
ABSOLUTE VALUE EQUATIONS

□ INTRODUCTION

An **absolute value equation** is just what it sounds like: an equation where at least one occurrence of the variable is contained within the absolute value sign. An example is the equation $|z + 5| = 11$.

But notice that the equation $x + |3| = 99$ (whose solution is $x = 96$) is not an absolute value equation because the variable, x , is not contained within the absolute value sign.

What does it mean for something to be a solution of an absolute value equation? Just what it's always meant: When that something is substituted into the equation, both sides of the equation result in the same number.

Homework

1. Is $x = 3$ a solution of the absolute value equation $|x| = 3$?
2. Is $n = -9$ a solution of the absolute value equation $|n| = 9$?
3. Is $c = -9$ a solution of the absolute value equation $|c| = -9$?
4. Is $a = 2$ a solution of the absolute value equation $|a + 7| = 9$?
5. Is $w = -10$ a solution of the absolute value equation $|w - 7| = 17$?
6. Is $y = -3$ a solution of the absolute value equation $|y - 5| = 2$?
7. Show that $x = 5$ and $x = -5$ are both solutions of $|x| = 5$.
8. Show that $z = 6$ and $z = -16$ are both solutions of $|z + 5| = 11$.
9. Explain why there is NO solution of the equation $|q - 6| = -10$.
10. Show that both $n = 3$ and $n = -3$ are solutions of $|n| - 10 = -7$.

□ EXAMPLES OF ABSOLUTE VALUE EQUATIONS

EXAMPLE 1: Solve for x : $|x| = 12$

Solution: What can you take the absolute value of, and get a result of 12? Well, the absolute value of 12 is 12, so x could be 12. But -12 also has an absolute value of 12, so x could be -12, too. In other words, since $|12| = 12$ and $|-12| = 12$, it appears that this equation has two solutions, 12 and -12.

$$x = 12, -12$$

EXAMPLE 2: Solve for n : $|n| = -5$

Solution: This equation is a statement that the absolute value of some number is -5. But the absolute value of any number is greater than or equal to zero; that is, the absolute value of any number can never be negative. [See: *Absolute Value, the Basics*] Thus, there is no number n that will work in this equation. Our conclusion:

No solution

EXAMPLE 3: Solve for w : $|w| = 0$

Solution: What number has an absolute value of 0? There's only one such number, and it's 0: $|0| = 0$. Therefore,

$$w = 0$$

EXAMPLE 4: Solve for x : $|8 - 2x| = 20$

Solution: Here's what we ask ourselves: "What has an absolute value of 20?" There are two numbers that have an absolute value

of 20, and they are 20 and -20 . This means that the entire quantity inside the absolute value sign must be either 20 or -20 . In other words, the quantity $8 - 2x$ must be either 20 or -20 .

This gives us two equations to solve:

$$\begin{array}{l|l} 8 - 2x = 20 & 8 - 2x = -20 \\ -2x = 12 & -2x = -28 \\ x = -6 & x = 14 \end{array}$$

Our absolute value equation has two solutions:

$$x = -6, 14$$

Check:

$$\underline{x = -6}: \quad |8 - 2x| = |8 - 2(-6)| = |8 + 12| = |20| = 20 \quad \checkmark$$

$$\underline{x = 14}: \quad |8 - 2x| = |8 - 2(14)| = |8 - 28| = |-20| = 20 \quad \checkmark$$

Homework

11. Solve each absolute value equation:

a. $|t| = 4$

b. $|n| = 0$

c. $|R| = -1$

d. $|x + 1| = 9$

e. $|x - 3| = 5$

f. $|2w + 8| = 0$

g. $|2 - 5x| = -3$

h. $|2x + 8| = 10$

i. $|3y - 6| = 9$

j. $|2a + 1| = 19$

k. $|7x + \sqrt{7}| = -\frac{\pi}{2}$

l. $|n - 4| = 9$

m. $|2x - 17| = 0$

n. $|2a + 3| = 21$

o. $|\sqrt{5} - \sqrt{7y}| = -\sqrt{\pi}$

The absolute value of a positive number is itself.

The absolute value of 0 is 0.

The absolute value of a negative number is its opposite.

Solutions

1. Yes
2. Yes
3. No; $|-9| = 9$, not -9
4. Yes
5. Yes; $|-10 - 7| = |-17| = 17$
6. No; letting $y = -3$ results in $8 = 2$.
7. Each of the numbers works in the equation.
8. Plug each one in (one at a time!), and the equation is satisfied.
9. The absolute value of any quantity is always ≥ 0 .
10. Plug them in and see what happens.

11. a. $t = 4, -4$ b. $n = 0$ c. No solution
- d. $x = 8, -10$ e. $x = 8, -2$ f. $w = -4$
- g. No solution h. $x = 1, -9$ i. $y = 5, -1$
- j. $a = 9, -10$ k. No solution l. $n = 13, -5$
- m. $x = \frac{17}{2}$ n. $a = 9, -12$ o. No solution



If $x \geq 0$, then $|x| = x$

If $x < 0$, then $|x| = -x$

*“Live as if you were
to die tomorrow.*

*Learn as if
you were to
live forever.”*



Mahatma Gandhi (1869–1948)