CH16 – MOTION PROBLEMS

□ INTRODUCTION

Whether it's the CHP pursuing a bank robber, or a physicist determining the velocity of a proton in a cyclotron, the concepts of time, distance, and speed are at the heart of all science and technology.

If you travel from L.A. to San Francisco, 400 miles away, and you travel for 8 hours at an average speed of 50 miles per hour, then

you traveled a **DISTANCE** of 400 miles at a **RATE** (speed) of 50 mph during a **TIME** interval of 8 hours.

Notice that in this example, if you multiply the rate by the time $(50 \text{ mph} \times 8 \text{ hrs})$, you get the distance (400 mi). This idea always holds:

Rate × Time = Distance

Homework

- 1. a. Moe traveled at a rate of 120 km/hr for 12 hours. Find Moe's distance.
 - b. Larry flew a distance of 3000 miles in 6 hours. What was Larry's rate?
 - c. Curly jogged 12 miles at a rate of 3 mph. How long was Curly jogging?



2. Which is the proper formula for distance?

a.
$$d = rt$$
 b. $d = \frac{r}{t}$ c. $d = \frac{t}{r}$ d. $d = r + t$

- 3. Two skaters leave the skate park and skate in opposite directions, one at 10 mph and the other at 8 mph. After some time, they are 18 miles apart. If d_1 is the distance traveled by the first skater, and if d_2 is the distance traveled by the second skater, write an appropriate equation.
- 4. A jet ski leaves the beach. Nine hours later a motorboat begins to pursue the jet ski and finally catches up with it. If d_1 is the distance the jet ski travels, and if d_2 is the distance the motorboat travels, write an appropriate equation.
- 5. A woodpecker traveled from the maple tree to the oak tree at 13 mph, and then made a return trip at 19 mph. If d_1 is the distance he traveled to the oak tree, and if d_2 is the distance from the oak back to the maple, write an appropriate equation.



7. Mutt and Jeff leave the mall at the same time and head in the same direction. Jeff's speed is 9 mph more than 6 times Mutt's speed. Four hours later Jeff is 1036 miles ahead of Mutt. If d_1 is the distance Mutt traveled, and if d_2 is the distance Jeff traveled, write an appropriate equation.

SOLVING A SYSTEM OF EQUATIONS BY SUBSTITUTION

A system of two equations in two variables is a pair of equations like

$$a+b = 10$$
$$a-b = 4$$

A solution of this system of equations is a pair of numbers a and b which make <u>both</u> equations true.

For example, if a = 5 and b = 5, then the first equation is true but the second is false, so this is not a solution of the <u>system</u> of equations.

Similarly, a = 12 and b = 8 is a solution to the second equation, but not the first; this pair is also not a solution of the system.

But what if a = 7 and b = 3? Then both equations are satisfied, and we conclude by saying that a = 7 and b = 3 is a solution of the system of equations.

There are many ways to solve a system of two equations in two unknowns, including the Elimination method. For the problems in this chapter, we'll use a form of the Substitution Method, but use any method you and your instructor like.

<u>EXAMPLE 1</u>: Solve the system $\begin{array}{c} x+y = 10\\ 2x-3y = 15 \end{array}$ by substitution.

<u>Solution</u>: To apply the substitution method, we select one of the two equations, and then select one of the two variables in that equation. We then solve for that variable and then *substitute* that result into the other equation. It's a lot easier to show than to explain.

First step: Select an equation. Let's choose the first equation because it looks a lot simpler than the second equation:

$$x + y = 10$$

Second step: Select a variable. For no particular reason, we'll choose the y.

Third step: Solve for that variable; that is, isolate it:

$$x + y = 10$$

 $\Rightarrow y = 10 - x$ [EQU 1] (subtract x from each side)

Fourth step: Substitute 10 - x for y in the other equation. Here's the other equation:

$$2x - 3y = 15$$

Now replace the y in this equation with its value of 10 - x: 2x - 3(10 - x) = 15 (since 10 - x and y are the same)

How is this equation any improvement over the original pair of equations? I'll tell you — it has only one variable in it! That's a good thing; we can solve for *x* rather easily:

	2x - 3(10 - x) = 15	
\Rightarrow	2x - 30 + 3x = 15	(distribute)
\Rightarrow	5x - 30 = 15	(combine like terms)
\Rightarrow	5x = 45	(add 30 to each side)
\Rightarrow	$\underline{x = 9}$	(divide each side by 5)

Now we know that x = 9. But what about *y*? To find the value of y, we could use either of the two original equations, but the easiest way to find y is to use EQU 1 -after all, it's already solved for y. So we write EQU 1, place the number we got for x, and figure out y:

$$y = 10 - x$$
(EQU 1)

$$\Rightarrow y = 10 - 9$$
(substitute 9 for x)

$$\Rightarrow y = 1$$

We're done. Now don't panic at how long it took to solve this problem. With a little practice, you'll be doing them quickly. We'll write our final answer as

$$x = 9 \& y = 1$$

Homework

8. Solve each system of equations by Substitution:

a. $\begin{array}{c} x+y=20\\ 4x-3y=-25 \end{array}$ b. $\begin{array}{c} y=2x+4\\ 3x+5y=7 \end{array}$ c. $\begin{array}{c} x-2y=-8\\ 4x+7y=28 \end{array}$

Opposite Directions

EXAMPLE 2:Mike and Sarah start from the burger stand
and skate in opposite directions.
Mike's speed is 5 less than 3 times
Sarah's speed. In 10 hours they are
70 miles apart. Find the speed of
both skaters.

Solution: Let's organize all the information in a table using the basic rt = d formula we're learning in this chapter. Down the first column are the names of our two skaters. Across the first row are the three components of motion, the rate (speed), the time, and the distance. We've written the formula to help us remember the basic relationship among these three concepts.

	Rate	× Time	=	Distance
Mike				
Sarah				

Since each skater's speed is being asked for (they're the unknowns), we'll let M stand for Mike's speed and S stand for Sarah's speed, and so these variables go into the Rate column.

As for the Time column, the problem states that each skater skated for exactly 10 hours, so each of their travel times is 10.

Since Distance = Rate × Time, the Distance column is simply the product of the Rate and Time columns for both Mike and Sarah.

	Rate	× Time =	Distance
Mike	M	10	10M
Sarah	S	10	10S

Since there are two unknowns in this problem, it's likely we'll need two equations. Let's look at the rates first: From the phrase in the problem "Mike's speed is 5 less than 3 times Sarah's speed" we create the equation

$$M = 3S - 5$$
 [Equation 1]

To determine the second equation, we have to picture where the skaters are going. They start in the same place and then skate in opposite directions and end up 70 miles from each other. Therefore, the <u>sum</u> of their individual distances must be 70. Well, Mike skated a distance of 10M miles while Sarah went 10S miles. So 70 must be the sum of 10M and 10S:

10M + 10S = 70 [Equation 2]

Now substitute the first equation into the second equation:

$$10(3S-5) + 10S = 70 \qquad (replaced M with 3S-5)$$

$$\Rightarrow \quad 30S-50 + 10S = 70 \qquad (distribute)$$

\Rightarrow	40S - 50 = 70	(combine like terms)
\Rightarrow	40S = 120	(add 50 to each side)
\Rightarrow	$\underline{S = 3}$	(divide each side by 40)

Recall that *S* stood for Sarah's speed, so we know for sure that Sarah skated 3 mph. To find Mike's speed we use Equation 1 and Sarah's speed:

$$M = 3S - 5 = 3(3) - 5 = 9 - 5 = 4$$

This shows that Mike skated at a rate of 4 mph. We now have the complete answer to the question:

Mike's speed was 4 mph and Sarah's speed was 3 mph.

- 9. Two pedestrians leave the same place and walk in opposite directions. The speed of one pedestrian is 5 mph less than 7 times the other. In 6 hours they are 354 miles apart. Find the speed of each pedestrian.
- 10. Two skaters leave the same place and skate in opposite directions. The speed of one skater is 8 mph less than 10 times the other. In 9 hours they are 819 miles apart. Find the speed of each skater.

- 11. Two joggers leave the same place and jog in opposite directions. The speed of one jogger is 9 mph more than 5 times the other. In 7 hours they are 357 miles apart. Find the speed of each jogger.
- 12. Two pedestrians leave the same place and walk in opposite directions. The speed of one pedestrian is 7 mph less than 9 times the other. In 10 hours they are 930 miles apart. Find the speed of each pedestrian.
- 13. Two skaters leave the same place and skate in opposite directions. The speed of one skater is 1 mph more than 7 times the other. In 9 hours they are 513 miles apart. Find the speed of each skater.

D PURSUIT

EXAMPLE 3: A camel leaves the oasis traveling 5 mph. Eight hours later a horse begins to pursue the camel at a speed of 45 mph. How many hours after the horse leaves the oasis will it catch up with the camel?

<u>Solution</u>: This problem gives us the rates of both animals, so those aren't an issue. In fact, the question is asking for the travel time of the horse. In addition, we don't know the travel time of the camel, either. So how about we let

- c = the travel *time* of the camel, and
- h = the travel *time* of the horse.

These variables will go into the Time column of our table. And since the rates (speeds) of the animals are given, we'll simply place them in the Rate column. As in the previous example, the

	Rate	× Time =	= Distance
camel	5	с	5c
horse	45	h	45h

Distance is found by multiplying the Rate by the Time. Here's the table with all the known and unknown information in it:

Again, two variables will require two equations. We'll start with the Time column. Notice that the horse left <u>after</u> the camel (by 8 hours). This implies that the horse's travel time was 8 hours <u>less</u> than the camel's. This observation (which is not very obvious) leads to the first equation:

$$h = c - 8$$
 [Equation 1]

To determine the second equation, we have to visualize where the animals are going. They start in the same place, leave one after the other, and then go in the same direction and end up in the same place. Thus, each of them went the <u>same distance</u> even though the camel left before the horse. This fact means that we can set the two distances in the table equal to each other:

5c = 45h [Equation 2]

Substituting Equation 1 into Equation 2:

	5c = 45(c - 8)	(since $h = c - 8$)
\Rightarrow	5c = 45c - 360	(distribute)
\Rightarrow	-40c = -360	(subtract $45c$ from each side)
\Rightarrow	c = 9	(divide each side by -40)

This tells us that the camel traveled for 9 hours. But the question asked for the horse's travel time, so we use Equation 1 to find h:

h = c - 8 = 9 - 8 = 1

We conclude that

It takes the horse 1 hour to catch up with the camel.

Homework

- 14. A camel leaves the oasis traveling 10 mph. Five hours later a dune buggy begins to pursue the camel at a speed of 15 mph. How many hours after the dune buggy leaves the oasis will it catch up with the camel?
- 15. A sailboat leaves the island traveling 14 mph. Five hours later a hydrofoil begins to pursue the sailboat at a speed of 24 mph. How many hours after the hydrofoil leaves the island will it catch up with the sailboat?
- 16. A robber leaves the bank traveling 15 mph. Four hours later a sheriff begins to pursue the robber at a speed of 35 mph. How many hours after the sheriff leaves the bank will he catch up with the robber?
- 17. A rowboat leaves the harbor traveling 26 mph. Seven hours later a speedboat begins to pursue the rowboat at a speed of 39 mph. How many hours after the speedboat leaves the harbor will it catch up with the rowboat?
- 18. A robber leaves the bank traveling 26 mph. Three hours later a sheriff begins to pursue the robber at a speed of 39 mph. How many hours after the sheriff leaves the bank will she catch up with the robber?

C ROUND TRIP

EXAMPLE 4: It takes a helicopter a total of 13 hours to travel from the mountain to the valley at a speed of 30 mph and return at a speed of 35 mph. How long does it take to get

from the mountain to the valley?

Solution: We'll let *x* represent the time it takes to go from the mountain to the valley (since this is what's being asked for). Let's also choose *y* to stand for the time it takes to return from the valley to the mountain. The two rates (speeds) are given, and we are getting pretty good at knowing that each distance is the product of the rate and the time. So here's the table:

	Rate :	× Time =	= Distance
to valley	30	x	30x
to mtn	35	у	35y

Since the <u>total</u> travel time is 13 hours, we get our first equation:

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x + y = 13 [Equation 1]
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Now what about the distances, 30x and 35y? Wouldn't you agree that the distance from the mountain to the valley is the same as the distance from the valley to the mountain? That is, 30x and 35y must be equal:

$$30x = 35y$$
 [Equation 2]

To solve this system of two equations in two unknowns, let's take Equation 1 and solve it for *y*:

	x + y = 13	(Equation 1)
\Rightarrow	y = 13 - x	(subtract <i>x</i> from each side)

We now replace the variable *y* in Equation 2 with the result just obtained:

	30x = 35(13 - x)	
\Rightarrow	30x = 455 - 35x	(distribute)
\Rightarrow	65x = 455	(add 35x to each side)
\Rightarrow	$\underline{x = 7}$	(divide each side by 65)

Now, what did x represent? Go back to the table and see that x represented the travel time from the mountain to the valley. But this is exactly what was being asked for in the problem, so we're done.

It takes 7 hours to travel from the mountain to the valley.

- 19. A helicopter traveled from the hospital to the battlefield at a speed of 22 mph and returned at a speed of 44 mph. If the entire trip took 18 hours, find the travel times to and from the battlefield.
- 20. A hang glider traveled from the beach to the mountaintop at a speed of 18 mph and returned at a speed of 21 mph. If the entire trip took 13 hours, find the travel times to and from the mountaintop.
- 21. A helicopter traveled from the hospital to the battlefield at a speed of 36 mph and returned at a speed of 24 mph. If the entire trip took 20 hours, find the travel times to and from the battlefield.
- 22. A tractor traveled from the wheat field to the chicken coop at a speed of 27 mph and returned at a speed of 36 mph. If the entire trip took 21 hours, find the travel times to and from the chicken coop.
- 23. A hang glider traveled from the beach to the mountaintop at a speed of 34 mph and returned at a speed of 51 mph. If the entire trip took 15 hours, find the travel times to and from the mountaintop.

Two-Part Journey

EXAMPLE 5:A limousine traveled at 29 mph
for the first part of a 540-mile trip,
and then increased its speed to 53
mph for the rest of the trip. How
many hours were traveled at each
rate if the total trip took 12 hours?



Solution: The rates for each part of the trip are given, so just put them in the table in the right places. Let *x* be the travel time for the first part of the trip and let *y* be the travel time for the second part of the trip. Finally, the Distance column is the product of the Rate and Time columns.

	Rate	× Time =	= Distance
1st part	29	x	29x
2nd part	53	У	53y

The total travel is given to be 12 hours. Therefore,

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x + y = 12
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Since the total distance traveled was 540 miles, adding the distance of the 1st part of the trip plus the distance of the 2nd part of the trip should give a total of 540:

$$29x + 53y = 540$$

Solving the first equation for y gives y = 12 - x. Substituting 12 - x for y in the second equation gives:

$$29x + 53(12 - x) = 540$$

$$\Rightarrow 29x + 636 - 53x = 540 \qquad (distribute)$$

$$\Rightarrow -24x + 636 = 540 \qquad (combine like terms)$$

 $\Rightarrow -24x = -96 \qquad (subtract 636)$ $\Rightarrow \underline{x = 4}$

This means that the first part of the limo trip took 4 hours. Using the equation y = 12 - x, we calculate the time for the rest of the trip as y = 12 - x = 12 - 4 = 8. In short,

4 hours at 29 mph and 8 hours at 53 mph

- 24. A 1096-mi trip took a total of 16 hours. The speed was 71 mph for the first part of the trip, and then decreased to 67 mph for the rest of the trip. How many hours were traveled at each speed?
- 25. A 730-mi trip took a total of 11 hours. The speed was 68 mph for the first part of the trip, and then decreased to 59 mph for the rest of the trip. How many hours were traveled at each speed?
- 26. A 664-mi trip took a total of 12 hours. The speed was 30 mph for the first part of the trip, and then increased to 68 mph for the rest of the trip. How many hours were traveled at each speed?
- 27. A 489-mi trip took a total of 9 hours. The speed was 45 mph for the first part of the trip, and then increased to 59 mph for the rest of the trip. How many hours were traveled at each speed?
- 28. A 556-mi trip took a total of 14 hours. The speed was 38 mph for the first part of the trip, and then increased to 42 mph for the rest of the trip. How many hours were traveled at each speed?

EXAMPLE 6:Six hours after Mary and Moe
leave the museum at the same
time and head in the same
direction, Moe is 252 miles
ahead of Mary. If Moe's speed is
8 mph less than 3 times Mary's
speed, find the speeds of Mary and Moe.

Solution: Mary and Moe left the museum at the same time and each traveled for 6 hours, so both times in the table must be 6. Since the rates are unknown, we let x represent Mary's speed and let y represent Moe's speed.

	Rate	× Time 🛛 =	= Distance
Mary	x	6	6x
Moe	У	6	6 <i>y</i>

First we need an equation relating *x* and *y*. The phrase "Moe's speed is 8 mph less than 3 times Mary's speed" translates to

$$y = 3x - 8.$$

Now, according to the table, Mary traveled 6x miles, while Moe traveled 6y miles. The problem says that at the end of the 6 hours, Moe is 252 miles ahead of Mary. This means that Moe's distance (6y) is 252 miles more than Mary's distance (6x), which translates into our second equation:

$$6y = 6x + 252$$

Substituting the first equation into the second equation gives us

$$6(3x - 8) = 6x + 252$$

 $\Rightarrow 18x - 48 = 6x + 252$ (distribute)

\Rightarrow	12x = 300	(subtract 6 <i>x</i> and add 48)
\Rightarrow	$\underline{x = 25}$	(divide by 12)

which implies that Moe's rate is y = 3x - 8 = 3(25) - 8 = 67.

Mary's speed was 25 mph and Moe's speed was 67 mph.

- 29. Moe and Curly leave the airport at the same time and head in the same direction. Curly's speed is 8 mph less than 3 times Moe's speed. Five hours later Curly is 430 miles ahead of Moe. Find the speeds of Moe and Curly.
- 30. Sally and Maria leave the mall at the same time and head in the same direction. Maria's speed is 6 mph less than 2 times Sally's speed. Ten hours later Maria is 410 miles ahead of Sally. Find the speeds of Sally and Maria.
- 31. Lucy and Ethyl leave the mall at the same time and head in the same direction. Ethyl's speed is 4 mph more than 4 times Lucy's speed. Four hours later Ethyl is 604 miles ahead of Lucy. Find the speeds of Lucy and Ethyl.
- 32. Sally and Maria leave the stadium at the same time and head in the same direction. Maria's speed is 3 mph less than 4 times Sally's speed. Eight hours later Maria is 1080 miles ahead of Sally. Find the speeds of Sally and Maria.
- 33. Mutt and Jeff leave the stadium at the same time and head in the same direction. Jeff's speed is 8 mph less than 3 times Mutt's speed. Ten hours later Jeff is 660 miles ahead of Mutt. Find the speeds of Mutt and Jeff.

Practice Problems

- 34. Two skaters leave from the same place and skate in opposite directions.The speed of one skater is 5 mph more than 9 times the other. In 10 hours they are 850 miles apart. Find the speed of each skater.
- 35. A camel leaves the oasis traveling 18 mph. Four hours later a dune buggy begins to pursue the camel at a speed of 26 mph. How many hours after the dune buggy leaves the oasis will it catch up with the camel?
- 36. A pickup truck traveled from the house to the ballpark at a speed of 16 mph and returned at a speed of 20 mph. If the entire trip took 18 hours, find the travel times to and from the ballpark.
- 37. A 450-mi trip took a total of 9 hours. The speed was 46 mph for the first part of the trip, and then increased to 64 mph for the rest of the trip. How many hours were traveled at each speed?
- 38. Sally and Maria leave the museum at the same time and head in the same direction. Maria's speed is 1 mph less than 10 times Sally's speed. Six hours later Maria is 1938 miles ahead of Sally. Find the speeds of Sally and Maria.
- 39. Mutt and Jeff leave the mall at the same time and head in the same direction. Jeff's speed is 9 mph more than 6 times Mutt's speed. Four hours later Jeff is 1036 miles ahead of Mutt. Find the speeds of Mutt and Jeff.
- 40. A 430-mi trip took a total of 11 hours. The speed was 35 mph for the first part of the trip, and then increased to 50 mph for the rest of the trip. How many hours were traveled at each speed?
- 41. A hang glider traveled from the beach to the mountaintop at a speed of 34 mph and returned at a speed of 17 mph. If the entire trip took 18 hours, find the travel times to and from the mountaintop.

- 42. A robber leaves the bank traveling 26 mph. Three hours later a sheriff begins to pursue the robber at a speed of 39 mph. How many hours after the sheriff leaves the bank will she catch up with the robber?
- 43. Two skaters leave from the same place and skate in opposite directions. The speed of one skater is 4 mph less than 6 times the other. In 4 hours they are 180 miles apart. Find the speed of each skater.
- 44. A robber leaves the bank traveling 15 mph. Seven hours later a sheriff begins to pursue the robber at a speed of 30 mph. How many hours after the sheriff leaves the bank will he catch up with the robber?
- 45. An 1127-mi trip took a total of 17 hours. The speed was 55 mph for the first part of the trip, and then increased to 79 mph for the rest of the trip. How many hours were traveled at each speed?
- 46. A helicopter traveled from the hospital to the battlefield at a speed of 45 mph and returned at a speed of 35 mph. If the entire trip took 16 hours, find the travel times to and from the battlefield.
- 47. Sally and Maria leave the museum at the same time and head in the same direction. Maria's speed is 4 mph less than 4 times Sally's speed. Three hours later Maria is 186 miles ahead of Sally. Find the speeds of Sally and Maria.
- 48. Two spaceships leave from the same place and fly in opposite directions. The speed of one spaceship is 6 mph more than 4 times the other. In 8 hours they are 248 miles apart. Find the speed of each spaceship.

Solutions

1.	a. 1440 km b. 500 r	mi/h	r c. 4 hrs		
2.	a. $d = rt$ or $rt = d$				
3.	$d_1 + d_2 = 18$ 4 . d_1	$= d_{s}$	5 . $d_1 = d_2$	6 .	$d_1 + d_2 = 1096$
7 .	$d_1 + 1036 = d_2 \text{ OR } d_2 - d_1 = 1036 \text{ OR } d_2 - 1036 = d_1$				
8.	x = 5 & y = 15	b. :	x = -1 & $y = 2$	с. <i>х</i>	x = 0 & y = 4
9 .	8 mph & 51 mph	10.	9 mph & 82 mph	11.	7 mph & 44 mph
12.	10 mph & 83 mph	13.	7 mph & 50 mph	14.	10 hours
15.	7 hours	16.	3 hours	17.	14 hours
18 .	6 hours	19 .	12 hrs & 6 hrs	20 .	7 hrs & 6 hrs
21 .	8 hrs & 12 hrs	22 .	12 hrs & 9 hrs	23 .	9 hrs & 6 hrs
24 .	6 hrs & 10 hrs	25 .	9 hrs & 2 hrs	26 .	4 hrs & 8 hrs
27 .	3 hrs & 6 hrs	28 .	8 hrs & 6 hrs	29 .	47 mph & 133 mph
30 .	47 mph & 88 mph	31.	49 mph & 200 mph	32.	46 mph & 181 mph
33.	37 mph & 103 mph	34.	8 mph & 77 mph	35.	9 hours
36 .	10 hrs & 8 hrs	37 .	7 hrs & 2 hrs	38 .	36 mph & 359 mph
39 .	50 mph & 309 mph	40 .	8 hrs & 3 hrs	41 .	6 hrs & 12 hrs
42 .	6 hours	43 .	7 mph & 38 mph	44 .	7 hours
45 .	9 hrs & 8 hrs	46 .	7 hrs & 9 hrs	47.	22 mph & 84 mph
48 .	5 mph & 26 mph				

"Formal education is but an incident in the lifetime of an individual. Most of us who have given the subject any study have come to realize that education is a continuous process ending only when ambition comes to a halt."

– R.I. Rees