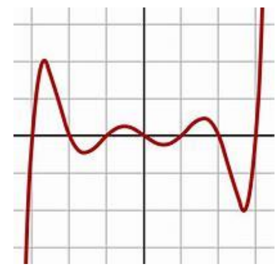
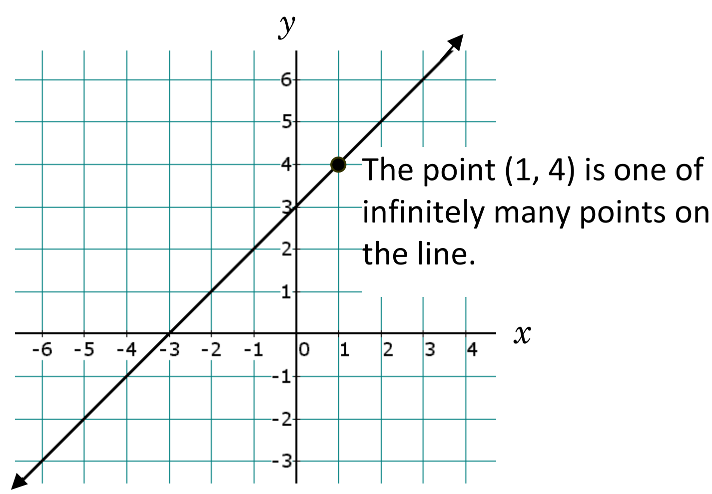
CH 2 – FROM GRAPH TO EQUATION

❑ INTRODUCTION



In this chapter I'm going to give you a graph, but only a small portion of the graph, generally around the origin. We will first analyze points that we can actually see (for instance, I will ask what y-value is associated with a certain x-value). Then I will ask you what y-value is associated with an x-value that you can't even see on the graph! How will we do this? We will use the given graph to create an *algebraic formula* to describe the graph, and then use that formula to predict y-values from any x-value I give you.

EXAMPLE 1: **Consider the following graph:**



a. Calculate the y-value when $x = 100$.

b. Calculate the y-value when $x = 2.7$.

c. Calculate the x-value when $y = 1000$.

Solution: We certainly don't want to extend (draw) the graph all the way to $x = 100$. So our approach for this problem, and throughout this chapter, will be to

1. determine the equation of the given graph by analyzing the relationship between x and y, and
2. use that formula to deduce the y-value for whatever x-value we might be given (and the other way around).

The first step will be to create an x-y table by reading the coordinates off the graph. For example, in the given graph, we see that when $x = 1$, $y = 4$; thus the point (1, 4) is on the graph (listed third from the bottom of the table).

x	y
−4	−1
−3	0
−2	1
−1	2
0	3
1	4
2	5
3	6

Remember that the arrowheads on the graph are used to indicate that the graph (the line) goes forever in both directions. It then follows that x can be any number at all. For example, although it's not easy to see from the graph, if we choose x to be 1.5, it can be guessed that $y = 4.5$. Similarly, we could let $x = \pi$ and then see that y would be a little bigger than 6.

Now let's examine the equation of the line. Looking at the x- and y-values in the table, we might see that the y-value is always 3 more than the x-value. We are therefore led to the formula

$$y = x + 3$$

Check this formula against all the x-y pairs in the table and you will see that it really works. Now we can answer the three questions:

> Guessing certainly isn't very accurate, and that's even when we have a graph to look at. It would be even more difficult to find points on the line if we had to extend the graph ourselves. This is why it's so very important to create an *equation* from the given graph. Assuming that we end up with the right equation, we are guaranteed exact y-values for any given x-value.

a. If $x = 100$, then $y = x + 3 = 100 + 3 =$ **103**.

b. If $x = 2.7$, then $y = x + 3 = 2.7 + 3 =$ **5.7**.

c. Be careful here; note that y is given and we want to find x.

$$y = x + 3 \quad \text{(the formula for the line)}$$

$$\Rightarrow \quad 1000 = x + 3 \quad \text{(let } y = 1000\text{)}$$

$$\Rightarrow \quad 1000 \underline{- 3} = x + 3 \underline{- 3} \quad \text{(subtract 3 from each side)}$$

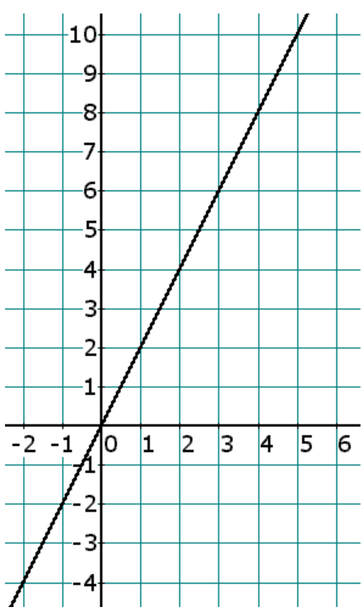
$$\Rightarrow \quad \mathbf{997} = x \quad \text{(solve for } x\text{)}$$

In summary, the following three points lie on the line:

(100, 103) (2.7, 5.7) (997, 1000)

Homework

1. Given the graph, create a formula and then use that formula to answer the questions:



a. When $x = 3$, $y =$ ____.

b. When $x = 0$, $y =$ ____.

c. When $x = -4$, $y =$ ____.

d. When $x = 278$, $y =$ ____.

e. When $x = -900$, $y =$ ____.

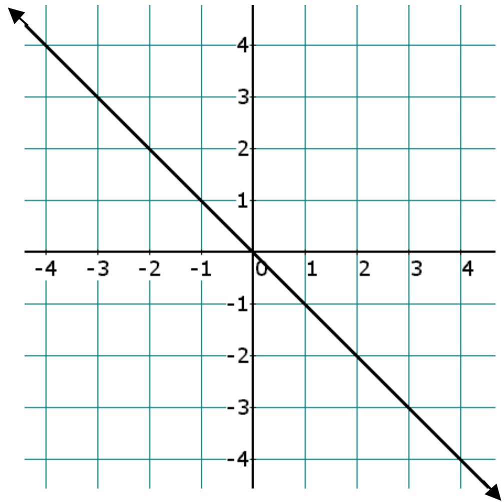
f. When $x = 18\pi$, $y =$ ____.

g. When $y = 10$, $x =$ ____.

h. When $y = 250$, $x =$ ____.

i. When $y = 22\pi$, $x =$ ____.

EXAMPLE 2: **Consider the following graph:**



a. **Calculate the y-value when $x = 1200$.**

b. **Calculate the y-value when $x = -123$.**

c. **Calculate the x-value when $y = -25\pi$.**

Solution: Again, the points in which we're interested are not visible on the graph, so let's create a table based on the graph, and then use that table to construct a formula.

x	-4	-3	-2	-1	0	1	2	3
y	4	3	2	1	0	-1	-2	-3

We see that the y-value is simply the ***opposite*** of the x-value:

$$y = -x$$

Notice that when $x = 0$, $y = -0 = 0$ (which yields the origin); therefore, 0 has an opposite, namely 0. We conclude that every number has an opposite. We can now answer the three questions:

a. If $x = 1200$, then $y = -x = \mathbf{-1200}$.

b. If $x = -123$, then $y = -x = -(-123) = \mathbf{123}$.

c. As in the previous example, the y-value is given and we're asked for the x-value. So let's begin with the formula, plug in the given value of y, and then solve for x:

$$y = -x \qquad \text{(the formula for the line)}$$

$$\Rightarrow \quad -25\pi = -x \qquad \text{(let } y = -25\pi\text{)}$$

$$\Rightarrow \quad 25\pi = x \qquad \text{(divide or multiply each side by } -1\text{)}$$

Therefore, the following three points lie on the line:

(1200, – 1200) (–123, 123) (25π, -25π)

Homework

2. The graph of $y = -x$ lies within Quadrants ___ and ___, and passes through the ___.

3. What is the **opposite** of each number?

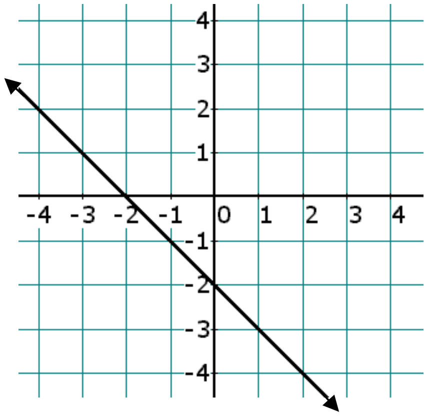
   a. 17 b. 0 c. –3.5 d. 8π e. $-\sqrt{2}$

4. a. T/F: Every number has an opposite.
   b. The opposite of 0 is ____.
   c. The opposite of a negative number is always _____.
   d. The opposite of a positive number is always _____.

5. Consider the formula $y = -x + 5$. Calculate the y-value for the given x-value:

   a. 8 b. 99 c. –10 d. –5 e. 0 f. π g. $-\pi$

6. Given the graph, create a formula and then use that formula to answer the questions:



a. When $x = 2$, $y =$ ___.

b. When $x = 0$, $y =$ ___.

c. When $x = -4$, $y =$ ___.

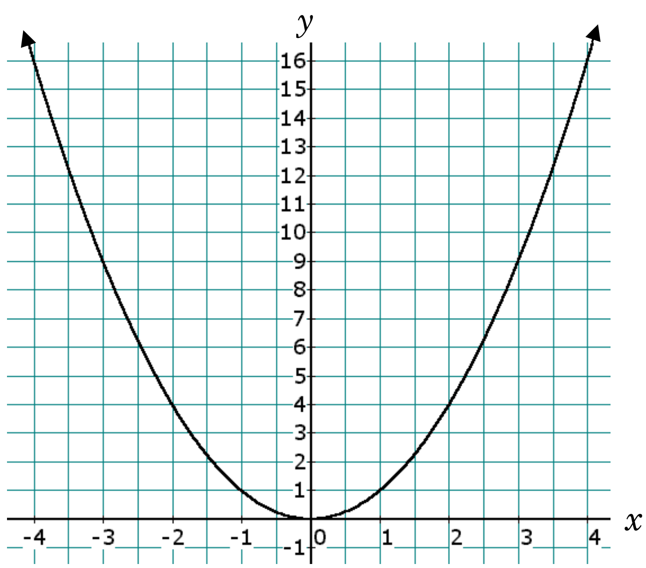
d. When $x = 25$, $y =$ ___.

e. When $x = -200$, $y =$ ___.

f. When $y = 32$, $x =$ ___.

g. When $y = -300$, $x =$ ___.

EXAMPLE 3: **Consider the following graph:**



a. **Calculate the y-value when $x = 1{,}000$.**

b. **Calculate the y-value when $x = 1.2$.**

c. **Calculate the x-values when $y = 625$.**

Solution: This graph is quite different from the previous graphs. It passes through the origin, and the rest of the graph resides only in Quadrants I and II. Like the bell-shaped curve from the homework in Chapter 1, it has symmetry with the y-axis. Let's try to read off some ordered pairs from the graph:

x	–4	–3	–2	–1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16

Now, how is the y-value related to the x-value? When $x = 0$, $y = 0$; this is rather useless, as is the pair (1, 1). But look at the ordered pairs (2, 4), (3, 9) and (4, 16). There's something going on here — it appears that the y-value is simply the *square* of the x-value:

$$y = x^2$$

Lest we jump to conclusions, let's double-check the rest of the pairs in the table. For example, is the square of –4 equal to 16? Yes. Is the square of –1 equal to 1? Yes, again. Is the square of 0 equal to 0? Yes, once more. Our formula $y = x^2$ is looking better and better. Notice that even though x can be any number at all, the y-value is never negative; its values start at 0 and go upwards toward positive infinity. Now to answer the original questions:

a. If $x = 1{,}000$, then $y = x^2 = 1{,}000^2 =$ **1,000,000**.

b. If $x = 1.2$, then $y = x^2 = 1.2^2 =$ **1.44**.

c. Notice that this part of the problem asks for the x-values, plural. I guess this means that a y-value of 625 might have two x-values associated with it. This seems reasonable from the table — if given a y-value of 9, we notice that both 3 and –3 are x-values that yield a y-value of 9. In other words, if $y = 9$, then $x = 3$ or –3. So, what two numbers, when squared, would yield a result of 625? Well, $25^2 = 625$, and of course $(-25)^2 = 625$.

In short, the following four ordered pairs lie on the curve:

(1000, 1000000)	(1.2, 1.44)	(25, 625)	(–25, 625)

Homework

7. The graph of $y = x^2$ lies within Quadrants ___ and ___, and passes through the ___.

8. Using the formula $y = x^2$, answer each question:

   a. If $x = 50$, then $y =$ ___.

   b. If $x = -25$, then $y =$ ___.

   c. If $x = 0$, then $y =$ ___.

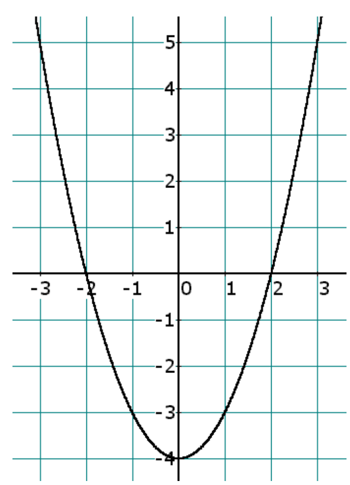
   d. If $y = 49$, then $x =$ ___ or ___.

   e. If $y = 144$, then $x =$ ___ or ___.

   f. If $y = -9$, then $x =$ ___.

9. If $y = 2x^2 - 3x - 1$, then what is y if $x = -5$?
   Recall: The Order of Operations requires that exponents be done before multiplication, which is to be done before any adding or subtracting.

10. Consider the graph at the right:



   a. Fill in the following *x*-*y* table.

x	–3	–2	–1	0	1	2	3
y							

b. Find a formula which relates x and y. Hint: There's an x^2 in the formula.

c. When $x = 100$, $y =$ ___.

d. When $x = -20$, $y =$ ___.

e. What is the lowest point (called the *minimum point*) on the graph?

f. How many y-values are associated with the x-value 1,000?

g. How many x-values are associated with the y-value 1,000?

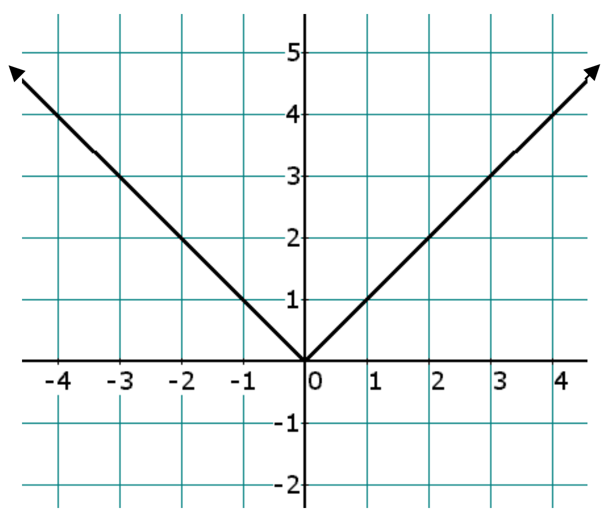
h. If $y = -3$, then $x =$ ___ or ___.

i. If $y = 96$, then $x =$ ___ or ___.

j. If $y = 221$, then $x =$ ___ or ___.

k. If $y = -6$, then $x =$ ___.

EXAMPLE 4: **Consider the following graph:**



a. Calculate the y-value when $x = 239$.

b. Calculate the y-value when $x = -777$.

c. Calculate the x-values when $y = 250$.

Solution: It's not a line and it's not curvy. So what is it? As usual, we begin with a table whose entries are read from the graph:

x	–4	–3	–2	–1	0	1	2	3	4
y	4	3	2	1	0	1	2	3	4

How is the y-value connected to the x-value? Well, when x is 0 or positive ($x \ge 0$), the y-value is the same as the x-value. But when x is negative ($x < 0$), the y-value is the opposite of the x-value.

Though the graph is straightforward (it's just the shape of the letter "V"), there's no simple, recognizable formula for the relationship between x and y. Or is there? You might recall from the Prologue the notion of absolute value: We say that

y is the ***absolute value*** of x,

and we write

$$y = |x|$$

Therefore, the vertical bar symbol around the x means "compute the absolute value of x," which means the following:

1. If $x \ge 0$, then $|x| = x$
   The absolute value of a positive number or 0 is the number itself.

2. If $x < 0$, then $|x| = -x$
   The absolute value of a negative number is the opposite of the number.

We should now be ready to answer the following questions:

a. For $x = 239$, $y = |x| = |239| = \mathbf{239}$.

b. For $x = -777$, $y = |x| = |-777| = \mathbf{777}$.

c. We are asked for the x-values (plural) when $y = 250$. In other words, we have to solve the equation

$$250 = |x|$$

which asks: What numbers have an absolute value of 250? Well, since $|250| = 250$ and $|-250| = 250$, the two x-values are **250** and **–250**. We conclude that the following four points lie on the graph:

(239, 239)	(–777, 777)	(250, 250)	(–250, 250)

Be sure you can visualize the location of these four points on the graph.

Homework

11. The graph of $y = |x|$ lies within Quadrants ___ and ___ and passes through the ___.

12. Find the **absolute value** of each number:

    a. 72   b. –99   c. 0   d. π e. $-\pi$ f. $-\sqrt{2}$

13. Evaluate each expression:

    a. $|17 - 7|$ b. $|3 - 25|$ c. $|2(3) - 6(1)|$ d. $|2\pi + 3\pi|$

14. Using the formula $y = |x|$, answer each question:

    a. If $x = 33$, then $y =$ ____.

    b. If $x = 0$, then $y =$ ____.

    c. If $x = -25$, then $y =$ ____.

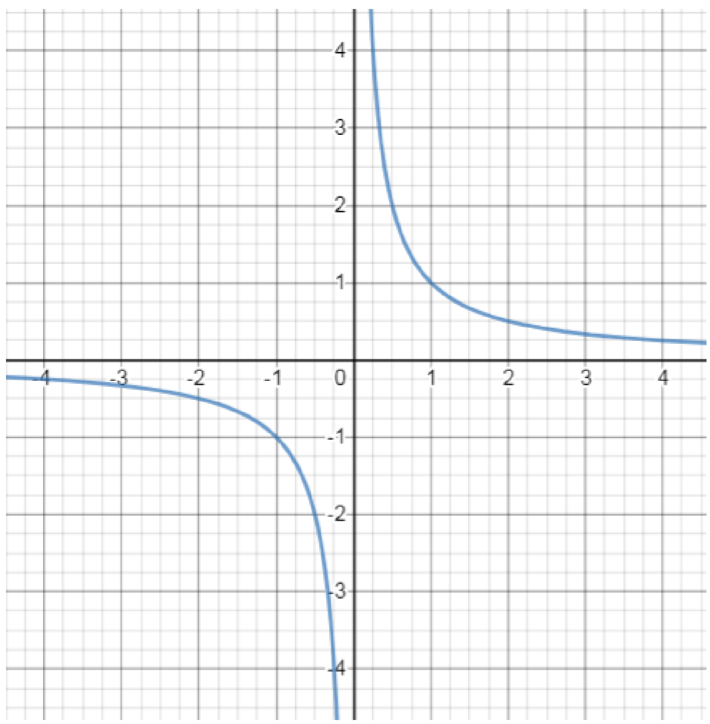
    d. If $y = 17$, then $x =$ ____ or ____.

e. If $y = 0$ then $x =$ ____.

f. If $y = -5$, then $x =$ ____.

15. Go to DESMOS and graph the function: $y = \sqrt{x^2}$.
There are two ways to do this: Type **`y = sqrt x^2`**, or click on the Keyboard icon in the lower-left corner of the screen and click the radical sign: $\sqrt{\ }$. Now look back in this chapter, and then tell me the proper way to simplify the expression $\sqrt{x^2}$.

EXAMPLE 5: **Consider the following graph:**



a. Calculate the y-value when $x = 50$.

b. Calculate the y-value when $x = -1/10$.

c. Calculate the y-value when $x = 0$.

<u>Solution:</u> We have another curvy graph, but this one's in two separate pieces. The most important property of this graph is that x is never 0; that is, the graph never touches the y-axis (even though it gets infinitely close!).

As in the two previous examples, the questions cannot be answered by looking at the graph — we must find a formula using the points we can see on the graph, and then use that formula to predict the y-values for the given x-values. To find some points on this graph, you'll have to trust me to a certain extent, because it's not easy reading fractional numbers from such a rough picture. See if you can agree with the following claims:

When $x = 1$, it's likely that $y = 1$. When $x = 2$, y looks like it's about 1/2 (or 0.5). And trust me, when $x = 3$, $y = 1/3$, and when $x = 4$, $y = 1/4$.

Now let $x = 1/2$; do you see that $y = 2$? Can we agree that when $x = 1/3$, $y = 3$? This nicely analyzes the first quadrant. Note again that x cannot be 0, because if you start at the origin (where $x = 0$), you can go up or down the y-axis as far as you like and you'll never run into the graph. It's time for a summary:

x	1/3	1/2	1	2	3	4
y	3	2	1	1/2	1/3	1/4

What is going on here? Using the point (3, 1/3) as a guide, we conjecture that the y-value is found by dividing 1 by the x-value; in other words, flip over the x-value to get the y-value:

$$y = \frac{1}{x}$$

Let's check our formula for $x = 1$ and $x = 1/3$:

$$x = 1 \Rightarrow y = \frac{1}{x} = \frac{1}{1} = 1 \checkmark$$

$$x = \frac{1}{3} \Rightarrow y = \frac{1}{x} = \frac{1}{\frac{1}{3}} = \frac{1}{1} \times \frac{3}{1} = 3 \checkmark$$

Now for a check of a negative *x*-value. Consider $x = -3$; the formula gives

$$y = \frac{1}{-3} = -\frac{1}{3}$$

which seems very reasonable from the graph. We are now convinced that we have the right formula: $y = \frac{1}{x}$. We call this the **reciprocal** formula, (called an *inverse variation* later in the course) and we'll use it to answer the original questions:

a. If $x = 50$, then $y = \frac{1}{x} = \mathbf{\frac{1}{50}}$, or $\mathbf{0.02}$.

b. If $x = -1/10$, then $y = \frac{1}{x} = \frac{1}{-\frac{1}{10}} = \frac{1}{1} \times -\frac{10}{1} = \mathbf{-10}$.

c. If $x = 0$, then $y = \frac{1}{x} = \frac{1}{0}$, which is **undefined** (see the Prologue). Notice that this is perfectly consistent with the earlier observation that *x* can never be 0 on the graph, and we see here that *y* can never be 0 in the formula, either.

Homework

16. The graph of $y = \frac{1}{x}$ lies entirely within Quadrants ___ and ___.

17. Using the previous problem's graph: As *x* grows larger and larger, *y* is always (positive, negative), but getting (larger, smaller).

18. Using the formula $y = \dfrac{1}{x}$, answer each question:

a. If $x = 14$, then $y =$ ____.

b. If $x = \frac{2}{3}$, then $y =$ ____.

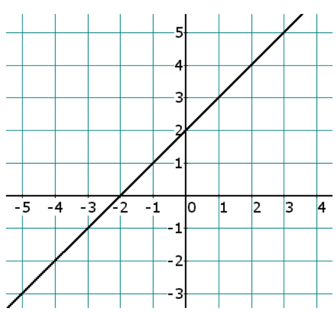
c. If $x = -99$, then $y =$ ____.

d. If $x = -\frac{5}{4}$, then $y =$ ____.

e. If $x = 0$, then $y =$ ____.

Practice Problems

19. Consider the following graph:



a. When $x = 3$, $y =$ _____.

b. When $x = 99$, $y =$ _____.

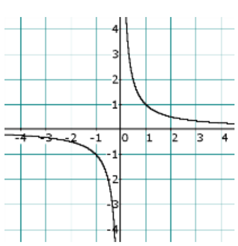
c. When $x = -45$, $y =$ _____.

d. When $y = 132$, $x =$ _____.

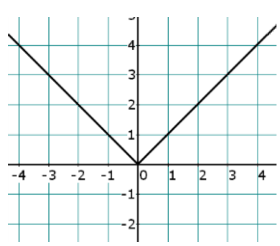
e. When $y = -33$, $x =$ _____.

20. What is the equation for each graph?

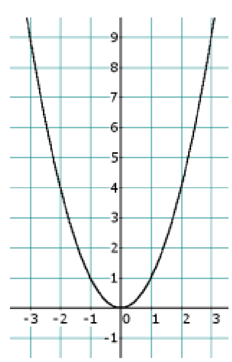
a.



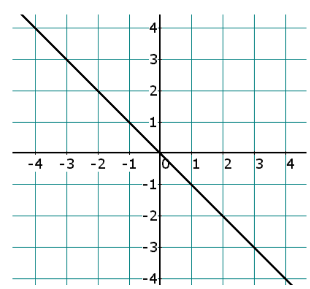
b.



c.



d.



21. Let $y = x^2$.

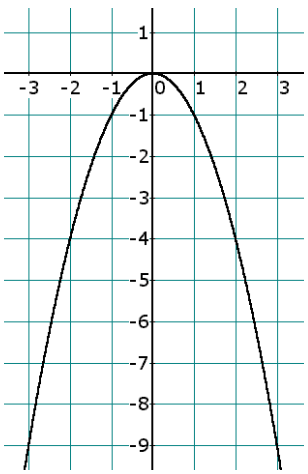
a. If $x = 17$, $y =$ _____ b. If $x = -13$, $y =$ _____

c. If $y = 16$, $x =$ _____ d. If $y = 0$, $x =$ _____

e. If $y = -9$, $x =$ _____ f. If $x = \pi$, $y =$ _____

g. Find the points on the graph where the two coordinates match.

22. Consider the graph:



a. If $x = 0$, then $y =$ _____

b. If $x = 10$, then $y =$ _____

c. If $x = -12$, then $y =$ _____

d. If $y = -225$, then $x =$ _____

e. If $y = 9$, then $x =$ _____

23. Evaluate: a. $|17|$ b. $|-\pi|$ c. $|0|$

24. Evaluate: $\dfrac{|-7| + |7|}{\frac{1}{2} + \frac{1}{3}}$

25. If $y = \dfrac{1}{x}$, and if $x = \dfrac{1}{10}$, what is y?

Solutions

1. a. 6 b. 0 c. −8 d. 556 e. −1800
f. 36π g. 5 h. 125 i. 11π

2. II and IV; origin

3. a. −17 b. 0 c. 3.5 d. −8π e. $\sqrt{2}$

4. a. True b. 0 c. positive d. negative

5. a. −3 b. −94 c. 15 d. 10
e. 5 f. −π + 5 g. π + 5

6. a. −4 b. −2 c. 2 d. −27
e. 198 f. −34 g. 298

7. I and II; origin

8. a. 2,500 b. 625 c. 0 d. 7, −7 e. 12, −12
f. No answer (no real number squared could be negative)

9. 64

10. a. 5 0 −3 −4 −3 0 5 b. $y = x^2 - 4$ c. 9,996
d. 396 e. (0, −4) f. 1 g. 2 h. −1 1 i. 10 −10
j. 15 −15 k. No answer

11. I and II; origin

12. a. 72 b. 99 c. 0 d. π e. π f. $\sqrt{2}$

13. a. 10 b. 22 c. 0 d. 5π

14. a. 33 b. 0 c. 25 d. 17, −17 e. 0 f. No answer

15. If you want to confirm your answer, you'll have to ask.

16. I and III

17. positive; smaller

18. a. $\frac{1}{14}$ b. $\frac{3}{2}$ c. $-\frac{1}{99}$ d. $-\frac{4}{5}$ e. Undefined

19. a. 5 b. 101 c. −43 d. 130 e. −35

20. a. $y = \frac{1}{x}$ b. $y = |x|$ c. $y = x^2$ e. $y = -x$

21. a. 289 b. 169 c. ±4 d. 0 e. Does not exist
f. π^2 g. Hint: There are two such points.

22. a. 0 b. −100 c. −144 d. ±15 e. Does not exist

23. a. 17 b. π c. 0

24. $\frac{84}{5}$ **25**. 10

“The greatest use of life is to spend it doing something that will outlast it.”

William James (the father of modern psychology)