CH 20 – FRACTIONS, PART I

❑ INTRODUCTION

No matter what a fraction is used for, or no matter how complicated it looks, a fraction ultimately represents division. For example, $\frac{6}{3}$ is a fraction, but it is also the division problem 6 ÷ 3, which is why we write

$$\frac{6}{3} = 2$$

The fraction $\frac{1}{4}$ is a division problem (it equals 0.25) even if we never actually carry out the division.

The top of a fraction is called the ***numerator***, while the bottom is called the ***denominator***.

The ***reciprocal*** of a fraction is obtained by swapping the numerator and denominator; that is, by "inverting" the fraction. As examples, the reciprocal of $\frac{2}{7}$ is $\frac{7}{2}$, the reciprocal of $-\frac{9}{5}$ is $-\frac{5}{9}$, the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$, the reciprocal of x is $\frac{1}{x}$, and the reciprocal of $\frac{1}{x}$ is x.

The only number which does not possess a reciprocal is 0 — the reciprocal of 0 would have to be $\frac{1}{0}$, but we've learned that this fraction is undefined.

Last, an interesting property of reciprocals is that the product of a number and its reciprocal is always 1. For example, $\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$.

Homework

1. True/False: Every real number has a reciprocal.

2. Find the **reciprocal** of each number:

a. $\frac{3}{2}$ b. $-\frac{4}{5}$ c. $-\frac{8}{7}$ d. 7 e. -14

f. $-\frac{1}{7}$ g. a h. $\frac{1}{x}$ i. $\frac{x}{y}$ j. $-\frac{y}{x}$

k. $-T$ l. $-\frac{1}{w}$ m. 1 n. -1 o. 0

3. a. What is the reciprocal of $-\frac{7}{19}$?

   b. What is the reciprocal of $-\frac{19}{7}$?

   c. Prove that $-\frac{17}{9}$ and $-\frac{9}{17}$ are reciprocals of each other by calculating their product (multiply them).

❑ *Reducing Number Fractions*

Most of us learned to reduce an arithmetic fraction by dividing the top and the bottom of the fraction by the same (non-zero) number. For example,

$$\frac{30}{75} = \frac{30 \div 15}{75 \div 15} = \frac{2}{5}$$

When it comes to algebra, though, we need to look at reducing a fraction a little differently, since it's kind of hard to divide letters by letters. So now we reduce the same fraction using a different method, a method more appropriate to algebraic fractions. Watch this:

$$\frac{30}{75} = \frac{2 \cdot 3 \cdot 5}{3 \cdot 5 \cdot 5} = \frac{2 \cdot \cancel{3} \cdot \cancel{5}}{\cancel{3} \cdot \cancel{5} \cdot 5} = \frac{2}{5},$$

which is, of course, the same answer as before. Here's what we did. First we factored the top and the bottom into prime factors. Then we "divided out" any factor that was common to both the top and the bottom (since any number divided by itself is 1). Whatever factors remain constitute the final answer. Let's do another example.

$$\frac{18}{54} = \frac{2\cdot3\cdot3}{2\cdot3\cdot3\cdot3} = \frac{\cancel{2}\cdot\cancel{3}\cdot\cancel{3}}{\cancel{2}\cdot\cancel{3}\cdot\cancel{3}\cdot3} = \frac{1}{3}$$

Notice that we don't leave a zero in the numerator just because every factor in the numerator canceled out. Remember that canceling is really dividing, so each time we cancel a pair of factors, we're really dividing some number by itself, which is always 1.

Homework

4. Reduce each fraction to lowest terms by factoring into primes:

a. $\frac{20}{22}$ b. $\frac{17}{51}$ c. $\frac{84}{174}$ d. $\frac{22}{33}$ e. $\frac{20}{41}$

f. $\frac{30}{50}$ g. $\frac{34}{51}$ h. $\frac{13}{39}$ i. $\frac{26}{65}$ j. $\frac{32}{128}$

❑ *Reducing Fractions by Factoring Out the GCF*

If we can reduce an arithmetic fraction by factoring and dividing out common factors, we should be able to reduce an algebraic fraction the same way. Here are some examples.

<u>EXAMPLE 1:</u> **Reduce each fraction to lowest terms:**

A. $$\frac{2x+8}{2a-10} = \frac{2(x+4)}{2(a-5)} = \frac{\cancel{2}(x+4)}{\cancel{2}(a-5)} = \boldsymbol{\frac{x+4}{a-5}}$$

B. $\dfrac{ax+bx}{xy-xz} = \dfrac{x(a+b)}{x(y-z)} = \dfrac{\cancel{x}(a+b)}{\cancel{x}(y-z)} = \mathbf{\dfrac{a+b}{y-z}}$

C. $\dfrac{bn+an}{aq+bq} = \dfrac{n(b+a)}{q(a+b)} = \dfrac{n(a+b)}{q(a+b)} = \dfrac{n\cancel{(a+b)}}{q\cancel{(a+b)}} = \mathbf{\dfrac{n}{q}}$

D. $\dfrac{ux-uw}{ax+aw} = \dfrac{u(x-w)}{a(x+w)}$

There's no common factor to cancel. So the original fraction is **not reducible**.

<u>EXAMPLE 2:</u> **Reduce each fraction to lowest terms:**

A. $\dfrac{x^2+x}{x} = \dfrac{x(x+1)}{x} = \dfrac{\cancel{x}(x+1)}{\cancel{x}} = \mathbf{x+1}$

B. $\dfrac{n^2-n}{n-1} = \dfrac{n(n-1)}{n-1} = \dfrac{n\cancel{(n-1)}}{\cancel{n-1}} = \mathbf{n}$

C. $\dfrac{a}{a^2-3a} = \dfrac{a}{a(a-3)} = \dfrac{\cancel{a}}{\cancel{a}(a-3)} = \mathbf{\dfrac{1}{a-3}}$

D. $\dfrac{u-4}{u^2-4u} = \dfrac{u-4}{u(u-4)} = \dfrac{\cancel{u-4}}{u\cancel{(u-4)}} = \mathbf{\dfrac{1}{u}}$

Here's a little trick that might help you understand better; we factor a 1 from the numerator:

$$\dfrac{u-4}{u^2-4u} = \dfrac{1(u-4)}{u(u-4)} = \dfrac{1\cancel{(u-4)}}{u\cancel{(u-4)}} = \mathbf{\dfrac{1}{u}}$$

The Two Steps to Reduce a Fraction to Lowest Terms

1) <u>Factor</u> the top and the bottom of the fraction.
2) <u>Divide</u> out (cancel) any factor common to the top and bottom.

We now have a technique for reducing fractions to lowest terms, but it may still be a little hard to believe, for instance, that $\frac{n^2-n}{n-1} = n$, as in part B of the previous example. Perhaps you'll feel a little more confident in this answer if we substitute a number for n and verify the equality ourselves. So let's choose $n = 10$; then

$$\frac{n^2-n}{n-1} = \frac{\mathbf{10}^2-\mathbf{10}}{\mathbf{10}-1} = \frac{100-10}{10-1} = \frac{90}{9} = 10, \text{ which does equals } n.$$

EXAMPLE 3: **Reduce each fraction to lowest terms:**

A. $\frac{w-u}{w-u}$

Since anything (except zero) divided by itself is 1, the fraction reduces to **1**. For example,

$$\frac{10-3}{10-3} = \frac{7}{7} = 1$$

B. $\frac{x-y}{y-x}$

The top and bottom are not the same, but if you look carefully, you'll see that they're *opposites* of each other. Our trick here will be to factor a -1 out of the bottom:

$$\frac{x-y}{y-x} = \frac{1(x-y)}{-1(-y+x)} = \frac{1(x-y)}{-1(x-y)} = \frac{1\cancel{(x-y)}}{-1\cancel{(x-y)}} = -\mathbf{1}$$

For instance,

$$\frac{7-2}{2-7} = \frac{5}{-5} = -1$$

C. $\dfrac{a+b}{a-b}$

Unlike part A, the top and bottom are not the same, so the final answer is not 1. Unlike part B, the top and bottom are not opposites of each other, so the answer is not –1. What relationship do the top and bottom have? Basically, none at all; there's nothing we can do here. Thus, the fraction is **not reducible**.

Here's an example: $\dfrac{10+1}{10-1} = \dfrac{11}{9}$, and that's the end of it.

Homework

5. Reduce each fraction to lowest terms:

a. $\dfrac{3x-12}{3n+21}$ b. $\dfrac{ax+bx}{ay+by}$ c. $\dfrac{tx+tz}{ty-tz}$

d. $\dfrac{xy+yz}{ay-by}$ e. $\dfrac{aR+aT}{bR+bT}$ f. $\dfrac{mx+my}{ax-ay}$

g. $\dfrac{ax+bx}{ax-cx}$ h. $\dfrac{ax-bx}{ay-by}$ i. $\dfrac{a}{am+an}$

j. $\dfrac{PT-QT}{T}$ k. $\dfrac{gn+hn}{gn-hn}$ l. $\dfrac{ab-ac}{cx-xy}$

6. Reduce each fraction to lowest terms:

a. $\dfrac{x^2+3x}{x}$ b. $\dfrac{n^2-n}{n-1}$ c. $\dfrac{z}{z+z^2}$ d. $\dfrac{Q-3}{QR-3R}$

e. $\dfrac{y^2-y}{y}$ f. $\dfrac{a^2+a}{a+1}$ g. $\dfrac{t-1}{t^2-t}$ h. $\dfrac{n}{n^2+9n}$

i. $\dfrac{a-3}{a^2-3a}$ j. $\dfrac{c+2}{c^2+c}$ k. $\dfrac{z^2-3z}{z-3}$ l. $\dfrac{Q^2-10Q}{Q}$

m. $\dfrac{c}{ac-c^2}$ n. $\dfrac{rx+x^2}{r+1}$ o. $\dfrac{ax+bx^2}{xy-xz^2}$ p. $\dfrac{an+b}{n}$

7. Reduce each fraction to lowest terms:

a. $\frac{x+z}{z+x}$ b. $\frac{Q-R}{R-Q}$ c. $\frac{b-a}{-a+b}$ d. $\frac{m+n}{m-n}$

e. $\frac{d+e}{-e-d}$ f. $\frac{w-z}{z+w}$ g. $\frac{f+g}{g-f}$ h. $\frac{c+d}{d-c}$

i. $\frac{t-n}{n-t}$ j. $\frac{t-n}{n+t}$ k. $\frac{c-x}{-x+c}$ l. $\frac{y-w}{w-y}$

❑ *REDUCING FRACTIONS BY STANDARD FACTORING*

<u>EXAMPLE 4:</u> **Reduce each fraction to lowest terms:**

A. $$\frac{x^2+5x+6}{x^2-4} = \frac{(x+3)(x+2)}{(x+2)(x-2)} = \frac{(x+3)\cancel{(x+2)}}{\cancel{(x+2)}(x-2)} = \mathbf{\frac{x+3}{x-2}}$$

B. $$\frac{6n^2+5n-21}{3n^2+22n+35} = \frac{(2n-3)(3n+7)}{(n+5)(3n+7)} = \frac{(2n-3)\cancel{(3n+7)}}{(n+5)\cancel{(3n+7)}} = \mathbf{\frac{2n-3}{n+5}}$$

C. $$\frac{x+5}{2x^2+7x-15} = \frac{x+5}{(2x-3)(x+5)} = \frac{\cancel{x+5}}{(2x-3)\cancel{(x+5)}} = \mathbf{\frac{1}{2x-3}}$$

D. $$\frac{3w^2-2w-5}{3w-5} = \frac{(3w-5)(w+1)}{3w-5} = \frac{\cancel{(3w-5)}(w+1)}{\cancel{3w-5}} = \mathbf{w+1}$$

E. $$\frac{a^2-25}{a^2+5a+6} = \frac{(a+5)(a-5)}{(a+3)(a+2)}$$ which contains no common factors.

Therefore, this fraction is **not reducible**.

EXAMPLE 5: **Reduce to lowest terms:** $\dfrac{\mathbf{5a^2 - 45}}{\mathbf{a^2 + 6a + 9}}$

Solution: We need to factor the numerator and the denominator. If we then see any common factors, we can divide them out.

$$\frac{5a^2-45}{a^2+6a+9} = \frac{5(a^2-9)}{a^2+6a+9} = \frac{5(a+3)(a-3)}{(a+3)(a+3)} = \frac{5\cancel{(a+3)}(a-3)}{\cancel{(a+3)}(a+3)}$$

Notice that factoring the numerator required two steps: pulling out the greatest common factor of 5, followed by factoring the $a^2 - 9$. If we hadn't factored out the 5 first, we would never have been able to divide out anything, and we would have reached the erroneous conclusion that the fraction is not reducible.

Thus, the final answer (after distributing the 5 to the $a - 3$) is

$$\boxed{\frac{5a-15}{a+3}}$$

EXAMPLE 6: **Reduce to lowest terms:** $\dfrac{\mathbf{x^3 - 5x^2 - 2x + 10}}{\mathbf{x^2 - 15x + 50}}$

Solution: Let's factor the top first, by grouping the first two terms and the last two terms:

$$x^3 - 5x^2 - 2x + 10$$
$$= x^2(x - 5) - 2(x - 5)$$
$$= (x - 5)(x^2 - 2)$$

This result, together with factoring the bottom gives the fraction

$\dfrac{(x-5)(x^2-2)}{(x-5)(x-10)}$, which then clearly reduces to

$$\boxed{\frac{x^2-2}{x-10}}$$

Homework

8. Reduce each fraction to lowest terms:

a. $\dfrac{n^2-9}{n^2+6n+9}$

b. $\dfrac{4a^2+4a+1}{2a^2+5a+2}$

c. $\dfrac{c+7}{c^2-49}$

d. $\dfrac{4w^2-9}{2w-3}$

e. $\dfrac{6x^2+11x-7}{2x^2+17x-9}$

f. $\dfrac{h^2+3h+2}{h^2-3h+2}$

g. $\dfrac{3k^2-17k+1}{3k^2-17k+1}$

h. $\dfrac{100-w^2}{w^2+10w}$

i. $\dfrac{10x^2+11x-6}{5x^2-12x+4}$

j. $\dfrac{x^2+7x+10}{x^2+3x+2}$

k. $\dfrac{n^2-9}{n^2+4n+3}$

l. $\dfrac{y-7}{y^2-14y+49}$

m. $\dfrac{w^2-81}{w+9}$

n. $\dfrac{m^2+10m+25}{m^2-25}$

o. $\dfrac{x^2+2x+1}{x^2-4}$

p. $\dfrac{6a^2+13a-5}{9a^2+12a-5}$

q. $\dfrac{k^2-6k+7}{7-6k+k^2}$

r. $\dfrac{16u^2+34u-15}{2u^2+3u-5}$

9. Reduce each fraction to lowest terms:

a. $\dfrac{n^2-4}{n^2-4n+4}$

b. $\dfrac{2x^2+8x+6}{6x^2+18x+12}$

c. $\dfrac{x^2-4x+1}{x^2-4x+1}$

d. $\dfrac{10y^2-30y+20}{5y^2-15y+10}$

e. $\dfrac{2x^2-2}{4x-4}$

f. $\dfrac{3n^2-3n-90}{3n^2+30n+75}$

g. $\dfrac{14x+98}{21x^2-63x-1470}$

h. $\dfrac{5a^2-30a-135}{10a^2-60a-270}$

10. Reduce each fraction to lowest terms:

a. $\dfrac{x^3 + 4x^2 + x + 4}{x^2 + x - 12}$

b. $\dfrac{n^2 + 2n + 1}{n^3 + n^2 + 3n + 3}$

c. $\dfrac{a^3 - a^2 - 5a + 5}{a^2 + 3a - 4}$

d. $\dfrac{w^2 - 25}{w^3 - 5w^2 + 3w - 15}$

❑ *ADDING AND SUBTRACTING FRACTIONS WITH THE SAME DENOMINATOR*

$$\frac{2}{9} + \frac{5}{9} = \frac{2+5}{9} = \frac{7}{9} \quad \Rightarrow \quad \frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

To **add** fractions with the same (common) denominator

i) the numerator of the answer is the sum of the two numerators

ii) the denominator of the answer is the common denominator

$$\frac{5}{11} - \frac{3}{11} = \frac{5-3}{11} = \frac{2}{11}$$

$$\frac{2}{13} - \frac{8}{13} = \frac{-6}{13} = -\frac{6}{13} \quad \Rightarrow \quad \frac{x}{y} - \frac{z}{y} = \frac{x-z}{y}$$

To **subtract** fractions with the same (common) denominator

i) the numerator of the answer is the difference of the two numerators

ii) the denominator of the answer is the common denominator

Homework

11. Perform the indicated operation:

a. $\frac{7}{10}+\frac{2}{10}$ b. $\frac{43}{101}+\frac{44}{101}$ c. $\frac{7}{8}-\frac{3}{8}$ d. $\frac{1}{5}-\frac{3}{5}$

e. $\frac{x}{y}+\frac{z}{y}$ f. $\frac{m}{u}-\frac{q}{u}$ g. $\frac{c}{a}+\frac{d}{a}$ h. $\frac{t}{w}-\frac{z}{w}$

i. $\frac{b}{c}+\frac{7}{c}$ j. $\frac{9}{Q}-\frac{7}{Q}$ k. $\frac{5}{R}-\frac{10}{R}$ l. $\frac{1}{3a}+\frac{7}{3a}$

m. $\frac{a}{bc}-\frac{d}{bc}$ n. $\frac{a}{x^2}+\frac{b}{x^2}$ o. $\frac{3}{10x}+\frac{4}{10x}$ p. $\frac{u}{a^3}-\frac{b}{a^3}$

❑ ADDING AND SUBTRACTING FRACTIONS WITH DIFFERENT DENOMINATORS

$$\frac{2}{3}+\frac{5}{7}$$
$$= \frac{2}{3}\left[\frac{7}{7}\right]+\frac{5}{7}\left[\frac{3}{3}\right]$$
$$= \frac{14}{21}+\frac{15}{21}$$
$$= \frac{29}{21}$$

$$\frac{a}{b}+\frac{c}{d}$$
$$= \frac{a}{b}\left[\frac{d}{d}\right]+\frac{c}{d}\left[\frac{b}{b}\right]$$
$$= \frac{ad}{bd}+\frac{bc}{bd}$$
$$= \frac{ad+bc}{bd}$$

$$\begin{aligned} & 3+\frac{4}{5} \\ =\; & \frac{3}{1}+\frac{4}{5} \\ =\; & \frac{3}{1}\left[\frac{5}{5}\right]+\frac{4}{5} \\ =\; & \frac{15}{5}+\frac{4}{5} \\ =\; & \frac{19}{5} \end{aligned}$$

$$\begin{aligned} & x+\frac{y}{z} \\ =\; & \frac{x}{1}+\frac{y}{z} \\ =\; & \frac{x}{1}\left[\frac{z}{z}\right]+\frac{y}{z} \\ =\; & \frac{xz}{z}+\frac{y}{z} \\ =\; & \frac{xz+y}{z} \end{aligned}$$

To **add** or **subtract** fractions with <u>different</u> denominators, each fraction must be written with the same denominator. This is accomplished by multiplying one or both fractions by a fraction equal to 1. A fraction is equal to 1 when the top and bottom are the same.

In the first example above, we see that we need to change each denominator into 21. We do this by multiplying the top and bottom of the first fraction by 7, and then multiplying the top and bottom of the second fraction by 3.

Similarly, in the second example, to achieve a common denominator we multiply the top and bottom of the first fraction by d, and the top and bottom of the second fraction by b. This converts both fractions into fractions with the same denominator, bd. Then they're ready to be added together.

Homework

12. Perform the indicated operation:

a. $\frac{1}{2}+\frac{1}{3}$ b. $\frac{2}{5}-\frac{1}{10}$ c. $\frac{1}{4}-\frac{5}{6}$ d. $\frac{1}{3}-\frac{9}{2}$

e. $\frac{a}{b}+\frac{w}{x}$ f. $\frac{c}{d}-\frac{g}{h}$ g. $\frac{m}{n}+\frac{m}{q}$ h. $\frac{a}{b}-\frac{a}{c}$

i. $\dfrac{6}{a}+\dfrac{3}{b}$ j. $\dfrac{a}{R}-\dfrac{3}{T}$ k. $a+\dfrac{b}{c}$ l. $w-\dfrac{x}{y}$

m. $\dfrac{k}{j}+n$ n. $\dfrac{w}{x}-z$ o. $\dfrac{x^2}{a}+\dfrac{y^2}{a}$ p. $\dfrac{w}{x}+\dfrac{y}{z}$

❑ MULTIPLYING AND DIVIDING FRACTIONS

$$\frac{7}{10}\cdot\frac{11}{15} = \frac{77}{150} \quad\Longrightarrow\quad \frac{m}{n}\cdot\frac{p}{q} = \frac{mp}{nq}$$

To **multiply** fractions

i) the numerator of the answer is the product of the numerators

ii) the denominator of the answer is the product of the denominators

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$$\frac{2}{7}\div\frac{5}{9} = \frac{2}{7}\times\frac{9}{5} = \frac{18}{35} \quad\Longrightarrow\quad \frac{c}{d}\div\frac{u}{w} = \frac{c}{d}\times\frac{w}{u} = \frac{cw}{du}$$

To **divide** fractions, multiply the first fraction by the reciprocal of the second fraction.

$$\frac{5}{6}\times\frac{13}{5} = \frac{\cancel{5}}{6}\times\frac{13}{\cancel{5}} = \frac{13}{6} \quad\Longrightarrow\quad \frac{x}{y}\times\frac{z}{x} = \frac{\cancel{x}}{y}\times\frac{z}{\cancel{x}} = \frac{z}{y}$$

Before actually multiplying the tops and bottoms, sometimes dividing out common factors can simplify the reducing of the final answer.

Homework

13. Perform the indicated operation:

a. $\frac{2}{3}\cdot\frac{5}{9}$ b. $\frac{1}{2}\times\frac{5}{7}$ c. $\left(\frac{8}{9}\right)\left(\frac{9}{10}\right)$ d. $\frac{4}{5}\cdot\frac{5}{4}$

e. $\frac{w}{x}\cdot\frac{w}{z}$ f. $\frac{a}{b}\times\frac{c}{b}$ g. $\frac{x}{y}\cdot\frac{c}{d}$ h. $\left[\frac{a}{b}\right]\left[\frac{b}{a}\right]$

i. $a\times\frac{b}{c}$ j. $\frac{m}{n}\times Q$ k. $z\div\frac{w}{z}$ l. $\frac{u}{w}\div a$

m. $\frac{x}{y}\div\frac{x}{y}$ n. $\frac{a}{b}\div\frac{b}{a}$ o. $\frac{a}{b}\div\frac{a}{c}$ p. $\frac{g}{h}\cdot\frac{g}{h}$

q. $3\cdot\frac{x}{y}$ r. $a\left(\frac{b}{c}\right)$ s. $\left(\frac{w}{7}\right)A$ t. $\frac{w}{7}+A$

14. Perform the indicated operation:

a. $\frac{K}{L}\div\frac{K}{M}$ b. $\frac{n}{b}+\frac{n}{b}$ c. $\frac{x}{a}-\frac{x}{a}$ d. $\frac{a}{b}\cdot\frac{b}{c}$

e. $\frac{G}{H}\div\frac{G}{H}$ f. $\frac{1}{a}+\frac{1}{b}$ g. $\frac{2}{c}-\frac{3}{d}$ h. $\frac{a}{w}-\frac{a}{z}$

i. $\frac{R}{T}\times\frac{T}{R}$ j. $6+\frac{x}{y}$ k. $\frac{u}{m}-n$ l. $\frac{x}{y}\div\frac{x}{z}$

m. $\left(\frac{b}{c}\right)\left(\frac{b}{c}\right)$ n. $\frac{b}{c}\times\frac{c}{b}$ o. $\frac{m}{n}\div\frac{p}{q}$ p. $\frac{p}{q}+\frac{m}{n}$

q. $a\left(\frac{b}{c}\right)$ r. $\frac{w}{n}\div 5$ s. $\frac{p}{m}+E$ t. $\frac{x}{y}+\frac{a}{b}\cdot\frac{b}{a}$

Practice Problems

15. Reduce each fraction to lowest terms:

a. $\frac{5x+20}{10x-45}$ b. $\frac{ax-bx}{cx+dx}$ c. $\frac{am+an}{a}$ d. $\frac{w}{w^2-3w}$

e. $\frac{t^2+t}{t+1}$ f. $\frac{R^2-4R}{R}$ g. $\frac{2x+8}{3x-12}$ h. $\frac{ax+ay}{bx+by}$

16. Reduce each fraction to lowest terms:

a. $\frac{5x+10}{10x+45}$ b. $\frac{a^2-ab}{ac+ad}$ c. $\frac{cm+cn}{c}$ d. $\frac{u}{u^2+7u}$

e. $\frac{t-1}{t^2-t}$ f. $\frac{R^2+99R}{R}$ g. $\frac{2x+8}{6x-12}$ h. $\frac{ax+ay}{bx-by}$

17. Reduce each fraction to lowest terms:

a. $\frac{6a^2-5a-21}{3a^2-4a-7}$ b. $\frac{18x+18}{14x^2+42x+28}$ c. $\frac{x^2-9}{x^2-4}$

d. $\frac{10a^2+29a-21}{5a^2-38a+21}$ e. $\frac{x-3}{x^2-9}$ f. $\frac{n^2+14n+49}{(n+7)^2}$

g. $\frac{x^3-2x^2-3x+6}{x^2-x-2}$ h. $\frac{u^2-14u+49}{u^3-7u^2+6u-42}$ i. $\frac{x^3+x^2+x+1}{1+x+x^2+x^3}$

18. a. What is the reciprocal of $-\frac{2}{9}$?

b. What is the reciprocal of 0?

19. Perform the indicated operation:

a. $\frac{a}{b}+\frac{c}{b}$ b. $\frac{x}{y}+\frac{y}{x}$ c. $a-\frac{w}{u}$ d. $\frac{m}{n}-A$

e. $\frac{x}{y}\cdot\frac{w}{z}$ f. $\frac{x}{z}\times\frac{z}{y}$ g. $\frac{a}{b}\div\frac{c}{d}$ h. $c\div\frac{d}{e}$

i. $\frac{g}{h}\div\pi$ j. $\frac{m}{\pi}\cdot\frac{\pi}{m}$ k. $h+\frac{k}{h}$ l. $\frac{a}{b}\left(\frac{b}{c}\right)$

m. $\left(\frac{a}{t}\right)\left(\frac{t}{a}\right)$ n. $\frac{a}{n^2}+\frac{b}{n^2}$ o. $\frac{8}{ab}+\frac{6}{ab}$ p. $\frac{a}{x^3}-\frac{b}{x^3}$

q. $\frac{w}{4c}+\frac{w}{4c}$ r. $\frac{a}{mn}-\frac{a}{mn}$ s. $\frac{A}{B}\cdot\frac{C}{D}$ t. $\frac{C}{D}\div\frac{B}{A}$

u. $\frac{s}{r}\div s$ v. $Q\div\frac{Q}{R}$ w. $\frac{6}{L}\div\frac{6}{L}$ x. $\frac{M}{Q}\div\frac{Q}{M}$

y. $\frac{ab}{xy}+\frac{c}{xy}$ z. $\frac{abc}{d}-\frac{def}{d}$

Solutions

1. False; 0 does not have a reciprocal.

2. a. $\frac{2}{3}$ b. $-\frac{5}{4}$ c. $-\frac{7}{8}$ d. $\frac{1}{7}$ e. $-\frac{1}{14}$

f. -7 g. $\frac{1}{a}$ h. x i. $\frac{y}{x}$ j. $-\frac{x}{y}$

k. $-\frac{1}{T}$ l. $-w$ m. 1 n. -1 o. Undefined

3. a. $-\frac{19}{7}$ b. $-\frac{7}{19}$ c. $\left(-\frac{17}{9}\right)\left(-\frac{9}{17}\right)=\left(-\frac{{}^{1}\cancel{17}}{{}_{1}\cancel{9}}\right)\left(-\frac{\cancel{9}^{1}}{\cancel{17}_{1}}\right)=1$ ✓

4. a. $\frac{20}{22} = \frac{\cancel{2}\cdot 2\cdot 5}{\cancel{2}\cdot 11} = \frac{10}{11}$ b. $\frac{1}{3}$ c. $\frac{14}{29}$ d. $\frac{2}{3}$ e. $\frac{20}{41}$

f. $\frac{3}{5}$ g. $\frac{2}{3}$ h. $\frac{1}{3}$ i. $\frac{2}{5}$ j. $\frac{1}{4}$

5. a. $\frac{x-4}{n+7}$ b. $\frac{x}{y}$ c. $\frac{x+z}{y-z}$ d. $\frac{x+z}{a-b}$

e. $\frac{a}{b}$ f. Not reducible g. $\frac{a+b}{a-c}$ h. $\frac{x}{y}$

i. $\frac{1}{m+n}$ j. $P-Q$ k. $\frac{g+h}{g-h}$ l. Not reducible

6. a. $x+3$ b. n c. $\frac{1}{1+z}$ d. $\frac{1}{R}$

e. $y-1$ f. a g. $\frac{1}{t}$ h. $\frac{1}{n+9}$

i. $\frac{1}{a}$ j. Not reducible k. z l. $Q-10$

m. $\frac{1}{a-c}$ n. Not reducible o. $\frac{a+bx}{y-z^2}$ p. Not reducible

7. a. 1 b. –1 c. 1 d. Not reducible e. –1 f. Not reducible

g. Not reducible h. Not reducible i. –1 j. Not reducible

k. 1 l. –1

8. a. $\frac{n-3}{n+3}$ b. $\frac{2a+1}{a+2}$ c. $\frac{1}{c-7}$ d. $2w+3$

e. $\frac{3x+7}{x+9}$ f. Not reducible g. 1 h. $\frac{10-w}{w}$

i. $\frac{2x+3}{x-2}$ j. $\frac{x+5}{x+1}$ k. $\frac{n-3}{n+1}$ l. $\frac{1}{y-7}$

m. $w-9$ n. $\frac{m+5}{m-5}$ o. Not reducible p. $\frac{2a+5}{3a+5}$

q. 1 r. $\frac{8u-3}{u-1}$

9. a. $\frac{n+2}{n-2}$ b. $\frac{x+3}{3x+6}$ c. 1 d. 2

e. $\frac{x+1}{2}$ f. $\frac{n-6}{n+5}$ g. $\frac{2}{3x-30}$ h. $\frac{1}{2}$

10. a. $\frac{x^2+1}{x-3}$ b. $\frac{n+1}{n^2+3}$ c. $\frac{a^2-5}{a+4}$ d. $\frac{w+5}{w^2+3}$

11. a. $\frac{9}{10}$ b. $\frac{87}{101}$ c. $\frac{1}{2}$ d. $-\frac{2}{5}$

e. $\frac{x+z}{y}$ f. $\frac{m-q}{u}$ g. $\frac{c+d}{a}$ h. $\frac{t-z}{w}$

i. $\frac{b+7}{c}$ j. $\frac{2}{Q}$ k. $-\frac{5}{R}$ l. $\frac{8}{3a}$

m. $\frac{a-d}{bc}$ n. $\frac{a+b}{x^2}$ o. $\frac{7}{10x}$ p. $\frac{u-b}{a^3}$

12. a. $\frac{5}{6}$ b. $\frac{3}{10}$ c. $-\frac{7}{12}$ d. $-\frac{25}{6}$

e. $\frac{ax+bw}{bx}$ f. $\frac{ch-dg}{dh}$ g. $\frac{mq+mn}{nq}$ h. $\frac{ac-ab}{bc}$

i. $\frac{6b+3a}{ab}$ j. $\frac{aT-3R}{RT}$ k. $\frac{ac+b}{c}$ l. $\frac{wy-x}{y}$

m. $\frac{k+jn}{j}$ n. $\frac{w-xz}{x}$ o. $\frac{x^2+y^2}{a}$ p. $\frac{wz+xy}{xz}$

13. a. $\frac{10}{27}$ b. $\frac{5}{14}$ c. $\frac{4}{5}$ d. 1

e. $\frac{w^2}{xz}$ f. $\frac{ac}{b^2}$ g. $\frac{cx}{dy}$ h. 1

i. $\frac{ab}{c}$ j. $\frac{mQ}{n}$ k. $\frac{z^2}{w}$ l. $\frac{u}{aw}$

m. 1 n. $\frac{a^2}{b^2}$ o. $\frac{c}{b}$ p. $\frac{g^2}{h^2}$

q. $\frac{3x}{y}$ r. $\frac{ab}{c}$ s. $\frac{Aw}{7}$ t. $\frac{w+7A}{7}$

14. a. $\frac{M}{L}$ b. $\frac{2n}{b}$ c. 0 d. $\frac{a}{c}$
e. 1 f. $\frac{a+b}{ab}$ g. $\frac{2d-3c}{cd}$ h. $\frac{az-aw}{wz}$
i. 1 j. $\frac{x+6y}{y}$ k. $\frac{u-mn}{m}$ l. $\frac{z}{y}$
m. $\frac{b^2}{c^2}$ n. 1 o. $\frac{mq}{np}$ p. $\frac{np+mq}{qn}$
q. $\frac{ab}{c}$ r. $\frac{w}{5n}$ s. $\frac{p+Em}{m}$ t. $\frac{x+y}{y}$

15. a. $\frac{x+4}{2x-9}$ b. $\frac{a-b}{c+d}$ c. $m+n$ d. $\frac{1}{w-3}$
e. t f. $R-4$ g. Not reducible h. $\frac{a}{b}$

16. a. $\frac{x+2}{2x+9}$ b. $\frac{a-b}{c+d}$ c. $m+n$ d. $\frac{1}{u+7}$
e. $\frac{1}{t}$ f. $R+99$ g. $\frac{x+4}{3x-6}$ h. Not reducible

17. a. $\frac{2a+3}{a+1}$ b. $\frac{9}{7x+14}$ c. Not reducible d. $\frac{2a+7}{a-7}$
e. $\frac{1}{x+3}$ f. 1 g. $\frac{x^2-3}{x+1}$ h. $\frac{u-7}{u^2+6}$
i. 1

18. a. $-\frac{9}{2}$ b. 0 does not have a reciprocal.

19. a. $\frac{a+c}{b}$ b. $\frac{x^2+y^2}{xy}$ c. $\frac{au-w}{u}$ d. $\frac{m-An}{n}$
e. $\frac{wx}{yz}$ f. $\frac{x}{y}$ g. $\frac{ad}{bc}$ h. $\frac{ce}{d}$
i. $\frac{g}{\pi h}$ j. 1 k. $\frac{h^2+k}{h}$ l. $\frac{a}{c}$
m. 1 n. $\frac{a+b}{n^2}$ o. $\frac{14}{ab}$ p. $\frac{a-b}{x^3}$

q. $\frac{w}{2c}$ r. 0 s. $\frac{AC}{BD}$ t. $\frac{AC}{BD}$

u. $\frac{1}{r}$ v. R w. 1 x. $\frac{M^2}{Q^2}$

y. $\frac{ab+c}{xy}$ z. $\frac{abc-def}{d}$

"It is far better to grasp the Universe as it really is than to persist in delusion, however satisfying and reassuring."

– Carl Sagan (1934-1996)