CH 26 – FRACTIONAL EXPONENTS

❑ *INTRODUCTION*

Let's begin by reviewing the **Five Laws of Exponents**:

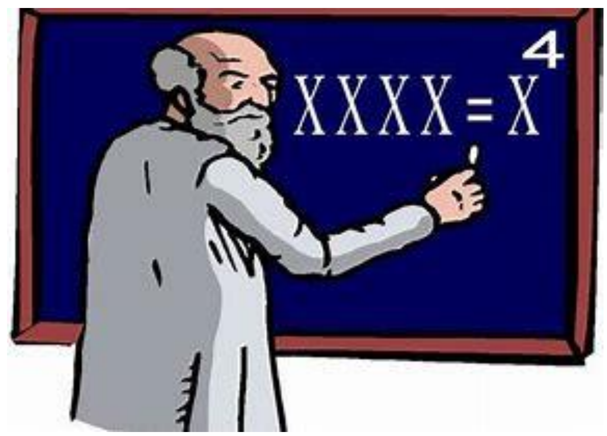
$$x^a x^b = x^{a+b}$$

$$\frac{x^a}{x^b} = x^{a-b}$$

$$(x^a)^b = x^{ab}$$

$$(x\,y)^a = x^a\,y^a$$

$$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$$



Homework

1. Simplify each expression:

a. x^3x^7 b. $\frac{a^{10}}{a^5}$ c. $(z^3)^5$ d. $\left(\frac{x}{y}\right)^9$ e. $(mn)^6$

f. r^4t^5 g. $a^2 + a^4$ h. $g^6 + g^6$ i. $(x+y)^2$ j. x^4x^{-3}

k. $(c^{-3})^{-5}$ l. $(ab)^{-4}$ m. $\frac{x^{-8}}{x^{-6}}$ n. $\frac{a^{-4}}{a^4}$ o. $\left(\frac{u}{w}\right)^{-3}$

p. $(a-b)^2$

❑ THE MEANING OF A FRACTIONAL EXPONENT

Here's what we've learned about exponents so far in this course:

$$x^3 = xxx \qquad y^1 = y \qquad z^0 = 1 \qquad w^{-4} = \frac{1}{w^4}$$

(where we assume that neither z nor w is 0). Now for a new kind of exponent: *fractional*. For example, is there any reasonable meaning for the for number *9 to the power of 1/2*; that is, $9^{1/2}$? To determine the meaning of this number, we can proceed like this:

	$9^{1/2}$	(the number we're trying to analyze)
=	$(3^2)^{1/2}$	(9 can certainly be written as the square of 3)
=	$3^{2\cdot\frac{1}{2}}$	(one of the laws of exponents: $(x^a)^b = x^{ab}$)
=	3^1	(since the product of 2 and $\frac{1}{2}$ is 1)
=	$\mathbf{3}$	(any number to the first power is itself)

We started with the unknown quantity $9^{1/2}$. Then, using just properties we know very well, we turned this mystery number into a 3. In short,

$$9^{1/2} = \mathbf{3} \qquad \text{See anything interesting yet?}$$

Let's see a second example. Let's calculate 100 to the $\frac{1}{2}$ power:

$$100^{1/2} = (10^2)^{1/2} = 10^{2\cdot\frac{1}{2}} = 10^1 = \mathbf{10}$$

For our third example, we'll try a different exponent and find the value of $64^{1/3}$:

$$64^{1/3} = (4^3)^{1/3} = 4^{3\cdot\frac{1}{3}} = 4^1 = \mathbf{4}$$

And a fourth example, using an exponent of $\frac{1}{4}$:

$$16^{1/4} = (2^4)^{1/4} = 2^{4\cdot\frac{1}{4}} = 2^1 = \mathbf{2}$$

Homework

2. At the top of the chapter it states that w cannot be 0 in the statement $w^{-4} = \frac{1}{w^4}$. Why such a restriction?

3. Using the methods described above, calculate each quantity:

a. $25^{1/2}$ b. $81^{1/2}$ c. $144^{1/2}$ d. $49^{1/2}$ e. $8^{1/3}$

f. $125^{1/3}$ g. $27^{1/3}$ h. $216^{1/3}$ i. $81^{1/4}$ j. $256^{1/4}$

❑ *SUMMARY AND EXAMPLES*

It's time for a recap and a conclusion as to the meaning of a **fractional exponent**. Here's what we know from the examples and the homework:

$9^{1/2} = 3$	$100^{1/2} = 10$	$25^{1/2} = 5$
$64^{1/3} = 4$	$8^{1/3} = 2$	$125^{1/3} = 5$
$16^{1/4} = 2$	$81^{1/4} = 3$	$256^{1/4} = 4$

Now, what's really going on here? It appears that an exponent of $\frac{1}{2}$ indicates square root, an exponent of $\frac{1}{3}$ indicates cube root, and an exponent of $\frac{1}{4}$ indicates fourth root. That is,

$$x^{1/2} = \sqrt{x} \qquad x^{1/3} = \sqrt[3]{x} \qquad x^{1/4} = \sqrt[4]{x}$$

<u>EXAMPLE 1</u>: **Evaluate each expression with a fractional exponent:**

$$x^{1/n} = \sqrt[n]{x}$$

A. $225^{1/2} = \sqrt{225} = \mathbf{15}$

B. $(-125)^{1/3} = \sqrt[3]{-125} = \mathbf{-5}$

C. $81^{1/4} = \sqrt[4]{81} = \mathbf{3}$

D. $(-32)^{1/5} = \sqrt[5]{-32} = \mathbf{-2}$

E. $-16^{1/2} = -\sqrt{16} = \mathbf{-4}$

F. $(-16)^{1/2} = \sqrt{-16} =$ **Not a real number**

G. $-81^{1/4} = -\sqrt[4]{81} = \mathbf{-3}$

H. $(-81)^{1/4} = \sqrt[4]{-81} =$ **Not a real number**

Homework

4. Explain why $25^{1/2}$ is a real number, but $(-25)^{1/2}$ is not.

5. Evaluate each expression:

a. $36^{1/2}$ b. $8^{1/3}$ c. $16^{1/4}$ d. $32^{1/5}$

e. $625^{1/2}$ f. $1^{1/3}$ g. $0^{1/4}$ h. $243^{1/5}$

i. $-25^{1/2}$ j. $-49^{1/2}$ k. $(-64)^{1/2}$ l. $(-16)^{1/4}$

m. $(-64)^{1/3}$ n. $(-1)^{1/5}$ o. $(-32)^{1/5}$ p. $(-1)^{1/4}$

6. Convert each expression to radical form:

a. $x^{1/2}$ b. $y^{1/3}$ c. $z^{1/4}$ d. $w^{1/5}$

e. $(ab)^{1/2}$ f. $ab^{1/2}$ g. $xy^{1/3}$ h. $(xy)^{1/3}$

i. $y+z^{1/2}$ j. $(y+z)^{1/2}$ k. $(a-b)^{1/3}$ l. $(Q+R-T)^{1/4}$

7. Convert each expression to exponent form:

a. $\sqrt{x}$ b. $\sqrt[4]{y}$ c. $\sqrt[3]{z}$ d. $\sqrt[5]{n}$

e. $a\sqrt{b}$ f. $\sqrt{ab}$ g. $x\sqrt[4]{y}$ h. $\sqrt[3]{tw}$

i. $\sqrt{x+y}$ j. $\sqrt[3]{p-q}$ k. $\sqrt[4]{a+n}$ l. $\sqrt[6]{x-x}$

8. True/False:

a. The expression $x^{1/2}$ is always defined.

b. The expression $x^{1/3}$ is always defined.

❑ *MORE FRACTIONAL EXPONENTS*

The previous problems each had a numerator of 1 in the fractional exponent. What about an expression like $27^{2/3}$? What could this mean? Let's dissect $27^{2/3}$ using our laws of exponents to determine the value of this number.

	$27^{2/3}$	(the power of 27 we're analyzing)
=	$27^{\frac{1}{3}\cdot 2}$	(certainly $\frac{1}{3}\cdot 2 = \frac{2}{3}$)
=	$\left(27^{1/3}\right)^2$	(law of exponents: $(x^a)^b = x^{ab}$)
=	$\left(\sqrt[3]{27}\right)^2$	(we know that a 1/3 exponent indicates cube root)

$= \quad 3^2$ (the cube root of 27 is 3)

$= \quad \mathbf{9}$ (3 squared is 9)

Here's another example. Let's calculate $16^{5/4}$:

$$16^{5/4} = 16^{\frac{1}{4}\cdot 5} = \left(16^{1/4}\right)^5 = \left(\sqrt[4]{16}\right)^5 = 2^5 = \mathbf{32}$$

And a third example:

$$243^{2/5} = 243^{\frac{1}{5}\cdot 2} = \left(243^{1/5}\right)^2 = \left(\sqrt[5]{243}\right)^2 = 3^2 = \mathbf{9}$$

Homework

9. Using the three examples above as a guide, find the value of each fractional power:

a. $8^{2/3}$ b. $4^{3/2}$ c. $9^{3/2}$ d. $16^{5/2}$

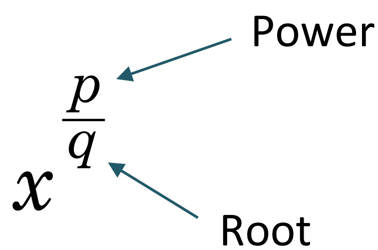
e. $27^{4/3}$ f. $27^{2/3}$ g. $16^{3/4}$ h. $32^{7/5}$

New Approach: There must be a simpler way to view a fractional exponent, and thus a simpler way to calculate one. If you look at the above examples and homework you just completed, you may have noticed that the denominator of the fractional exponent indicated a root, while the numerator denoted a power. For example, in the first example above, we showed that $27^{2/3}$ was calculated by first taking the cube root of 27 (which is 3), and then squaring that result, producing **9**.

Thus, a problem like $16^{5/2}$ is calculated quickly as the square root of 16, raised to the fifth power, which is 4^5, which is **1024**.

In summary,

$$x^{p/q} = \left(\sqrt[q]{x}\right)^{p}$$



EXAMPLE:

$$8^{5/3}$$
$$= \left(\sqrt[3]{8}\right)^{5}$$
$$= 2^{5}$$
$$= \mathbf{32}$$

One more example for this section: To calculate $64^{2/3}$, think cube root of 64, raised to the second power, which is 4^2, which is **16**.

Homework

10. Using the root-power idea, find the value of each fractional power:

a. $8^{4/3}$	b. $4^{1/2}$	c. $9^{5/2}$	d. $16^{3/2}$
e. $27^{2/3}$	f. $27^{4/3}$	g. $16^{5/4}$	h. $32^{6/5}$
i. $8^{2/3}$	j. $4^{3/2}$	k. $9^{3/2}$	l. $16^{5/2}$
m. $27^{4/3}$	n. $125^{2/3}$	o. $16^{3/4}$	p. $32^{7/5}$

❑ NEGATIVE FRACTIONAL EXPONENTS

Since a <u>negative</u> exponent indicates <u>reciprocal</u> (see Chapter 21), we can combine the negative exponents we learned about earlier with the fractional exponents we're learning now. Recalling that $x^{-n} = \frac{1}{x^n}$, we can work the following examples.

<u>EXAMPLE 2:</u> **Evaluate each expression:**

A. $9^{-1/2} = \frac{1}{9^{1/2}} = \frac{1}{\sqrt{9}} = \mathbf{\frac{1}{3}}$

B. $-27^{-1/3} = -\frac{1}{27^{1/3}} = -\frac{1}{\sqrt[3]{27}} = -\mathbf{\frac{1}{3}}$

C. $8^{-2/3} = \frac{1}{8^{2/3}} = \frac{1}{\left(\sqrt[3]{8}\right)^2} = \frac{1}{2^2} = \mathbf{\frac{1}{4}}$

D. $9^{-5/2} = \frac{1}{9^{5/2}} = \frac{1}{\left(\sqrt{9}\right)^5} = \frac{1}{3^5} = \mathbf{\frac{1}{243}}$

E. $-16^{-3/4} = -\frac{1}{16^{3/4}} = -\frac{1}{\left(\sqrt[4]{16}\right)^3} = -\frac{1}{2^3} = -\mathbf{\frac{1}{8}}$

F. $(-16)^{-3/2} = \frac{1}{(-16)^{3/2}} = \frac{1}{\left(\sqrt{-16}\right)^3} =$ **Not a real number**

G. $(-8)^{-2/3} = \frac{1}{(-8)^{2/3}} = \frac{1}{\left(\sqrt[3]{-8}\right)^2} = \frac{1}{(-2)^2} = \mathbf{\frac{1}{4}}$

Homework

11. Evaluate each expression:

a. $9^{-3/2}$ b. $27^{-2/3}$ c. $8^{-4/3}$ d. $8^{-1/3}$

e. $16^{-5/4}$ f. $81^{-1/4}$ g. $81^{-3/4}$ h. $32^{-1/5}$

i. $-8^{-4/3}$ j. $(-25)^{-3/2}$ k. $(-64)^{-2/3}$ l. $(-32)^{-6/5}$

12. Convert each expression to radical form:

a. $x^{2/3}$ b. $y^{5/4}$ c. $z^{1/7}$

d. $(a+b)^{3/2}$ e. $(x-y)^{4/3}$ f. $(xy)^{4/5}$

g. $uw^{2/3}$ h. $x+y^{4/5}$ i. $(ab+c)^{5/6}$

13. Convert each expression to exponent form:

a. $\sqrt[3]{t}$ b. $\sqrt[5]{z}$ c. $\sqrt[4]{ab}$

d. $x\sqrt[3]{y}$ e. $\left(\sqrt{a}\right)^3$ f. $\left(\sqrt[3]{p}\right)^2$

g. $\left(\sqrt[4]{w}\right)^5$ h. $\left(\sqrt[7]{a+b}\right)^3$ i. $\left(\sqrt{x-y}\right)^{10}$

❑ *THE LAWS OF EXPONENTS REVISITED*

The same five laws of exponents we've used with all the previous exponents still work just fine with fractional exponents.

<u>EXAMPLE 3:</u> **Simplify each expression:**

A. $a^{1/2}a^{1/3} = a^{1/2+1/3} = a^{3/6+2/6} = \boldsymbol{a^{5/6}}$

(add the exponents)

B. $y^{1/2}y^{1/2} = y^{1/2+1/2} = y^1 = \boldsymbol{y}$

(add the exponents)

C. $\left(x^{3/4}\right)^{7/2} = x^{(3/4)(7/2)} = \boldsymbol{x^{21/8}}$

(multiply the exponents)

D. $\dfrac{n^{5/6}}{n^{2/3}} = n^{5/6-2/3} = n^{5/6-4/6} = \boldsymbol{n^{1/6}}$

(subtract the exponents)

E. $(ab)^{4/5} = \boldsymbol{a^{4/5}\,b^{4/5}}$

(raise each factor to the 4/5 power)

F. $\left(\dfrac{a}{b}\right)^{4/5} = \boldsymbol{\dfrac{a^{4/5}}{b^{4/5}}}$

(raise top and bottom to the 4/5 power)

G. $\dfrac{w^{1/2}}{w^{4/5}} = w^{1/2-4/5} = w^{5/10-8/10} = w^{-3/10} = \boldsymbol{\dfrac{1}{w^{3/10}}}$

(subtract the exponents) (LCD) (a negative exponent means reciprocal)

Homework

14. Simplify each expression:

a. $x^{1/2}x^{2/3}$ b. $\dfrac{y^{4/5}}{y^{1/5}}$ c. $\dfrac{w^{1/2}}{w^{2/3}}$ d. $\left(a^{2/7}\right)^7$

e. $\left(\dfrac{p}{q}\right)^{3/8}$ f. $t^{4/5}t^{1/3}$ g. $\left(k^{5/2}\right)^{2/5}$ h. $(wz)^{2/3}$

<u>EXAMPLE 4:</u> **Simplify each expression:**

A. $x^{-1/2}x^{-2/3} = x^{-1/2-2/3} = x^{-3/6-4/6} = x^{-7/6} = \mathbf{\dfrac{1}{x^{7/6}}}$

B. $\dfrac{n^{1/2}}{n^{-4/5}} = n^{1/2-(-4/5)} = n^{1/2+4/5} = n^{5/10+8/10} = \mathbf{n^{13/10}}$

C. $\left(x^{2/3}\right)^{-1/4} = x^{(2/3)(-1/4)} = x^{-1/6} = \mathbf{\dfrac{1}{x^{1/6}}}$

D. $(xy)^{-2/3} = x^{-2/3}y^{-2/3} = \dfrac{1}{x^{2/3}}\cdot\dfrac{1}{y^{2/3}} = \mathbf{\dfrac{1}{x^{2/3}y^{2/3}}}$

E. $\left(\dfrac{g}{h}\right)^{-4/3} = \dfrac{g^{-4/3}}{h^{-4/3}} = \dfrac{\frac{1}{g^{4/3}}}{\frac{1}{h^{4/3}}} = \dfrac{1}{g^{4/3}}\times\dfrac{h^{4/3}}{1} = \mathbf{\dfrac{h^{4/3}}{g^{4/3}}}$

Homework

15. Simplify each expression:

a. $x^{4/5}x^{-3/5}$ b. $y^{1/3}y^{-5/3}$ c. $\left(a^{-1/2}\right)^{-2/3}$

d. $(abc)^{-3/4}$ e. $\left(\frac{w}{z}\right)^{-2/5}$ f. $\frac{n^{-1/2}}{n^{-2/3}}$

g. $\frac{a^{-3}}{a^{5/2}}$ h. $\frac{x}{x^{-2/3}}$ i. $\left(\left(c^{1/2}\right)^{-4/3}\right)^{-3/2}$

Practice Problems

16. Convert $\sqrt[4]{x+y}$ to exponent form.

17. Convert $ab^{2/3}$ to radical form.

18. Convert $(a+b)^{5/2}$ to radical form.

19. Convert $\sqrt{a^3-x^3}$ to exponent form.

20. Evaluate: a. $9^{1/2}$ b. $64^{1/3}$ c. $81^{1/4}$ d. $32^{1/5}$

21. Evaluate: a. $8^{2/3}$ b. $27^{4/3}$ c. $32^{2/5}$ d. $16^{3/4}$

22. Evaluate: a. $-9^{1/2}$ b. $(-9)^{1/2}$ c. $(-8)^{1/3}$ d. $(-16)^{1/4}$

23. Evaluate: a. $9^{-3/2}$ b. $8^{-4/3}$ c. $125^{-4/3}$ d. $-81^{-1/4}$

24. Simplify: a. $x^{1/2}x^{4/5}$ b. $\frac{a^{1/3}}{a^{2/5}}$ c. $(ab)^{4/7}$

25. Simplify: a. $\left(\frac{a}{b}\right)^{2/7}$ b. $\left(x^{2/3}\right)^{3/5}$ c. $a^{1/2}a^{1/3}a^{1/4}$

26. Simplify: a. $y^{1/2}y^{-1/2}$ b. $\frac{n^{1/3}}{n^{-4/3}}$ c. $(PQ)^{-2/3}$

27. Simplify: a. $\left(\frac{x}{w}\right)^{-7/10}$ b. $\left(p^{-2/3}\right)^{5/6}$ c. $x^{2/3}+x^{1/3}$

Solutions

1. a. x^{10} b. a^5 c. z^{15} d. $\frac{x^9}{y^9}$ e. m^6n^6

f. As is g. As is h. $2g^6$ i. $x^2 + 2xy + y^2$

j. x k. c^{15} l. $\frac{1}{a^4b^4}$ m. $\frac{1}{x^2}$ n. $\frac{1}{a^8}$

o. $\frac{w^3}{u^3}$ p. $a^2 - 2ab + b^2$

2. If w were 0, we'd have $0^{-4} = \frac{1}{0^4} = \frac{1}{0} =$ Undefined

3. a. $25^{1/2} = (5^2)^{1/2} = 5^{2\cdot\frac{1}{2}} = 5^1 = 5$ b. same idea; result is 9

c. same idea; result is 12 d. same idea; result is 7

e. $8^{1/3} = (2^3)^{1/3} = 2^{3\cdot\frac{1}{3}} = 2^1 = 2$ f. same idea; result is 5

g. same idea; result is 3 h. same idea; result is 6

i. $81^{1/4} = (3^4)^{1/4} = 3^{4\cdot\frac{1}{4}} = 3^1 = 3$ j. same idea; result is 4

4. $25^{1/2} = \sqrt{25} = 5$, a real number. But $(-25)^{1/2} = \sqrt{-25}$, not a real number.

5. a. 6 b. 2 c. 2 d. 2 e. 25 f. 1 g. 0 h. 3 i. –5

j. -7 k. Not real l. Not real m. -4 n. -1 o. -2
p. Not real

6. a. $\sqrt{x}$ b. $\sqrt[3]{y}$ c. $\sqrt[4]{z}$ d. $\sqrt[5]{w}$ e. $\sqrt{ab}$
f. $a\sqrt{b}$ g. $x\sqrt[3]{y}$ h. $\sqrt[3]{xy}$ i. $y+\sqrt{z}$ j. $\sqrt{y+z}$
k. $\sqrt[3]{a-b}$ l. $\sqrt[4]{Q+R-T}$

7. a. $x^{1/2}$ b. $y^{1/4}$ c. $z^{1/3}$ d. $n^{1/5}$ e. $ab^{1/2}$ f. $(ab)^{1/2}$
g. $xy^{1/4}$ h. $(tw)^{1/3}$ i. $(x+y)^{1/2}$ j. $(p-q)^{1/3}$ k. $(a+n)^{1/4}$ l. 0

8. a. False; if $x = -9$, for instance, then $(-9)^{1/2} = \sqrt{-9}$ which is not a real number, which means that $x^{1/2}$ is undefined in this class when $x = -9$. In fact, $x^{1/2}$ is undefined whenever x is a negative number.
b. True; since $x^{1/3} = \sqrt[3]{x}$, and since the cube root is defined whether x is positive, zero, or negative, $x^{1/3}$ is always defined.

9. a. $8^{2/3} = 8^{\frac{1}{3}\cdot 2} = \left(8^{1/3}\right)^2 = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$
b. 8 c. 27 d. 1024 e. 81 f. 9 g. 8
h. $32^{7/5} = 32^{\frac{1}{5}\cdot 7} = \left(32^{1/5}\right)^7 = \left(\sqrt[5]{32}\right)^7 = 2^7 = 128$

10. a. $8^{4/3}$ is the cube root of 8, raised to the 4th power: $2^4 = 16$
b. 2
c. $9^{5/2}$ is the square root of 9, raised to the 5th power: $3^5 = 243$
d. 64 e. 9 f. 81 g. 32
h. $32^{6/5}$ is the fifth root of 32, raised to the 6th power: $2^6 = 64$
i. 4 j. 8 k. 27 l. 1024 m. 81 n. 25 o. 8 p. 128

11. a. $\frac{1}{27}$ b. $\frac{1}{9}$ c. $\frac{1}{16}$ d. $\frac{1}{2}$ e. $\frac{1}{32}$ f. $\frac{1}{3}$
g. $\frac{1}{27}$ h. $\frac{1}{2}$ i. $-\frac{1}{16}$ j. Not real k. $\frac{1}{16}$ l. $\frac{1}{64}$

12. a. $\left(\sqrt[3]{x}\right)^2$ b. $\left(\sqrt[4]{y}\right)^5$ c. $\sqrt[7]{z}$ d. $\left(\sqrt{a+b}\right)^3$ e. $\left(\sqrt[3]{x-y}\right)^4$

f. $\left(\sqrt[5]{xy}\right)^4$ g. $u\left(\sqrt[3]{w}\right)^2$ h. $x+\sqrt[5]{y^4}$ i. $\left(\sqrt[6]{ab+c}\right)^5$

13. a. $t^{1/3}$ b. $z^{1/5}$ c. $(ab)^{1/4}$ d. $xy^{1/3}$ e. $a^{3/2}$

f. $p^{2/3}$ g. $w^{5/4}$ h. $(a+b)^{3/7}$ i. $(x-y)^{10/2} = (x-y)^5$

14. a. $x^{7/6}$ b. $y^{3/5}$ c. $\dfrac{1}{w^{1/6}}$ d. a^2

e. $\dfrac{p^{3/8}}{q^{3/8}}$ f. $t^{17/15}$ g. k h. $w^{2/3}z^{2/3}$

15. a. $x^{1/5}$ b. $\dfrac{1}{y^{4/3}}$ c. $a^{1/3}$ d. $\dfrac{1}{a^{3/4}b^{3/4}c^{3/4}}$

e. $\dfrac{z^{2/5}}{w^{2/5}}$ f. $n^{1/6}$ g. $\dfrac{1}{a^{11/2}}$ h. $x^{5/3}$

i. c

16. $(x+y)^{1/4}$ **17.** $a\left(\sqrt[3]{b}\right)^2$

18. $\left(\sqrt{a+b}\right)^5$ or $\sqrt{(a+b)^5}$

19. $(a^3-x^3)^{1/2}$

20. a. 3 b. 4 c. 3 d. 2

21. a. 4 b. 81 c. 4 d. 8

22. a. –3 b. Not real c. –2 d. Not real

23. a. $\dfrac{1}{27}$ b. $\dfrac{1}{16}$ c. $\dfrac{1}{625}$ d. $-\dfrac{1}{3}$

24. a. $x^{13/10}$ b. $\dfrac{1}{a^{1/15}}$ c. $a^{4/7}b^{4/7}$

25. a. $\dfrac{a^{2/7}}{b^{2/7}}$ b. $x^{2/5}$ c. $a^{13/12}$

26. a. 1 b. $n^{5/3}$ c. $\dfrac{1}{P^{2/3}Q^{2/3}}$

27. a. $\dfrac{w^{7/10}}{x^{7/10}}$ b. $\dfrac{1}{p^{5/9}}$ c. As is

"If you limit your choices only to what seems possible or reasonable, you disconnect yourself from what you truly want, and all that is left is a compromise."

- Robert Fritz