CH 11 – POLYNOMIALS

❑ INTRODUCTION

It's very difficult to define what a **polynomial** is at this point in your algebra studies, because we haven't come across many things that aren't polynomials. Suffice it to say that a typical polynomial looks like

$$3x^5 - \pi x^3 + x^2 - 9x + \frac{4}{5}$$

The major theme of a polynomial is that all the exponents on the x (or whatever variable) must be one of the whole numbers 0, 1, 2, 3, ... The following are not polynomials: $8x^{-2}$ and $3x^{1/2}$, because the exponents –2 and 1/2 are not whole numbers.

❑ WORKING WITH POLYNOMIALS

A polynomial with one term is called a ***monomial***. The expressions $7n$ and $10x^2$ are monomials. The key to multiplying monomials is that each monomial is a single term whose final operation is multiplication.

For example, to find the product $(7x)(9x)$, we proceed the long way — you never have to do it this way, but it's important to see.

	$(7x)(9x)$	(the original expression)
=	$7 \cdot 9 \times x \cdot x$	(it's all multiplication)
=	$(7 \cdot 9) \times (x \cdot x)$	(regroup the factors)
=	$63 \times x^2$	(something times itself is squaring)
=	$63x^2$	(no need for the multiplication sign)

Another example is $3(-10n) = (3 \cdot -10)n = -30n$.

But don't forget that adding and subtracting don't follow the same rules as multiplication. Two monomials can be added or subtracted only if they're **like terms**. See if the homework sorts all of this out.

Homework

1. Simplify each expression:

a. $3(7L)$	b. $-5(2x)$	c. $-6(-2T)$
d. $20(-3w)$	e. $3 + 7L$	f. $-5 + 2x$
g. $-6 - 2T$	h. $20 - 3w$	i. $(7y)5$
j. $(-2p)(-5)$	k. $(-3a)(10)$	l. $(5n)(-2)$
m. $7y + 5$	n. $(4x)(3x)$	o. $4x + 3x$
p. $(2n)(-3n)$	q. $2n - 3n$	r. $(-8x)(-7x)$
s. $(7u)(-u)$	t. $(-4c)(4c)$	u. $-4c + 4c$
v. $(7m)(6n)$	w. $7m - 6n$	x. $(13k)(-13k)$
y. $13k - 13k$	z. $-14x + 20x$	

2. Suppose a friend believed that $4n^2$ and $7n$ were like terms, and that their sum should be $11n^3$. Prove your friend wrong by letting $n = 2$, and then showing that

$$4n^2 + 7n \neq 11n^3$$

3. Simplify each expression by combining like terms:

a. $3x^2 - 7x + 5x^2 + 9$	b. $n^2 - 9 + 9 - n^2$
c. $1 - 3u - u^2 - 3u^2 + 7u - 1$	d. $7a^2 - 8a + 7 - 9a^2 + 7a - 7$

e. $x^2 - 3x - 1 + 7x^2 - 3x + 1$

f. $3y^2 - 2 + 3y^2 - 2$

g. $1 - 3x - x^2 + 5 - 7x + x^2$

h. $-5w^2 + 2 - 3w + 8w - 2 - w^2$

4. Simplify each expression by distributing and then combining like terms:

a. $(3c^2 - 2c - 1) + 2(c^2 + 5c - 7)$
b. $3(x^2 - 8x + 1) - 5(2x^2 + 7x - 1)$
c. $-(a^2 - a - 1) + 3(-a^2 + a)$
d. $7w^2 - 13w + 8 - (5w^2 - 3w - 2)$
e. $-(7u^2 - 7u - 6) - (-6u^2 + 3u + 5)$
f. $(3x^2 - x - 1) - (3x^2 - x - 1)$
g. $-2(x^2 - 3x + 7) - (3x^2 + 10x - 1)$
h. $-(3n^2 + 8n - 1) - 3(n^2 + 2n - 1)$

❑ *THE DOUBLE DISTRIBUTIVE PROPERTY*

As stated before, a polynomial with one term is called a ***monomial***; just as a bicycle has two wheels, a polynomial with two terms is called a ***binomial***. A problem where we must multiply a monomial by a binomial is the following:

$\mathbf{3x}(2x + 10)$. (3x is the monomial and $2x + 10$ is the binomial)

Finding the product of these two polynomials is pretty easy — just distribute the $3x$ to the $2x$ and then distribute the $3x$ to the 10:

$\mathbf{3x}(2x) + \mathbf{3x}(10)$

$= 6x^2 + 30x$, and it's done.

What we need now is a way to multiply <u>two binomials</u> together. For example, how do we simplify the product $(x + 7)(x + 5)$? The **Double Distributive Property** says, in a nutshell,

Multiply each term in the first binomial
by each term in the second binomial.

EXAMPLE 2: **Multiply out (simplify): $(x + 7)(x + 5)$**

Solution: Multiply each term in the first binomial by each term in the second binomial:

i)	Multiply the first x by the second x:	x^2
ii)	Multiply the first x by the 5:	$5x$
iii)	Multiply the 7 by the second x:	$7x$
iv)	Multiply the 7 by the 5:	35

Add the four terms together: $x^2 + 5x + 7x + 35$, and then combine like terms

$$\boxed{x^2 + 12x + 35}$$

EXAMPLE 3: **Simplify each expression:**

A. $(2n + 1)(n - 8)$

$= 2n^2 - 16n + n - 8$ (double distribute)

$= \mathbf{2n^2 - 15n - 8}$ (combine like terms)

B. $(7a - 3)(4a - 5)$

$= 28a^2 - 35a - 12a + 15$ (double distribute)

$= \mathbf{28a^2 - 47a + 15}$ (combine like terms)

C. $(6k - 7)(6k + 7)$

$= 36k^2 + 42k - 42k - 49$ (double distribute)

$= \mathbf{36k^2 - 49}$ (combine like terms)

D. $(10 + y)(10 - y)$

$= 100 - 10y + 10y - y^2$ (double distribute)

$= \mathbf{100 - y^2}$ (combine like terms)

E. $(2x + 9)^2$ The square of a quantity is the product of the quantity with itself:

$(2x + 9)^2$

$= (2x + 9)(2x + 9)$ (since $N^2 = N \cdot N$)

$= 4x^2 + 18x + 18x + 81$ (double distribute)

$= \mathbf{4x^2 + 36x + 81}$ (combine like terms)

<u>EXAMPLE 4:</u> **Simplify: $(2x + 1)(x - 5) - (x - 4)^2$**

<u>Solution:</u> The Order of Operations tells us to multiply and square first, and subtract last:

$(2x + 1)(x - 5) - (x - 4)^2$

$= (2x^2 - 10x + x - 5) - (x^2 - 4x - 4x + 16)$ (multiply and square)

[Notice how parentheses still enclose the result of the squaring.]

$= (2x^2 - 9x - 5) - (x^2 - 8x + 16)$ (combine like terms)

$= 2x^2 - 9x - 5 - x^2 + 8x - 16$ (distribute the –1)

$= \boxed{x^2 - x - 21}$ (combine like terms)

Homework

5. Simplify each expression by double distributing:

a. $(x + y)(w + z)$ b. $(c + d)(a - b)$ c. $(x + 2)(y + 3)$

d. $(x + 3)(x + 4)$ e. $(n - 4)(n - 1)$ f. $(a + 3)(a - 7)$

g. $(y + 9)(y - 9)$ h. $(u - 3)(u + 3)$ i. $(t - 20)(t - 19)$

j. $(z + 3)(z + 3)$ k. $(v - 4)(v - 4)$ l. $(N + 1)(N - 1)$

6. Simplify each expression by double distributing:

a. $(3a + 7)(a - 9)$ b. $(2n - 3)(n + 4)$ c. $(3n - 8)(n - 1)$

d. $(5x + 7)(5x + 6)$ e. $(7w + 2)(7w - 2)$ f. $(x + 12)(x - 12)$

g. $(2y + 1)(2y + 1)$ h. $(7x + 3)(6x - 7)$ i. $(q + 7)(3q - 7)$

j. $(3n + 1)(3n + 1)$ k. $(3x - 7)(6x + 5)$ l. $(u - 7)(u - 7)$

7. Square and simplify each expression:

a. $(y + 4)^2$ b. $(z - 9)^2$ c. $(3x + 5)^2$

d. $(2a - 1)^2$ e. $(n + 12)^2$ f. $(6t - 7)^2$

g. $(q - 15)^2$ h. $(5b + 3)^2$ i. $(7u - 1)^2$

j. $(2x + 1)^2$ k. $(3h - 12)^2$ l. $(5y - 5)^2$

8. Simplify each expression:

a. $(a + b)(c - d)$ b. $(2x - 3)(2x + 3)$ c. $(3n - 1)^2$

d. $(3t + 1)(2t - 3)$ e. $(2x + 4)(3x - 6)$ f. $(n + 1)(n - 1)$

g. $(7a - 10)(6a - 10)$ h. $(10c + 7)^2$ i. $(L + 4)^2$

j. $(7x - 3)(3x + 7)$ k. $(13n - 7)(13n + 7)$ l. $(12d - 20)^2$

9. Simplify each expression:

a. $(2n + 1)(n + 1) + (n - 1)(n + 1)$

b. $(x + 1)^2 + (x + 2)^2$

c. $(3a + 2)(a - 1) - (a + 1)(a + 2)$

d. $(4w + 1)^2 - (w - 1)(w - 3)$

e. $(y + 2)(y - 3) - (2y - 1)^2$

f. $(2y + 1)^2 - (2y - 1)^2$

10. Prove that $(a + b)^2 \neq a^2 + b^2$ in two ways:
i) Plug in numbers.
ii) Simplify $(a + b)^2$ the correct way.

11. Use numbers to prove that $(x + y)^3 \neq x^3 + y^3$

❑ *TRINOMIALS*

Just as a *trio* consists of three musicians, a ***trinomial*** is a polynomial consisting of three terms. Here are a couple of problems where we subtract some trinomials and multiply with a trinomial.

EXAMPLE 5: **Simplify each expression:**

A. $(2x^2 - x + 1) - (x^2 - 7x + 2)$ (difference of 2 **trinomials**)

$= 2x^2 - x + 1 - x^2 + 7x - 2$ (distribute the minus sign)

$= 2x^2 - x^2 - x + 7x + 1 - 2$ (rearrange the terms)

$= \mathbf{x^2 + 6x - 1}$ (combine like terms)

B. $(a-3)(a^2+2a-5)$ (the product of a **binomial** and a **trinomial**)

The secret here is to multiply each of the terms in the binomial by each of the terms in the trinomial:

Multiply a by all three terms: a^3+2a^2-5a

Multiply -3 by all three terms: $-3a^2-6a+15$

Now combine like terms: $\boldsymbol{a^3-a^2-11a+15}$

Homework

12. Simplify each expression:

a. $(3n^2-14n+2)+(2n^2+2n-1)$

b. $(4x^2-x-1)-(x^2-1)$

c. $(x+2)(x^2+3x+4)$

d. $(y-1)(y^2-1)$

e. $(z+3)(2z^2-z-1)$

f. $(2x-5)(x^2-5x+5)$

g. $(4w^2-3w-1)(2w+5)$

h. $(x+3)(x^2-3x+9)$

i. $(x^2+1)(x^2+2)$

j. $(2a+1)(a^2+1)$

k. $(x-3)(x^2+7x-1)$

l. $(3t^2-5t+3)(2t-3)$

❑ *CUBING A BINOMIAL*

EXAMPLE 6: **Cube the binomial $2x+5$. That is, simplify the expression $(2x+5)^3$.**

Solution: The cube of anything is found by multiplying three of those anythings together: A^3 means $A \times A \times A$. Therefore, the expression

$$(2x+5)^3$$

can be expanded to get

$$(2x+5)(2x+5)(2x+5)$$

We know that one way to multiply three things together is to multiply the first two of them, and then multiply that result by the 3rd thing. (For example, (2)(3)(4) = (6)(4) = 24.) Multiplying the first two factors together gives

$$(4x^2 + 10x + 10x + 25)(2x + 5) \qquad \text{(double distribute)}$$

$$= (4x^2 + 20x + 25)(2x + 5) \qquad \text{(combine like terms)}$$

We now have a trinomial times a binomial. What do we do? Most students find that reversing the trinomial and the binomial makes things a little easier to keep track of, so let's do it.

$$= (2x + 5)(4x^2 + 20x + 25) \qquad \text{(commutative property)}$$

We multiply each term in the binomial by each term in the trinomial:

$$= \mathbf{2x}(4x^2) + \mathbf{2x}(20x) + \mathbf{2x}(25) + \underline{5}(4x^2) + \underline{5}(20x) + \underline{5}(25)$$

$$= 8x^3 + 40x^2 + 50x + 20x^2 + 100x + 125$$

$$= \boxed{8x^3 + 60x^2 + 150x + 125}$$

Homework

13. Simplify each expression:

a. $(x + 3)^3$ b. $(y + 1)^3$ c. $(n - 5)^3$

d. $(2a + 4)^3$ e. $(3m - 2)^3$ f. $(5q + 3)^3$

14. Prove that $(x + y)^3 \neq x^3 + y^3$ by cubing the binomial.

❑ Preview of a Future Chapter

Consider simplifying (expanding) the expression $(a + b)^9$. You should realize that the answer is <u>not</u> $a^9 + b^9$.

First of all, earlier examples have shown us that $(a + b)^2$ is <u>not</u> equal to $a^2 + b^2$. And the previous example showed us that $(2x + 5)^3$ is <u>not</u> equal to $(2x)^3 + 5^3$. It therefore seems reasonable that $(a + b)^9$ would <u>not</u> be equal to $a^9 + b^9$.

Second, watch what happens when we test the *conjecture* that $(a + b)^9 = a^9 + b^9$. Let a and b both take on the value 1. Then

$$(a + b)^9 = (1 + 1)^9 = 2^9 = \mathbf{512};$$

$$\text{but,} \quad a^9 + b^9 = 1^9 + 1^9 = 1 + 1 = \mathbf{2} \text{ — not even close!!}$$

We therefore conclude that $(a + b)^9 \neq a^9 + b^9$. So how <u>do</u> we raise the sum of a and b to the 9th power? Here's the hard way:

$$(a + b)(a + b)(a + b)(a + b)(a + b)(a + b)(a + b)(a + b)(a + b)$$

Start with the first two binomials; multiply that result by the third binomial, and so on and so on. You'd be done in a few hours (most likely with errors), but there's a much quicker way that we'll learn about at the very end of this book.

❑ Dividing a Polynomial by a Monomial

Just as $\frac{1}{7} + \frac{3}{7} = \frac{4}{7}$, we can do the problem $\frac{a}{b} + \frac{c}{b}$ by adding the numerators, and placing that sum over the common denominator b:

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$$

By reversing this reasoning we can take the fraction $\frac{a+c}{b}$ and, if we like, split it into the sum of two fractions:

$$\frac{a+c}{b} = \frac{a}{b} + \frac{c}{b}$$

This is the trick we need to divide a polynomial by a monomial.

<u>EXAMPLE 7:</u> **Divide:** $\dfrac{\mathbf{12x^2y^3 - 8x^3y^2 + 7xy^3}}{\mathbf{2x^2y}}$

<u>Solution:</u> Split the fraction into three separate fractions:

$$\frac{12x^2y^3}{2x^2y} - \frac{8x^3y^2}{2x^2y} + \frac{7xy^3}{2x^2y},$$

and then simplify (reduce) each fraction:

$$\boxed{6y^2 - 4xy + \frac{7y^2}{2x}}$$

Homework

15. Perform each division problem, where the divisor is a monomial:

a. $\dfrac{x^3 - x^2 + x}{x}$ b. $\dfrac{14xy + 21x^2y - 28xy^2}{7xy}$ c. $\dfrac{x^2 + 3x + 1}{x}$

d. $\dfrac{a+b}{b}$ e. $\dfrac{x-y}{y}$ f. $\dfrac{ax+bx}{x}$

❑ Dividing a Polynomial by a Polynomial

First we need the right terminology. When written as a fraction, a division problem has two parts:

$$\frac{\text{dividend}}{\text{divisor}}$$

When written in the standard "long division" format, we write

$$\text{divisor}\overline{)\text{dividend}}$$

The result of dividing is called the ***quotient***, and the leftover is called the ***remainder***. For example,

$$\begin{array}{r} 5 \\ 3\overline{)17} \\ \underline{15} \\ 2 \end{array}$$

dividend = 17
divisor = 3
quotient = 5
remainder = 2

We can then write the answer as $5+\frac{2}{3}$ $\left(\text{dividend} + \frac{\text{remainder}}{\text{divisor}}\right)$, which is written as the mixed number $5\frac{2}{3}$ when we're dealing with numbers.

Think back when you were a kid and learned long division of numbers. Though I've seen different ways of doing this, the method we'll use here boils down to a 4-step process, a process that is repeated until the problem is finished:

1. Divide the divisor into the first part of the dividend
2. Multiply the part of the quotient calculated in step 1 by the divisor
3. Subtract
4. Bring down the next digit

And then repeat steps 1 – 4 as many times as necessary until there's nothing left to bring down.

We use the same process for polynomial long division in algebra.

EXAMPLE 8: **Perform the long division:** $\dfrac{\mathbf{3x^3 - 5x - 2}}{\mathbf{x - 1}}$

Solution: The first step is to fill in the missing term in the dividend. Since there is no x^2 term, we put in the "place-holder" $0x^2$ between the cubic term and the linear term, giving us a dividend of $3x^3 + 0x^2 - 5x - 2$. So our long division problem is

$$x-1\overline{\big)\,3x^3+0x^2-5x-2}$$

1. Divide x into $3x^3$; it goes in $3x^2$ times (since $3x^2 \cdot x = 3x^3$):

$$\begin{array}{r|l} & \mathbf{3x^2} \\ \hline \mathbf{x}-1 & \mathbf{3x^3}+0x^2-5x-2 \end{array}$$

2. Multiply $3x^2$ by the divisor, $x - 1$:

$$\begin{array}{r|l} & \mathbf{3x^2} \\ \hline \mathbf{x-1} & 3x^3+0x^2-5x-2 \\ & \underline{\mathbf{3x^3-3x^2}} \end{array}$$

3. Subtract; $3x^3 - 3x^3 = 0$; $0x^2 - (-3x^2) = 3x^2$:

$$\begin{array}{r|l} & 3x^2 \\ \hline x-1 & \mathbf{3x^3}+\mathbf{0x^2}-5x-2 \\ & \underline{-(\mathbf{3x^3-3x^2})} \\ & \quad\mathbf{0}\ +\mathbf{3x^2} \end{array}$$

4. Bring down the next term, $-5x$:

$$\begin{array}{r|l} & 3x^2 \\ \hline x-1 & 3x^3+0x^2-\mathbf{5x}-2 \\ & \underline{-(3x^3-3x^2)}\ \downarrow \\ & \quad 0\ +3x^2-\mathbf{5x} \end{array}$$

1. And repeat: Divide x into $3x^2$:

$$\begin{array}{r|l} & 3x^2 \quad + \mathbf{3x} \\ \hline \boldsymbol{x}-1 & 3x^3+0x^2-5x-2 \\ & -(3x^3-3x^2) \\ \hline & 0 \;+\mathbf{3x^2}-5x \end{array}$$

2. Multiply $3x$ by $x - 1$, the divisor:

$$\begin{array}{r|l} & 3x^2 \quad + \mathbf{3x} \\ \hline \boldsymbol{x}-\mathbf{1} & 3x^3+0x^2-5x-2 \\ & -(3x^3-3x^2) \\ \hline & 0 \;+3x^2-5x \\ & \quad \mathbf{3x^2-3x} \end{array}$$

3. Subtract; $3x^2 - 3x^2 = 0$; $-5x - (-3x) = -2x$:

$$\begin{array}{r|l} & 3x^2 \quad + 3x \\ \hline x-1 & 3x^3+0x^2-5x-2 \\ & -(3x^3-3x^2) \\ \hline & 0 \;+\mathbf{3x^2}-\mathbf{5x} \\ & \quad -(\mathbf{3x^2}-\mathbf{3x}) \\ \hline & \quad \mathbf{0} \;\; -\mathbf{2x} \end{array}$$

4. Bring down the next (and last) term, -2:

$$\begin{array}{r|l} & 3x^2 \quad + 3x \\ \hline x-1 & 3x^3+0x^2-5x-\mathbf{2} \\ & -(3x^3-3x^2) \\ \hline & 0 \;+3x^2-5x \\ & \quad -(3x^2-3x) \downarrow \\ \hline & \quad 0 \;\; -2x-\mathbf{2} \end{array}$$

1. <u>Divide</u> x into $-2x$:

$$\begin{array}{r|l} & 3x^2 + 3x - \mathbf{2} \\ \hline \boldsymbol{x} - 1 & 3x^3 + 0x^2 - 5x - 2 \\ & -(3x^3 - 3x^2) \\ & 0 + 3x^2 - 5x \\ & \quad -(3x^2 - 3x) \\ & \qquad 0 - \mathbf{2}\boldsymbol{x} - 2 \end{array}$$

2. <u>Multiply</u> -2 by $x - 1$:

$$\begin{array}{r|l} & 3x^2 + 3x - \mathbf{2} \\ \hline \boldsymbol{x} - \mathbf{1} & 3x^3 + 0x^2 - 5x - 2 \\ & -(3x^3 - 3x^2) \\ & 0 + 3x^2 - 5x \\ & \quad -(3x^2 - 3x) \\ & \qquad 0 - 2x - 2 \\ & \qquad\quad - \mathbf{2}\boldsymbol{x} + \mathbf{2} \end{array}$$

3. <u>Subtract</u>; $-2x - (-2x) = 0$; $-2 - (+2) = -4$

$$\begin{array}{r|l} & 3x^2 + 3x - 2 \\ \hline x - 1 & 3x^3 + 0x^2 - 5x - 2 \\ & -(3x^3 - 3x^2) \\ & 0 + 3x^2 - 5x \\ & \quad -(3x^2 - 3x) \\ & \qquad 0 - \mathbf{2}\boldsymbol{x} - \mathbf{2} \\ & \qquad\quad -(-\mathbf{2}\boldsymbol{x} + \mathbf{2}) \\ & \qquad\qquad \mathbf{0} - \mathbf{4} \end{array}$$

There are no terms left to bring down in the dividend, so we write the remainder (the -4) over the divisor and add it to the quotient. The final answer to the long division problem is

$$\boxed{3x^2 + 3x - 2 + \frac{-4}{x - 1}}$$

Homework

16. Perform each polynomial long division problem, expressing any remainder as a fraction added to the quotient:

a. $\frac{x^2+5x+6}{x+3}$ b. $\frac{x^2-9}{x-3}$ c. $\frac{x^2+2x+1}{x+1}$

d. $\frac{n^2+n-4}{n+5}$ e. $\frac{2a^2-5a+2}{a+3}$ f. $\frac{3w^2+10}{w+5}$

g. $\frac{6b^2+b-15}{2b+3}$ h. $\frac{3y^2-9}{y+5}$ i. $\frac{10x^2+3x-7}{2x-1}$

j. $\frac{x^3+1}{x+1}$ Hint: $x^3 + 1 = x^3 + 0x^2 + 0x + 1$

k. $\frac{n^3-8}{n-2}$ l. $\frac{a^3+27}{a^2-3a+9}$

17. Perform each polynomial long division problem (Hint: there is no remainder):

a. $\frac{40x^3+97x^2+60x+27}{5x+9}$ b. $\frac{8w^3+22w^2+13w+2}{2w^2+5w+2}$

c. $\frac{40r^3-4r^2-7r-3}{8r^2+4r+1}$ d. $\frac{63m^3+43m^2+13m+1}{7m^2+4m+1}$

Practice Problems

18. Simplify each expression:

a. $7x^2 - 3x + 7 - 7x^2 - 3x - 7$
b. $-8(3y^2 - 4y - 1)$
c. $2(a^2 - 8) - (a^2 - 2a - 1)$
d. $-(4n^2 - 4n) - (4n - 4n^2)$
e. $3(4g^2 - g + 3) - 2(6g^2 + g - 1)$
f. $(x + y)(w + z)$
g. $(3x)(-4x)$
h. $10(3y)$
i. $-3n + 4n$
j. $-2(x^2 - 3x - 1)$
k. $3(x^2 - x - 2) - (2x^2 + 7x + 8)$
l. $10x^2 + 29x$

19. Simplify each expression:

a. $(x + 9)(x + 8)$
b. $(y - 1)(y - 8)$
c. $(2z + 5)(2z - 5)$
d. $(N + 10)(N - 10)$
e. $(x - 9)^2$
f. $(a + 5)^2$
g. $(t + 9)(t - 5)$
h. $(a - 22)(a + 1)$
i. $(a - 11)(a + 2)$
j. $(2x + 1)(x - 5)$
k. $(3x + 8)(2x - 5)$
l. $(6x + 5)(x - 3)$
m. $(6a + 17)(a - 1)$
n. $(R + 12)(R - 12)$
o. $(5n - 3)^2$
p. $(1 - a)(2 - a)$
q. $(7w + 5)^2$
r. $(3a - 1)(3a - 2)$
s. $(9a - 1)(a - 2)$
t. $(x + 18)(x - 2)$
u. $(x + 36)(x + 1)$
v. $(5c - 1)(6c - 1)$
w. $(8a + 1)(2a - 1)$
x. $(6q + 5)^2$
y. $(3 + n)(3 - n)$
z. $(16n - 9)(2n - 3)$

20. Prove that $(u + w)^4 \neq u^4 + w^4$. [Letting both u and w equal 1 will do the trick.]

21. Simplify each expression:

a. $(2n - 5)(3n - 1)$
b. $(8x + 3)(8x - 3)$
c. $(7z - 5)^2$
d. $(8 - 7a)(8 + 7a)$
e. $(2x - 1)(3x + 4) - (4x - 1)^2$

22. Simplify each expression:

a. $(w - 5)(3w^2 - 2w - 1)$ b. $(2x - 5)^3$

23. True/False, and prove your answer:

a. $(a - b)^2 = a^2 + b^2$ b. $(x - y)^3 = x^3 - y^3$

24. Prove that $(a + b)^5 \neq a^5 + b^5$

25. Divide: $\dfrac{4x^3 - 8x^2 + 6x - 10}{4x^2}$

26. Divide: $\dfrac{x^2 + 9}{x - 5}$

27. Divide: $\dfrac{x^3 - 3x + 8}{x + 3}$

28. Divide: $\dfrac{x^4 - 1}{x + 1}$

29. Divide: $\dfrac{n^3 + 8}{n + 2}$

Solutions

1. a. $21L$ b. $-10x$ c. $12T$ d. $-60w$ e. As is f. As is
g. As is h. As is i. $35y$ j. $10p$ k. $-30a$ l. $-10n$
m. As is n. $12x^2$ o. $7x$ p. $-6n^2$ q. $-n$ r. $56x^2$
s. $-7u^2$ t. $-16c^2$ u. 0 v. $42mn$ w. As is x. $-169k^2$
y. 0 z. $6x$

2. $4n^2 + 7n = 4(\mathbf{2})^2 + 7(\mathbf{2}) = 4(4) + 7(2) = 16 + 14 = 30$,
whereas $11n^3 = 11(\mathbf{2})^3 = 11(8) = 88$
Therefore, $4n^2 + 7n \neq 11n^3$

3. a. $8x^2 - 7x + 9$ b. 0 c. $-4u^2 + 4u$ d. $-2a^2 - a$
e. $8x^2 - 6x$ f. $6y^2 - 4$ g. $-10x + 6$ h. $-6w^2 + 5w$

4. a. $5c^2 + 8c - 15$ b. $-7x^2 - 59x + 8$ c. $-4a^2 + 4a + 1$
d. $2w^2 - 10w + 10$ e. $-u^2 + 4u + 1$ f. 0
g. $-5x^2 - 4x - 13$ h. $-6n^2 - 14n + 4$

5. a. $xw + xz + wy + yz$ b. $ac - bc + ad - bd$ c. $xy + 3x + 2y + 6$
d. $x^2 + 7x + 12$ e. $n^2 - 5n + 4$ f. $a^2 - 4a - 21$
g. $y^2 - 81$ h. $u^2 - 9$ i. $t^2 - 39t + 380$
j. $z^2 + 6z + 9$ k. $v^2 - 8v + 1$ 6 l. $N^2 - 1$

6. a. $3a^2 - 20a - 63$ b. $2n^2 + 5n - 12$ c. $3n^2 - 11n + 8$
d. $25x^2 + 65x + 42$ e. $49w^2 - 4$ f. $x^2 - 144$
g. $4y^2 + 4y + 1$ h. $42x^2 - 31x - 21$ i. $3q^2 + 14q - 49$
j. $9n^2 + 6n + 1$ k. $18x^2 - 27x - 35$ l. $u^2 - 14u + 49$

7. a. $y^2 + 8y + 16$ b. $z^2 - 18z + 81$ c. $9x^2 + 30x + 25$
d. $4a^2 - 4a + 1$ e. $n^2 + 24n + 144$ f. $36t^2 - 84t + 49$
g. $q^2 - 30q + 225$ h. $25b^2 + 30b + 9$ i. $49u^2 - 14u + 1$
j. $4x^2 + 4x + 1$ k. $9h^2 - 72h + 144$ l. $25y^2 - 50y + 25$

8. a. $ac - ad + bc - bd$ b. $4x^2 - 9$ c. $9n^2 - 6n + 1$
d. $6t^2 - 7t - 3$ e. $6x^2 - 24$ f. $n^2 - 1$
g. $42a^2 - 130a + 100$ h. $100c^2 + 140c + 49$ i. $L^2 + 8L + 16$
j. $21x^2 + 40x - 21$ k. $169n^2 - 49$ l. $144d^2 - 480d + 400$

9. a. $3n^2 + 3n$ b. $2x^2 + 6x + 5$ c. $2a^2 - 4a - 4$
d. $15w^2 + 12w - 2$ e. $-3y^2 + 3y - 7$ f. $8y$

10. i) By letting $a = 3$ and $b = 4$, for instance, we get:

$(a + b)^2 = (\mathbf{3} + \mathbf{4})^2 = 7^2 = 49$, whereas

$a^2 + b^2 = \mathbf{3}^2 + \mathbf{4}^2 = 9 + 16 = 25.$

Clearly, $(a + b)^2 \neq a^2 + b^2$

ii) $(a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$

11. Choosing, for example, $x = 1$ and $y = 2$, we would get the following results:

$$(x + y)^3 = (\mathbf{1} + \mathbf{2})^3 = 3^3 = 27$$

On the other hand, $x^3 + y^3 = \mathbf{1}^3 + \mathbf{2}^3 = 1 + 8 = 9$

12. a. $5n^2 - 12n + 1$ b. $3x^2 - x$
c. $x^3 + 5x^2 + 10x + 8$ d. $y^3 - y^2 - y + 1$
e. $2z^3 + 5z^2 - 4z - 3$ f. $2x^3 - 15x^2 + 35x - 25$
g. $8w^3 + 14w^2 - 17w - 5$ h. $x^3 + 27$
i. $x^4 + 3x^2 + 2$ j. $2a^3 + a^2 + 2a + 1$
k. $x^3 + 4x^2 - 22x + 3$ l. $6t^3 - 19t^2 + 21t - 9$

13. a. $x^3 + 9x^2 + 27x + 27$ b. $y^3 + 3y^2 + 3y + 1$
c. $n^3 - 15n^2 + 75n - 125$ d. $8a^3 + 48a^2 + 96a + 64$
e. $27m^3 - 54m^2 + 36m - 8$ f. $125q^3 + 225q^2 + 135q + 27$

14. $(x + y)^3 = (x + y)(x + y)(x + y) = (x + y)(x^2 + 2xy + y^2)$
$= x^3 + 3x^2y + 3xy^2 + y^3,$
which is most likely not equal to $x^3 + y^3$ for all values of x and y.

15. a. $x^2 - x + 1$ b. $2 + 3x - 4y$ c. $x + 3 + \frac{1}{x}$
d. $\frac{a}{b} + 1$ e. $\frac{x}{y} - 1$ f. $a + b$

16. a. $x + 2$ b. $x + 3$ c. $x + 1$
d. $n - 4 + \frac{16}{n+5}$ e. $2a - 11 + \frac{35}{a+3}$ f. $3w - 15 + \frac{85}{w+5}$
g. $3b - 4 + \frac{-3}{2b+3}$ h. $3y - 15 + \frac{66}{y+5}$ i. $5x + 4 + \frac{-3}{2x-1}$
j. $x^2 - x + 1$ k. $n^2 + 2n + 4$ l. $a + 3$

17. a. $8x^2 + 5x + 3$ b. $4w + 1$ c. $5r - 3$
d. $9m + 1$

18. a. $-6x$ b. $-24y^2 + 32y + 8$ c. $a^2 + 2a - 15$
d. 0 e. $-5g + 11$ f. $xw + xz + wy + yz$
g. $-12x^2$ h. $30y$ i. n
j. $-2x^2 + 6x + 2$ k. $x^2 - 10x - 14$ l. As is

19. a. $x^2 + 17x + 72$ b. $y^2 - 9y + 8$ c. $4z^2 - 25$
d. $N^2 - 100$ e. $x^2 - 18x + 81$ f. $a^2 + 10a + 25$
g. $t^2 + 4t - 45$ h. $a^2 - 21a - 22$ i. $a^2 - 9a - 22$
j. $2x^2 - 9x - 5$ k. $6x^2 + x - 40$ l. $6x^2 - 13x - 15$
m. $6a^2 + 11a - 17$ n. $R^2 - 144$ o. $25n^2 - 30n + 9$
p. $a^2 - 3a + 2$ q. $49w^2 + 70w + 25$ r. $9a^2 - 9a + 2$
s. $9a^2 - 19a + 2$ t. $x^2 + 16x - 36$ u. $x^2 + 37x + 36$
v. $30c^2 - 11c + 1$ w. $16a^2 - 6a - 1$ x. $36q^2 + 60q + 25$
y. $9 - n^2$, or $-n^2 + 9$ z. $32n^2 - 66n + 27$

20. $(1 + 1)^4 = 2^4 = 16$; whereas $1^4 + 1^4 = 1 + 1 = 2$.

21. a. $6n^2 - 17n + 5$ b. $64x^2 - 9$ c. $49z^2 - 70z + 25$
d. $64 - 49a^2$ e. $-10x^2 + 13x - 5$

22. a. $3w^3 - 17w^2 + 9w + 5$ b. $8x^3 - 60x^2 + 150x - 125$

23. a. False; let $a = 5$ and $b = 2$:

$$(a - b)^2 = (5 - 2)^2 = 3^2 = 9$$
$$a^2 + b^2 = 5^2 + 2^2 = 25 + 4 = 29$$

b. False; let $a = 2$ and $b = 3$

$(x - y)^3 = (5 - 4)^3 = 1^3 = 1$

$x^3 - y^3 = 5^3 - 4^3 = 125 - 64 = 61$

24. Let $a = 2$ and $b = 3$

$(2 + 3)^5 = 5^5 = 3125$

$2^5 + 3^5 = 32 + 243 = 275$

25. $x - 2 + \frac{3}{2x} - \frac{5}{2x^2}$

26. $x + 5 + \frac{34}{x - 5}$

27. $x^2 - 3x + 6 + \frac{-10}{x + 3}$

28. $x^3 - x^2 + x - 1$

29. $n^2 - 2n + 4$

"When one door closes, another opens; but we often look so long and so regretfully upon the closed door that we do not see the one which has opened for us."

- *Alexander Graham Bell*