+b}}$.

$$\boxed{x^3x^4 = x^7}$$

II. For our second example, let's raise a power to a power:

$$(x^4)^2 = (xxxx)^2 = (xxxx)(xxxx) = xxxxxxxx = x^8$$

We appear to have a shortcut at hand. Simply multiply the two exponents together and we're done. So, to raise a power to a power, we can write a general rule: $\boldsymbol{(x^a)^b = x^{ab}}$.

$$\boxed{(x^4)^2 = x^8}$$

III. Now we're to try raising a product to a power; for instance,

$$(ab)^5 = (ab)(ab)(ab)(ab)(ab) = (aaaaa)(bbbbb) = a^5b^5$$

In general, when raising a product to a power, raise each factor to the power: $\boldsymbol{(xy)^n = x^ny^n}$.

$$\boxed{(ab)^5 = a^5b^5}$$

Note that the quantity in the parentheses is a single term; there's no adding or subtracting in the parentheses. In fact, if there are two or more terms in the parentheses, this law of exponents does not apply.

IV. Next we divide powers of the same base. We'll need two examples for this concept.

A. $$\frac{x^6}{x^2} = \frac{xxxxxx}{xx} = \frac{\cancel{x}\,\cancel{x}\,xxxx}{\cancel{x}\,\cancel{x}} = x^4$$

B. $$\frac{y^3}{y^6} = \frac{yyy}{yyyyyy} = \frac{\cancel{y}\,\cancel{y}\,\cancel{y}}{\cancel{y}\,\cancel{y}\,\cancel{y}\,yyy} = \frac{1}{y^3}$$

$$\frac{x^6}{x^2} = x^4$$

$$\frac{y^3}{y^6} = \frac{1}{y^3}$$

In general, when dividing powers of the same base, <u>subtract</u> the exponents, leaving the remaining factors on the top if the top exponent is bigger, and on the bottom if the bottom exponent is bigger: $\boldsymbol{\frac{x^a}{x^b} = x^{a-b}}$

V. Our last example in this section is the process of raising a quotient to a power. As usual, we stretch and squish; then we generalize to a law of exponents.

$$\left(\frac{a}{b}\right)^4 = \left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right) = \frac{aaaa}{bbbb} = \frac{a^4}{b^4}$$

$$\left(\frac{a}{b}\right)^4 = \frac{a^4}{b^4}$$

In general, we can raise a quotient to a power by raising both the top and bottom to the power: $\boldsymbol{\left(\frac{a}{b}\right)^n = \frac{a^b}{b^n}}$.

Homework

4. Use the stretch-and-squish technique to simplify each expression:

a. x^2x^4 b. yy^5 c. $a^4a^3a^2$ d. $z^{10}z^{10}$

e. $(a^2)^3$ f. $(b^3)^2$ g. $(y^3)^3$ h. $(w^4)^2$

i. $(xy)^3$ j. $(ab)^2$ k. $(cd)^4$ l. $(wz)^5$

m. $\frac{a^6}{a^2}$ n. $\frac{x^{10}}{x^9}$ o. $\frac{b^5}{b^8}$ p. $\frac{y}{y^7}$

q. $\left(\frac{x}{y}\right)^2$ r. $\left(\frac{a}{b}\right)^3$ s. $\left(\frac{w}{z}\right)^5$ t. $\left(\frac{g}{h}\right)^6$

❑ *SUMMARY OF THE FIVE LAWS OF EXPONENTS*

Exponent Law	Example
$x^a x^b = x^{a+b}$	$x^2 x^6 = x^8$
$(x^a)^b = x^{ab}$	$(a^4)^3 = a^{12}$
$(xy)^n = x^n y^n$	$(wz)^7 = w^7 z^7$
For $a > b$, $\frac{x^a}{x^b} = x^{a-b}$ For $b > a$, $\frac{x^a}{x^b} = \frac{1}{x^{b-a}}$	$\frac{x^{10}}{x^2} = x^8$ $\frac{a^3}{a^7} = \frac{1}{a^4}$
$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$	$\left(\frac{a}{b}\right)^5 = \frac{a^5}{b^5}$

❑ *SINGLE-STEP EXAMPLES*

EXAMPLE 1:

A. $A^7A^5 = A^{7+5} = A^{12}$

The bases are the same, and it's a multiplication problem. So we can simply write the base and add the exponents.

B. $x^2x^3x^4 = x^{2+3+4} = x^9$

All the bases are the same, and it's a multiplication problem, and so we simply add the exponents.

C. $(x+y)^4(x+y)^9 = (x+y)^{13}$

It doesn't matter what the base is, as long as we're multiplying powers of the <u>same</u> base.

EXAMPLE 2:

A. $(c^{10})^2 = c^{20}$

Raising a base to a power, and then raising that result to a further power requires simply that we multiply the exponents.

B. $\left((x^2)^3\right)^4 = x^{24}$

Power to a power to a power? Just multiply all three exponents.

EXAMPLE 3:

A. $(ax)^5 = a^5x^5$

It's a power of a product (a single term). So just raise each factor to the 5th power.

B. $(abc)^7 = a^7b^7c^7$

Even a term with three factors can be raised to the 7th power by raising each factor to the 7th power.

EXAMPLE 4:

A. $\frac{x^7}{x^5} = x^2$

Since 7 > 5, we divide powers of the same base by subtracting the exponents.

B. $\frac{w^{15}}{w^{25}} = \frac{1}{w^{10}}$

Since 25 > 15, we divide the powers of w by subtracting the exponents, leaving the result on the bottom.

EXAMPLE 5:

A. $\left[\frac{x}{z}\right]^7 = \frac{x^7}{z^7}$

To raise a quotient to a power, just raise both the top and bottom to the 7th power.

B. $\left(\frac{a+b}{u-w}\right)^{23} = \frac{(a+b)^{23}}{(u-w)^{23}}$

Just raise top and bottom to the 23rd power.

Homework

5. Use the Five Laws of Exponents to simplify each expression:

a. a^3a^4 b. $x^5x^6x^2$ c. y^3y^3 d. $z^{12}z$

e. $(x^3)^4$ f. $(z^8)^2$ g. $(n^{10})^{10}$ h. $(a^1)^7$

i. $(ab)^3$ j. $(xyz)^5$ k. $(RT)^1$ l. $(math)^5$

m. $\frac{a^8}{a^2}$ n. $\frac{b^3}{b^9}$ o. $\frac{w^5}{w^5}$ p. $\frac{Q^{100}}{Q^{50}}$

q. $\left(\frac{k}{w}\right)^4$ r. $\left(\frac{a}{b}\right)^{99}$ s. $\left(\frac{1}{m}\right)^{20}$ t. $a(bc)^2$

6. Use the Five Laws of Exponents to simplify each expression:

a. a^3a^5 b. $u^5u^7u^2$ c. $y^{30}y^{30}$ d. $z^{14}z$

e. $(x^4)^5$ f. $(z^9)^2$ g. $(n^{100})^{10}$ h. $(a^1)^9$

i. $(xy)^4$ j. $(abc)^{17}$ k. $(pn)^1$ l. $(love)^4$

m. $\frac{a^{10}}{a^2}$ n. $\frac{b^3}{b^{12}}$ o. $\frac{w^9}{w^9}$ p. $\frac{Q^{100}}{Q^{20}}$

q. $\left(\frac{x}{w}\right)^3$ r. $\left(\frac{a}{b}\right)^{999}$ s. $\left(\frac{1}{z}\right)^{22}$ t. $w(xy)^3$

❑ *WHEN **NOT** TO USE THE FIVE LAWS OF EXPONENTS*

a^5b^6 cannot be simplified. Although the first law of exponents demands that the expressions be multiplied — and they are — it also requires that the bases be the same — and they aren't.

$x^3 + x^4$ cannot be simplified. Even though the bases are the same, the first law of exponents requires that the two powers of x be multiplied.

$w^3 + w^3$ can be simplified, but not by the first law of exponents, since the powers of w are not being multiplied. But the two terms are like terms, which means we simply add them together to get $2w^3$.

$(a + b)^{23}$ does not equal $a^{23} + b^{23}$. The third law of exponents $(xy)^n = x^n y^n$ does not apply because xy is a single term, whereas $a + b$ consists of two terms. However, Chapter 41 will show us a pretty cool way to consider expanding $(a + b)^{23}$, which actually expands out to 24 terms.

Homework

7. Simplify each expression:

a. y^4y^4	b. a^3b^4	c. $x^4x^3x^2$	d. $p^3t^2p^2$
e. $a^3 + a^5$	f. a^3a^5	g. $n^4 + n^4$	h. $x^3 - x^3$
i. $(x + y)^{55}$	j. $Q^2 + Q^2$	k. u^5w^6	l. $h^6 - h^2$
m. $(a - b)^2$	n. $(ab)^2$	o. $(x^3)^3$	p. $x^4 + x^5$
q. $x^{14} + x^{14}$	r. $y^{12} - y^{12}$	s. $a^8 + a^9$	t. $a^{10} + a^{10}$
u. $(xy)^2$	v. $(x + y)^2$	w. a^3b^4	x. $a^3 + b^4$
y. $a(a^2)(b^2)b$	z. $n^6 + n^6$		

❑ MULTI-STEP EXAMPLES

EXAMPLE 6:

A. $(-3x^2y^3)(-5xy^7) = (-3)(-5)\,(x^2x)(y^3y^7) = 15x^3y^{10}$

B. $-2x^2y(xy - 4x^3y^4) = -2x^3y^2 + 8x^5y^5$

C. $\left(2a^2b^3\right)^4 = 2^4(a^2)^4(b^3)^4 = 16a^8b^{12}$

D. $7\left(xy^{10}\right)^5 = 7x^5(y^{10})^5 = 7x^5y^{50}$

E. $\left(\dfrac{a^2}{b^3}\right)^7 = \dfrac{(a^2)^7}{(b^3)^7} = \dfrac{a^{14}}{b^{21}}$

F. $\left(\dfrac{x^3y^9}{xy^{12}}\right)^5 = \left(\dfrac{x^2}{y^3}\right)^5 = \dfrac{(x^2)^5}{(y^3)^5} = \dfrac{x^{10}}{y^{15}}$

Homework

8. Simplify each expression:

a. $(-5a^3b^4)(-2a^2b)$

b. $(7xy)(-7x^2y^5)$

c. $(-2uw)(2uw)$

d. $x^3(2x^2 - x - 1)$

e. $3y^2(3y^2 - y + 3)$

f. $\left(a^2b^3\right)^4$

g. $\left(-5m^3n^{10}\right)^3$

h. $\left[-3p^3q^3\right]^4$

i. $4\left(xy^7\right)^{10}$

j. $-10(-2c^3y^4)^3$

k. $\left(\dfrac{a^3}{c^2}\right)^{10}$

l. $\left[\dfrac{2x^3}{3xy^4}\right]^3$

m. $\left(\dfrac{a^2b^3}{a^4b}\right)^5$

n. $2(3x^2y^3)^4$

❑ *ZERO AS AN EXPONENT*

Approach #1: Now for the interesting exponent of zero — what in the world could 2^0, for instance, possibly mean? If your first instinct is 0, then I might be inclined to agree with you, but we'd both be wrong! Let's make a list of known ***powers of* 2**, determine the pattern which exists, and then extend that pattern to figure out what 2^0 is.

$$\begin{aligned} 2^4 &= 16 \\ 2^3 &= 8 \\ 2^2 &= 4 \\ 2^1 &= 2 \end{aligned}$$

The exponents in the first column are clearly decreasing by 1 at each step, and each number in the right column is one-half of the number above it.

Look at the pattern in the powers of 2 in the first column: The exponents are decreasing one at a time; the next power of 2 in the sequence ought to be 2^0. Now look at the sequence of numbers 16, 8, 4, and 2 in the second column. Each number is one-half the preceding number; that is, we're dividing by 2 at each step. Therefore, the next number in the sequence should be 1 (which is half of 2). So, continuing the two patterns extends the list above to the following:

$$\begin{aligned} 2^4 &= 16 \\ 2^3 &= 8 \\ 2^2 &= 4 \\ 2^1 &= 2 \\ \hline \mathbf{2^0} &= \mathbf{1} \end{aligned}$$

This leads to the conclusion that $2^0 = 1$, a very strange result, indeed.

<u>Approach #2:</u> Here's a more powerful way to deduce the meaning of the zero exponent: Consider the expression

$$x^3x^0, \text{ where we assume } x \neq 0.$$

To figure out the meaning of x^0, we can use the first law of exponents to calculate

$$x^3x^0 = x^{3+0} = x^3$$

That is,

$$x^3x^0 = x^3$$

Now "isolate" the x^0, since that's what we're trying to find the value of. We do this by dividing each side of the equation by x^3:

$$\frac{x^3x^0}{x^3} = \frac{x^3}{x^3}$$

It's legal to divide by x^3, since we've stipulated that $x \neq 0$.

which implies that

$$x^0 = 1$$

and we're done:

Any number (except 0) raised to the zero power is 1.

<u>EXAMPLE 7:</u>

A. $(x - 3y + z)^0 = \mathbf{1}$ (any quantity ($\neq 0$) to the zero power is 1)

B. $(abc)^0 = \mathbf{1}$ (any quantity ($\neq 0$) to the zero power is 1)

C. $a + b^0 = \boldsymbol{a + 1}$ (the exponent is on the b only)

D. $uw^0 = u(1) = \boldsymbol{u}$ (the exponent is on the w only)

E. $(-187)^0 = \mathbf{1}$ (the exponent is on the -187)

F. $-14^0 = \mathbf{-1}$ (the exponent is on the 14, not on the minus sign)

Homework

9. Evaluate each expression:

a. $2^0 + 2^0$ b. $2^0 \cdot 2^0$ c. $2^0 + 2^1 + 2^2 + 2^3 + 2^4$

d. $(1 + 1)^0$ e. $2^5 - 2^3 + 2^1 - 2^0$ f. $(8 - 6)^0 + (10 - 8)^1$

g. $2^0 \times 2^1 \times 2^2 \times 2^3 \times 2^4$

h. $\left(\frac{12}{6}\right)^0 + \left(\frac{100}{50}\right)^0 - (20 - 15 - 3)^0 + \left(3^2 - 7\right)^1$

10. Simplify each expression:

a. x^0 b. xy^0 c. $x + y^0$ d. $(x + y)^0$

e. $(ab)^0$ f. $\left(\frac{a^2}{b^3}\right)^0$ g. $\frac{(x^2)^0}{y^3}$ h. $m^0 m$

i. $x^0 + x^0$ j. $Q^0 Q^0$ k. $a^0 - a^0$ l. $\left(\frac{2x^2y^0}{-3ab^{10}}\right)^0$

Practice Problems

11. Simplify: $\left(-3x^3y^4x^7\right)^3$

12. Simplify: $-3\left(x^5x^4x^8\right)^3$

13. Simplify: $\frac{a^2b^3c^9}{ab^4c^3}$

14. Simplify: $-2y^3(3y^4 - 2y^3 + 1)$

15. Simplify: $x^{12} + x^{14}$

16. Simplify: $u^{22} + u^{22}$

17. Simplify: $abcd^0e^0$

18. Simplify: $\left[\dfrac{10^0a^0b^{14}}{a^3b^7}\right]^5$

Solutions

1. a. n^4 b. x^3 c. a^5 d. y^3z^3 e. a^3b^2 f. $10q^3$

2. a. 27 b. 1024 c. 1 d. 0 e. 256 f. 125
g. –8 h. 81 i. 9 j. –1 k. 1 l. 256
m. This is not the square of –5 (there are no parentheses). It's the opposite of 5^2; so the answer is –25.
n. –1000 o. –16

3. a. $2^2 \times 2^3 = 4 \times 8 = 32$ b. 243 c. 256
d. $(3^2)^3 = 9^3 = 729$ e. 512 f. 25
g. $(2 \times 3)^3 = 6^3 = 216$ h. 144 i. 1,000,000
j. $\dfrac{2^7}{2^5} = \dfrac{128}{32} = 4$ k. 1000 l. $\dfrac{1}{9}$
m. $\left(\dfrac{2}{5}\right)^3 = \dfrac{2}{5}\times\dfrac{2}{5}\times\dfrac{2}{5} = \dfrac{8}{125}$ n. $\dfrac{1}{64}$ o. $\dfrac{243}{1024}$

4. a. $x^2x^4 = (xx)(xxxx) = xxxxxx = x^6$
b. y^6 c. a^9 d. z^{20}
e. $(a^2)^3 = (aa)^3 = (aa)(aa)(aa) = aaaaaa = a^6$
f. b^6 g. y^9 h. w^8
i. $(xy)^3 = (xy)(xy)(xy) = xxxyyy = x^3y^3$
j. a^2b^2 k. c^4d^4 l. w^5z^5

m. $\dfrac{a^6}{a^2} = \dfrac{aaaa\not{a}\not{a}}{\not{a}\not{a}} = aaaa = a^4$ n. x

o. $\dfrac{b^5}{b^8} = \dfrac{\not{b}\not{b}\not{b}\not{b}\not{b}}{\not{b}\not{b}\not{b}\not{b}\not{b}bbb} = \dfrac{1}{bbb} = \dfrac{1}{b^3}$ p. $\dfrac{1}{y^6}$

q. $\left(\dfrac{x}{y}\right)^2 = \left(\dfrac{x}{y}\right)\left(\dfrac{x}{y}\right) = \dfrac{x^2}{y^2}$ r. $\dfrac{a^3}{b^3}$ s. $\dfrac{w^5}{z^5}$ t. $\dfrac{g^6}{h^6}$

5. a. a^7 b. x^{13} c. y^6 d. z^{13}
e. x^{12} f. z^{16} g. n^{100} h. a^7
i. a^3b^3 j. $x^5y^5z^5$ k. RT l. $m^5a^5t^5h^5$
m. a^6 n. $\dfrac{1}{b^6}$ o. 1 p. Q^{50}
q. $\dfrac{k^4}{w^4}$ r. $\dfrac{a^{99}}{b^{99}}$ s. $\dfrac{1}{m^{20}}$ t. ab^2c^2

6. a. a^8 b. u^{14} c. y^{60} d. z^{15}
e. x^{20} f. z^{18} g. n^{1000} h. a^9
i. x^4y^4 j. $a^{17}b^{17}c^{17}$ k. pn l. $l^4o^4v^4e^4$
m. a^8 n. $\dfrac{1}{b^9}$ o. 1 p. Q^{80}
q. $\dfrac{x^3}{w^3}$ r. $\dfrac{a^{999}}{b^{999}}$ s. $\dfrac{1}{z^{22}}$ t. wx^3y^3

7. a. y^8 b. As is c. x^9 d. p^5t^2
e. As is f. a^8 g. $2n^4$ h. 0
i. As is (for now) j. $2Q^2$ k. As is l. As is
m. $a^2 - 2ab + b^2$ n. a^2b^2 o. x^9 p. As is
q. $2x^{14}$ r. 0 s. As is t. $2a^{10}$
u. x^2y^2 v. $x^2 + 2xy + y^2$ w. As is
x. As is y. a^3b^3 z. $2n^6$

8. a. $10a^5b^5$ b. $-49x^3y^6$ c. $-4u^2w^2$ d. $2x^5 - x^4 - x^3$
e. $9y^4 - 3y^3 + 9y^2$ f. a^8b^{12} g. $-125m^9n^{30}$

h. $81p^{12}q^{12}$ i. $4x^{10}y^{70}$ j. $80c^9y^{12}$ k. $\frac{a^{30}}{c^{20}}$

l. $\frac{8x^6}{27y^{12}}$ m. $\frac{b^{10}}{a^{10}}$ n. $162x^8y^{12}$

9. a. 2 b. 1 c. 31 d. 1
e. 25 f. 3 g. 1024 h. 3

10. a. 1 b. x c. $x + 1$ d. 1 e. 1 f. 1
g. $\frac{1}{y^3}$ h. m i. 2 j. 1 k. 0 l. 1

11. $-27x^{30}y^{12}$ **12**. $-3x^{51}$ **13**. $\frac{ac^6}{b}$

14. $-6y^7 + 4y^6 - 2y^3$ **15**. As is **16**. $2u^{22}$

17. abc **18**. $\frac{b^{35}}{a^{15}}$

"What sculpture is to a block of marble, education is to the human soul."

Joseph Addison