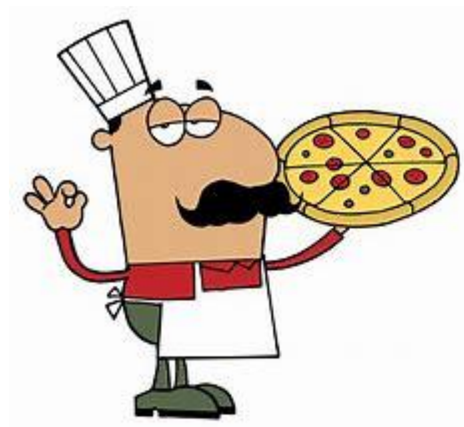
CH 22 – FRACTIONS, PART II

❑ INTRODUCTION

We continue our analysis of algebraic fractions. Now the fractions will have more stuff in their numerators and denominators, but all the basic rules still apply. To **add** or **subtract** fractions, we must have a common denominator; to **multiply** fractions, we still multiply the tops and multiply the bottoms; and we continue to **divide** by multiplying by the reciprocal.



❑ MORE ADDING AND SUBTRACTING

EXAMPLE 1: **Add:** $\frac{4u-21}{u+7} + \frac{u^2}{u+7}$

Solution: We are adding fractions with the same denominator (u + 7). So all we have to do is add the numerators. The common denominator becomes the denominator of the sum:

$$\frac{4u-21+u^2}{u+7}$$

We've added the two fractions so that it's now a single fraction, but all fractional answers need to be reduced — we'll rearrange the terms in the numerator so it's easier to factor:

$$\frac{u^2+4u-21}{u+7}$$

Factor the numerator and divide out the common factor:

$$\frac{(u+7)(u-3)}{u+7} = \frac{\cancel{(u+7)}(u-3)}{\cancel{u+7}} = \boxed{u-3}$$

EXAMPLE 2: **Subtract:** $\dfrac{\mathbf{12x-10}}{\mathbf{x^2-2x-8}} - \dfrac{\mathbf{11x-12}}{\mathbf{x^2-2x-8}}$

Solution: Like addition, subtraction requires a common denominator. The fractions in this example already have the same denominator, so there's nothing to worry about here.

The tricky part is subtracting the numerators. We must subtract the second numerator (all of it!) from the first numerator — this is where parentheses come to the rescue:

$$\frac{12x-10}{x^2-2x-8} - \frac{11x-12}{x^2-2x-8} \quad \text{(the original problem)}$$

$$= \frac{(12x-10)-(11x-12)}{x^2-2x-8} \quad \text{(one numerator minus the other)}$$

$$= \frac{12x-10-11x+12}{x^2-2x-8} \quad \text{(remove the parentheses)}$$

$$= \frac{x+2}{x^2-2x-8} \quad \text{(combine like terms)}$$

$$= \frac{\cancel{x+2}}{\cancel{(x+2)}(x-4)} \quad \text{(factor and divide)}$$

$$= \boxed{\frac{1}{x-4}}$$

<u>EXAMPLE 3:</u> **Subtract:** $\dfrac{10}{a^6} - \dfrac{7}{a^2}$

<u>Solution:</u> The denominators are different, so we can't add the fractions yet. To make the denominators the same, the second denominator (a^2) needs to be built up to match the first denominator (a^6). We ask, what should we multiply a^2 by to get a^6? Since a^2 is 2 factors of a, and a^6 is 6 factors of a, we would need 4 more factors of a to make them match. So we'll multiply the second fraction, top and bottom, by a^4.

$$\frac{10}{a^6} - \frac{7}{a^2}$$

$$= \frac{10}{a^6} - \frac{7}{a^2}\left[\frac{a^4}{a^4}\right] \quad \left(\left[\frac{a^4}{a^4}\right] = 1\right)$$

$$= \underbrace{\frac{10}{a^6} - \frac{7a^4}{a^6}}_{\text{same denominator}}$$

$$= \boxed{\frac{10 - 7a^4}{a^6}}$$

<u>EXAMPLE 4:</u> **Add:** $\dfrac{2}{x^2y^3} + \dfrac{7}{y^7z}$

<u>Solution:</u> This problem also requires a common denominator. To make the denominators the same, we notice that the first denominator needs four more factors of y and a factor of z; the second denominator needs an x^2 factor in it. Here's how we do it:

$$\frac{2}{x^2y^3} + \frac{7}{y^7z}$$

$$= \frac{2}{x^2y^3}\left[\frac{y^4z}{y^4z}\right] + \frac{7}{y^7z}\left[\frac{x^2}{x^2}\right]$$

$=1$ $\qquad$ $=1$

$$= \frac{2y^4z}{x^2y^7z} + \frac{7x^2}{x^2y^7z}$$

same denominator

$$= \boxed{\frac{2y^4z + 7x^2}{x^2y^7z}}$$

EXAMPLE 5: **Subtract:** $\dfrac{6k+4}{3} - \dfrac{2k+3}{2}$

Solution: The least common denominator (LCD) is 6, so we'll multiply the top and bottom of the first fraction by 2 and the top and bottom of the second fraction by 3:

$$\frac{6k+4}{3} - \frac{2k+3}{2} \qquad \text{(the original problem)}$$

$$= \left[\frac{2}{2}\right]\frac{6k+4}{3} - \left[\frac{3}{3}\right]\frac{2k+3}{2} \qquad \text{(create the LCD)}$$

$$= \frac{12k+8}{6} - \frac{6k+9}{6} \qquad \text{(multiply)}$$

$$= \frac{(12k+8)-(6k+9)}{6} \qquad \text{(combine the fractions)}$$

$$= \frac{12k+8-6k-9}{6} \qquad \text{(remove parentheses)}$$

$$\boxed{\frac{6k-1}{6}} \qquad \text{(simplify the numerator)}$$

Homework

1. Perform the indicated operation:

a. $\frac{d^2}{d-9} - \frac{d+72}{d-9}$

b. $\frac{3p+1}{p^2-6p-27} - \frac{2p-2}{p^2-6p-27}$

c. $\frac{n^2}{n-6} - \frac{7n-6}{n-6}$

d. $\frac{16x+63}{x+7} + \frac{x^2}{x+7}$

e. $\frac{-w-9}{w^2+8w-48} - \frac{-2w-5}{w^2+8w-48}$

f. $\frac{-6a-11}{a^2-10a-24} - \frac{-7a+1}{a^2-10a-24}$

2. Perform the indicated operation:

a. $\frac{8}{5w^6} - \frac{3}{2w^4}$

b. $\frac{3}{b^6} + \frac{9}{5b^3}$

c. $\frac{2}{k^2} + \frac{7}{4k^5}$

d. $\frac{1}{9w^2} - \frac{4}{5w^4}$

e. $\frac{2}{5r^2} + \frac{5}{4r^4}$

f. $\frac{1}{z^3} - \frac{7}{6z^6}$

3. Perform the indicated operation:

a. $\frac{2}{c^2m^2} + \frac{3}{m^2w^6}$

b. $\frac{4}{a^5c^5} - \frac{2}{c^6v^2}$

c. $\frac{5}{a^3b^2} + \frac{5}{b^5m^5}$

d. $\frac{2}{b^3h^2} - \frac{6}{h^3x^4}$

e. $\frac{5}{a^3c^2} + \frac{5}{c^4s^6}$

f. $\frac{6}{k^3t^4} - \frac{4}{t^5v^6}$

4. Perform the indicated operation:

a. $\frac{2z-9}{8} + \frac{2z-9}{5}$

b. $\frac{8b+6}{6} - \frac{6b-8}{2}$

c. $\dfrac{-9m-9}{9} - \dfrac{-5m-9}{7}$

d. $\dfrac{4s-9}{3} + \dfrac{5s-3}{3}$

e. $\dfrac{-7u+9}{5} - \dfrac{4u-1}{6}$

f. $\dfrac{-2k-5}{4} + \dfrac{9k+1}{5}$

❑ *MORE MULTIPLYING AND DIVIDING*

EXAMPLE 6: $$\frac{\boldsymbol{a^4k^5}}{\boldsymbol{u^2b^2}} \times \frac{\boldsymbol{a^3k^4}}{\boldsymbol{u^2b}} = \frac{a^4k^5a^3k^4}{u^2b^2u^2b} = \frac{a^7k^9}{u^4b^3}$$

EXAMPLE 7: $$\frac{\boldsymbol{k^4u^6}}{\boldsymbol{ws^2}} \div \frac{\boldsymbol{k^6u}}{\boldsymbol{ws^4}}$$

$$= \frac{k^4u^6}{ws^2} \cdot \frac{ws^4}{k^6u} = \frac{\cancel{k^4}\,u^{\cancel{6}5}}{\cancel{w}\,\cancel{s^2}} \cdot \frac{\cancel{w}s^{\cancel{4}2}}{k^{\cancel{6}2}\,\cancel{u}} = \frac{s^2u^5}{k^2}$$

EXAMPLE 8: $$\frac{\boldsymbol{x^2-9}}{\boldsymbol{x^2-6x+9}} \cdot \frac{\boldsymbol{x+2}}{\boldsymbol{x^2+5x+6}}$$

Solution: Factor everything possible:

$$\frac{(x+3)(x-3)}{(x-3)(x-3)} \cdot \frac{x+2}{(x+2)(x+3)}$$

$$= \frac{\cancel{(x+3)}\,\cancel{(x-3)}}{(x-3)\cancel{(x-3)}} \cdot \frac{\cancel{x+2}}{\cancel{(x+2)}\,\cancel{(x+3)}}$$

$$= \boxed{\frac{1}{x-3}}$$

EXAMPLE 9: $$\frac{\boldsymbol{x+10}}{\boldsymbol{x^2+6x-7}} \div \frac{\boldsymbol{x-12}}{\boldsymbol{x^2-13x+12}}$$

Solution:

$$\frac{x+10}{x^2+6x-7} \div \frac{x-12}{x^2-13x+12}$$

$$= \frac{x+10}{x^2+6x-7} \times \frac{x^2-13x+12}{x-12} \quad \text{(invert and multiply)}$$

$$= \frac{x+10}{(x+7)(x-1)} \times \frac{(x-1)(x-12)}{x-12} \quad \text{(factor everything)}$$

$$= \frac{x+10}{(x+7)\cancel{(x-1)}} \times \frac{\cancel{(x-1)}\cancel{(x-12)}}{\cancel{x-12}} \quad \text{(cross-cancel)}$$

$$= \boxed{\frac{x+10}{x+7}}$$

Homework

5. Perform the indicated operation:

a. $\frac{x^2k}{t^4u^4} \div \frac{t^4u^5}{x^4k^3}$ b. $\frac{r^6a^6}{v^5u} \div \frac{r^4a^4}{vu^6}$ c. $\frac{w^2r^5}{k^6x} \div \frac{kx^6}{w^2r^5}$

d. $\frac{r^5z^5}{b^2n^4} \cdot \frac{b^6n}{r^6z^4}$ e. $\frac{u^4w}{v^3m^2} \div \frac{uw^6}{v^6m}$ f. $\frac{ku^2}{v^6w^3} \cdot \frac{v^6w^5}{k^4u^3}$

6. Perform the indicated operation:

a. $\frac{c-2}{c^2} \cdot \frac{c^2+4c}{c+4}$ b. $\frac{m}{m^2-2m-35} \cdot \frac{m^2-m-42}{m+6}$

c. $\dfrac{c-2}{c^2} \div \dfrac{c-3}{c^2-4c}$ d. $\dfrac{v}{v^2+4v-21} \div \dfrac{v+10}{v^2-2v-63}$

e. $\dfrac{q+6}{q^2+6q} \div \dfrac{q}{q^2+6q}$ f. $\dfrac{v-1}{v^2-v-6} \cdot \dfrac{v^2-2v-3}{v+1}$

7. Perform the indicated operation:

a. $\dfrac{m-6}{m^2-21m+108} \div \dfrac{m+5}{m^2-4m-45}$ b. $\dfrac{c+7}{c^2-18c+80} \cdot \dfrac{c^2-6c-40}{c+4}$

c. $\dfrac{z-1}{z^2-15z+44} \cdot \dfrac{z^2+2z-24}{z+6}$ d. $\dfrac{z+4}{z^2+8z+7} \div \dfrac{z-4}{z^2-3z-4}$

e. $\dfrac{m+8}{m^2+22m+120} \div \dfrac{m+3}{m^2+18m+80}$ f. $\dfrac{u-8}{u^2-6u-7} \div \dfrac{u-10}{u^2-9u-10}$

❑ And Some More Adding and Subtracting

EXAMPLE 10: **Subtract:** $\dfrac{7}{x+3} - \dfrac{3}{x-2}$

Solution: The two denominators have no factor in common. (Yes, they both have an x, but this is a term, not a factor.) The LCD is the product of the two denominators: $(x+3)(x-2)$. So we'll multiply the top and bottom of the first fraction by $(x-2)$, and multiply the top and bottom of the second fraction by $(x+3)$:

$$\frac{7}{x+3} - \frac{3}{x-2} \qquad \text{(the original problem)}$$

$$= \frac{7}{x+3}\underbrace{\left[\frac{x-2}{x-2}\right]}_{=1} - \frac{3}{x-2}\underbrace{\left[\frac{x+3}{x+3}\right]}_{=1} \qquad \text{(build up to the LCD)}$$

$$= \frac{7(x-2)}{(x+3)(x-2)} - \frac{3(x+3)}{(x-2)(x+3)} \quad \text{(the bottoms are the same)}$$

same denominator

$$= \frac{7(x-2)-3(x+3)}{(x-2)(x+3)} \quad \text{(combine the two fractions)}$$

$$= \frac{7x-14-3x-9}{x^2+x-6} \quad \text{(distribute)}$$

$$= \boxed{\frac{4x-23}{x^2+x-6}} \quad \text{(combine like terms)}$$

EXAMPLE 11: **Add:** $\frac{5}{m^2+5m+6} + \frac{3}{m+2}$

Solution: This is a tricky one. In order to add these fractions, we have to multiply tops and bottoms by factors that will result in the LCD. But to know what factors to multiply by, we have to know what factors are already in the denominators. So, we factor the denominators:

$$\frac{5}{m^2+5m+6} + \frac{3}{m+2} = \frac{5}{(m+2)(m+3)} + \frac{3}{m+2}$$

Now we can easily see the factors in the denominators. To make the denominators the same, we need to multiply the top and bottom of the second fraction by $m + 3$. Then we can add the fractions:

$$\frac{5}{(m+2)(m+3)} + \frac{3}{m+2}\left[\frac{m+3}{m+3}\right] = \frac{5}{(m+2)(m+3)} + \frac{3(m+3)}{(m+2)(m+3)}$$

$$= \frac{5+3(m+3)}{(m+2)(m+3)} = \frac{5+3m+9}{m^2+5m+6} = \boxed{\frac{3m+14}{m^2+5m+6}}$$

EXAMPLE 12: **Subtract:** $\dfrac{8}{u^2+3u-4} - \dfrac{7-5u}{u^2-3u-28}$

Solution: First factor all the denominators:

$$\frac{8}{(u-1)(u+4)} - \frac{7-5u}{(u+4)(u-7)}$$

Now build up the fractions so that the LCD is created:

$$= \frac{8}{(u-1)(u+4)}\left[\frac{u-7}{u-7}\right] - \frac{7-5u}{(u+4)(u-7)}\left[\frac{u-1}{u-1}\right]$$

$$= \frac{8(u-7)}{(u-1)(u+4)(u-7)} - \frac{(7-5u)(u-1)}{(u-4)(u-7)(u-1)}$$

Notice that the denominators are the same.

$$= \frac{8(u-7) - (7-5u)(u-1)}{(u-1)(u+4)(u-7)}$$ (subtract the numerators)

$$= \frac{8u-56-(7u-7-5u^2+5u)}{(u-1)(u+4)(u-7)}$$ (distribute)

Notice the parentheses

$$= \frac{8u-56-(-5u^2+12u-7)}{(u-1)(u+4)(u-7)}$$ (combine like terms)

$$= \frac{8u-56+5u^2-12u+7}{(u-1)(u+4)(u-7)}$$ (distribute the minus sign)

$$= \boxed{\frac{5u^2-4u-49}{(u-1)(u+4)(u-7)}}$$ (combine like terms)

You may be wondering why we left the denominator in factored form. It's just a matter of opinion, but many teachers figure that the problem's hard enough without having to multiply out those three binomials.

Homework

8. Perform the indicated operation:

a. $\dfrac{6}{d+1} + \dfrac{-4}{6d+5}$

b. $\dfrac{-6}{5h-2} - \dfrac{5}{4h-1}$

c. $\dfrac{-6}{q+1} + \dfrac{3}{5q+4}$

d. $\dfrac{3}{2t+5} - \dfrac{-6}{2t+1}$

9. Perform the indicated operation:

a. $\dfrac{3}{z-3} + \dfrac{1}{z^2+12z+32}$

b. $\dfrac{-3}{u^2+u-72} - \dfrac{3}{u+9}$

c. $\dfrac{-2}{w^2-19w+90} - \dfrac{2}{w-10}$

d. $\dfrac{4}{h+7} + \dfrac{-5}{h^2+11h+28}$

10. Perform the indicated operation:

a. $\dfrac{7}{b^2+5b-6} + \dfrac{3b-1}{b^2-4b-60}$

b. $\dfrac{-6}{a^2-16} - \dfrac{-5a+3}{a^2-5a+4}$

c. $\dfrac{-1}{s^2-s-42} - \dfrac{6s-7}{s^2-4s-21}$

d. $\dfrac{9}{v^2-9} + \dfrac{2v+1}{v^2+13v+30}$

Practice Problems

11. Perform the indicated operation:

a. $\frac{z^2}{z-2} - \frac{-2z+8}{z-2}$

b. $\frac{-7u+1}{u^2-u-42} - \frac{-8u+8}{u^2-u-42}$

c. $\frac{-11x-8}{x^2+3x-18} + \frac{12x+5}{x^2+3x-18}$

d. $\frac{m^2}{m-3} - \frac{4m-3}{m-3}$

e. $\frac{3s-10}{s-2} + \frac{s^2}{s-2}$

f. $\frac{11r-5}{r^2+r-30} - \frac{10r-11}{r^2+r-30}$

g. $\frac{9}{7v^2} - \frac{7}{4v^2}$

h. $\frac{3}{c^3} - \frac{5}{8c^2}$

i. $\frac{4}{9t^6} + \frac{8}{3t^6}$

j. $\frac{6}{r^5v^3} - \frac{4}{v^6z^5}$

k. $\frac{5}{c^2n^2} - \frac{2}{n^2v^5}$

l. $\frac{6}{n^2v^4} - \frac{6}{v^2x^6}$

m. $\frac{5}{h^6n^2} + \frac{4}{n^6u^3}$

n. $\frac{6}{m^4s^2} - \frac{3}{s^5z^4}$

o. $\frac{3}{k^3u^6} + \frac{1}{u^4v^2}$

p. $\frac{-4n+4}{9} + \frac{-5n-3}{6}$

q. $\frac{-4d+7}{9} - \frac{9d-5}{4}$

r. $\frac{4a+5}{4} - \frac{8a+2}{9}$

s. $\frac{5a-8}{2} - \frac{-2a-8}{7}$

t. $\frac{-6r-1}{8} + \frac{8r-8}{7}$

u. $\frac{-4u+5}{6} + \frac{7u+1}{9}$

v. $\frac{9h+8}{h^2-5h-50} - \frac{8h+3}{h^2-5h-50}$

w. $\dfrac{4-5n}{n-1} + \dfrac{n^2}{n-1}$

x. $\dfrac{2}{b^2d^3} - \dfrac{1}{d^6z^4}$

y. $\dfrac{7-4h}{8} + \dfrac{8-8h}{4}$

z. $\dfrac{4t+5}{7} - \dfrac{7t+3}{6}$

12. Perform the indicated operation:

a. $\dfrac{c^2d}{y^3} \div \dfrac{dc^2}{y^3}$

b. $\dfrac{a^3c}{xy} \cdot \dfrac{yx}{a^3c}$

c. $\dfrac{x^2-4}{x+3} \cdot \dfrac{x+3}{x+2}$

d. $\dfrac{n^2-5n+6}{n^2-4n+4} \div \dfrac{n^2-6n+9}{n-2}$

e. $\dfrac{x^2-x}{x} \cdot \dfrac{1}{x-1}$

f. $\dfrac{abc}{c^2-9} \div \dfrac{b^2c^2}{c+3}$

g. $\dfrac{x^2+5x+6}{x^2+4x+3} \cdot \dfrac{x^2+3x+2}{x^2-2x-8}$

h. $\dfrac{x^2-9}{x^2-1} \div \dfrac{x-3}{x-1}$

i. $\dfrac{2a^2+a-1}{a^2-1} \div \dfrac{4a^2-4a+1}{a^2-2a+1}$

Solutions

1. a. $d + 8$ b. $\dfrac{1}{p-9}$ c. $n - 1$

d. $x + 9$ e. $\dfrac{1}{w+12}$ f. $\dfrac{1}{a+2}$

2. a. $\dfrac{16-15w^2}{10w^6}$ b. $\dfrac{15+9b^3}{5b^6}$ c. $\dfrac{8k^3+7}{4k^5}$

d. $\dfrac{5w^2-36}{45w^4}$ e. $\dfrac{8r^2+25}{20r^4}$ f. $\dfrac{6z^3-7}{6z^6}$

3. a. $\dfrac{2w^6+3c^2}{c^2m^2w^6}$ b. $\dfrac{4cv^2-2a^5}{a^5c^6v^2}$ c. $\dfrac{5b^3m^5+5a^3}{a^3b^5m^5}$

d. $\dfrac{2hx^4-6b^3}{b^3h^3x^4}$ e. $\dfrac{5c^2s^6+5a^3}{a^3c^4s^6}$ f. $\dfrac{6tv^6-4k^3}{k^3t^5v^6}$

4. a. $\dfrac{26z-117}{40}$ b. $\dfrac{-5b+15}{3}$ c. $\dfrac{-2m+2}{7}$

d. $3s-4$ e. $\dfrac{-62u+59}{30}$ f. $\dfrac{26k-21}{20}$

5. a. $\dfrac{x^6k^4}{t^8u^9}$ b. $\dfrac{r^2a^2u^5}{v^4}$ c. $\dfrac{w^4r^{10}}{k^7x^7}$

d. $\dfrac{zb^4}{rn^3}$ e. $\dfrac{u^3v^3}{w^5m}$ f. $\dfrac{w^2}{k^3u}$

6. a. $\dfrac{c-2}{c}$ b. $\dfrac{m}{m+5}$ c. $\dfrac{c^2-6c+8}{c^2-3c}$

d. $\dfrac{v^2-9v}{v^2+7v-30}$ e. $\dfrac{q+6}{q}$ f. $\dfrac{v-1}{v+2}$

7. a. $\dfrac{m-6}{m-12}$ b. $\dfrac{c+7}{c-8}$ c. $\dfrac{z-1}{z-11}$

d. $\dfrac{z+4}{z+7}$ e. $\dfrac{m^2+16m+64}{m^2+15m+36}$ f. $\dfrac{u-8}{u-7}$

8. a. $\dfrac{32d+26}{(d+1)(6d+5)}$ or $\dfrac{32d+26}{6d^2+11d+5}$ b. $\dfrac{-49h+16}{(5h-2)(4h-1)}$

c. $\dfrac{-27q-21}{(q+1)(5q+4)}$ d. $\dfrac{18t+33}{(2t+5)(2t+1)}$

9. a. $\dfrac{3z^2+37z+93}{(z+4)(z+8)(z-3)}$ b. $\dfrac{-3u+21}{(u+9)(u-8)}$

c. $\dfrac{-2w+16}{(w-10)(w-9)}$ d. $\dfrac{4h+11}{(h+7)(h+4)}$

10. a. $\frac{3b^2+3b-69}{(b-1)(b+6)(b-10)}$ b. $\frac{5a^2+11a-6}{(a+4)(a-4)(a-1)}$

c. $\frac{-6s^2-30s+39}{(s+6)(s-7)(s+3)}$ d. $\frac{2v^2+4v+87}{(v-3)(v+3)(v+10)}$

11. a. $z+4$ b. $\frac{1}{u+6}$ c. $\frac{1}{x+6}$ d. $m-1$

e. $s+5$ f. $\frac{1}{r-5}$ g. $\frac{-13}{28v^2}$ h. $\frac{24-5c}{8c^3}$

i. $\frac{28}{9t^6}$ j. $\frac{6v^3z^5-4r^5}{r^5v^6z^5}$ k. $\frac{5v^5-2c^2}{c^2n^2v^5}$

l. $\frac{6x^6-6n^2v^2}{n^2v^4x^6}$ m. $\frac{5n^4u^3+4h^6}{h^6n^6u^3}$ n. $\frac{6s^3z^4-3m^4}{m^4s^5z^4}$

o. $\frac{3v^2+k^3u^2}{k^3u^6v^2}$ p. $\frac{-23n-1}{18}$ q. $\frac{-97d+73}{36}$

r. $\frac{4a+37}{36}$ s. $\frac{39a-40}{14}$ t. $\frac{22r-71}{56}$

u. $\frac{2u+17}{18}$ v. $\frac{1}{h-10}$ w. $n-4$

x. $\frac{2d^3z^4-b^2}{b^2d^6z^4}$ y. $\frac{-20h+23}{8}$ z. $\frac{-25t+9}{42}$

12. a. 1 b. 1 c. $x-2$ d. $\frac{1}{n-3}$ e. 1

f. $\frac{a}{bc(c-3)}$, or $\frac{a}{bc^2-3bc}$ g. $\frac{x+2}{x-4}$ h. $\frac{x+3}{x+1}$ i. $\frac{a-1}{2a-1}$

"The illiterate of the 21st century will not be those who cannot read and write, but those who cannot learn, unlearn, and relearn."

– Alvin Toffler