ABSOLUTE VALUE: THE BASICS

❑ THE MEANING OF ABSOLUTE VALUE

Earlier in the book we talked about the **distance** from a point on the number line to the origin. For example, when asked what numbers on the line are 9 units away from the origin, we answered by saying that both of the numbers **9** and –**9** satisfy that condition.

We're now ready to learn and use the official term and notation for this distance idea: The ***absolute value*** of a number is the **distance** from that number to the origin. So, for example, the absolute value of 7 is 7, the absolute value of –11 is 11, and the absolute value of 0 is 0.

> In computers, the absolute value of *n* would be written
>
> **abs(*n*)**

Instead of writing the words *absolute value* all the time, we do what we always do in math: condense the concept into symbols. To represent the absolute value of a number, we put vertical bars around the number. Thus,

The absolute value of 7 is 7: $|7| = 7$

The absolute value of –11 is 11: $|-11| = 11$

The absolute value of 0 is 0: $|0| = 0$

If a number is greater than or equal to 0, then its absolute value is that <u>same</u> number. If a number is less than 0 (which means it's a negative number), then its absolute value is the <u>opposite</u> of that number (which will then turn into a positive number). We can write this in the following way; it looks confusing, but it's totally valid:

If $x \geq 0$, then $|x| = x$

If $x < 0$, then $|x| = -x$

This definition ensures that the absolute value of a quantity is never negative.

Here are some more examples of absolute value:

$|9| = 9$ $\qquad |0| = 0$ $\qquad |-13| = 13$

$|\pi| = \pi$ $\qquad |-\sqrt{7}| = \sqrt{7}$ $\qquad |-3\pi| = 3\pi$

Note:

The absolute value of a quantity is either positive or zero:

$$\textbf{If } \boldsymbol{x} \textbf{ is ANY quantity, then } |\boldsymbol{x}| \geq \mathbf{0}.$$

In other words, the absolute value of a quantity is <u>never</u> negative.

To illustrate this point, though you may not be able to calculate

$$\left|\sin^2(\pi/6) + \ln(e-1)\right|$$

until pre-calculus, you should still be able to understand that the answer to this problem -- whatever it is -- is <u>greater than or equal to zero</u>. Equivalently, the answer is <u>not negative</u>.

<u>EXAMPLE 1:</u> **Evaluate each expression:**

A. $|7-2| = |5| = \mathbf{5}$

B. $|3^2 - 15| = |9-15| = |-6| = \mathbf{6}$

C. $-|2-7| = -|-5| = -\mathbf{5}$

D. $|-3-4| - |10-2| = |-7| - |8| = 7 - 8 = \mathbf{-1}$

Homework

1. The absolute value of any number is ______.

2. If x represents any number, then

   a. $|x| > 0$ b. $|x| < 0$ c. $|x| \geq 0$ d. $|x| \leq 0$

3. Which one of the following inequalities has NO solution?

   a. $|x| > 0$ b. $|x| < 0$ c. $|x| \geq 0$ d. $|x| \leq 0$

4. True/False:

   a. Every number has an absolute value.
   b. There is a number whose absolute value is 0.
   c. There is a number whose absolute value is negative.
   d. There are two different numbers whose absolute value is 9.

5. Simplify each expression:

   a. $|4-14|$ b. $|2(-3)-7|$ c. $-|-9|$ d. $|2^0-\pi^0|$

The *absolute value* of a positive number is itself.

The *absolute value* of 0 is 0.

The *absolute value* of a negative number is its opposite.

Solutions

1. "greater than or equal to 0"
OR "never negative"
OR " ≥ 0 "

2. c.

3. b.

4. a. T b. T c. F d. T

5. a. 10 b. 13 c. –9 d. 0

"Who questions much, shall learn much, and retain much."

– Francis Bacon