CONSECUTIVE INTEGERS

❑ *INTRODUCTION*

Consecutive Integers

An *integer* is either a positive whole number, zero, or a negative whole number; in other words, it's the collection of numbers:

$$. . . -4, -3, -2, -1, 0, 1, 2, 3, 4 . . . \qquad \text{The set of } \textbf{\textit{integers}}$$

An example of four **consecutive integers** might be 11, 12, 13, 14. Notice that consecutive integers differ by 1 -- each integer is 1 more than the previous integer. This means that if we have four unknown consecutive integers, and if we call the smallest one x, then the next one would be written $x + 1$, and the one after that, $x + 2$, and the largest one, $x + 3$.

x
$x + 1$
$x + 2$ Four **consecutive** integers
$x + 3$

Consecutive Even Integers

An example of three **consecutive <u>even</u> integers** might be 8, 10, 12. Notice that consecutive even integers differ by 2 and that we usually write them in ascending order (smallest to largest). Thus, in a sequence of consecutive even integers, each number is 2 larger than the preceding number. So, assuming the smallest of them is called x, they could be written

x
$x + 2$ Three **consecutive even** integers
$x + 4$

Consecutive Odd Integers

An example of five **consecutive <u>odd</u> integers** might be 5, 7, 9, 11, 13. Notice that consecutive odd integers differ by 2. Now comes a curious little fact: Notice that if n is an odd integer, then the odd integer that comes right after n must be $n + 2$, since they must differ by 2. (For example, 11 is odd, and the next odd integer is $11 + 2 = 13$.) Therefore, if x represents the smallest of five consecutive odd integers, for instance, they could be denoted like this:

x
$x + 2$
$x + 4$ Five **consecutive odd**
$x + 6$ integers
$x + 8$

Don't be fooled into thinking – just because we're talking about consecutive ODD integers – that the expressions to the left must somehow contain odd numbers. Consecutive ODD integers still differ by 2, just as consecutive EVEN integers do.

❑ *EXAMPLES*

EXAMPLE 1:   **Four consecutive odd integers have a sum of 32. Find the four numbers.**

Solution: If we let n represent the smallest of the four consecutive odd integers, then the four integers can be represented like this: n, $n + 2$, $n + 4$, and $n + 6$. Since the given condition is that their sum must be 32, the following equation seems appropriate:

$$n + (n + 2) + (n + 4) + (n + 6) = 32$$

Dropping the parentheses (which weren't really necessary):

$$4n + 12 \ = \ 32 \ \Rightarrow \ 4n \ = \ 20 \ \Rightarrow \ n = 5$$

Since n, the smallest of the four numbers is 5, our final answer is

$$\boxed{5,\ 7,\ 9,\ 11}$$

EXAMPLE 2:   **Find three consecutive integers such that the largest is 18 less than 3 times the smallest.**

Solution: We can represent the three consecutive integers with a, $a + 1$, and $a + 2$. Is it clear that a is the smallest and $a + 2$ is the largest? The relationship between the largest and the smallest as stated in the problem can be transformed into the following equation:

$$a + 2 \ = \ 3a - 18$$

$$\Rightarrow \quad -2a \ = \ -20$$

$$\Rightarrow \quad a \ = \ 10$$

Since the smallest of the three consecutive integers is 10, the three numbers must be

$$\boxed{10,\ 11,\ 12}$$

EXAMPLE 3:   **If the <u>mean</u> (average) of five consecutive even integers is 2, find the largest of the integers.**

Solution: We can represent the five consecutive even integers with x, $x + 2$, $x + 4$, $x + 6$, and $x + 8$. Recall that the mean of five numbers is found by dividing the sum of the numbers by 5:

$$\frac{x + (x + 2) + (x + 4) + (x + 6) + (x + 8)}{5} \ = \ 2$$

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$$\Rightarrow \quad \frac{5x + 20}{5} = 2$$

$$\Rightarrow \quad 5x + 20 = 10$$

$$\Rightarrow \quad 5x = -10$$

$$\Rightarrow \quad x = -2$$

Since $x = -2$, and the largest of the five integers is $x + 8$, it follows that the largest integer is $-2 + 8 = \boxed{-6}$
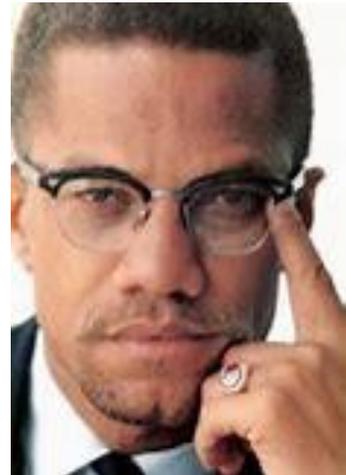
Homework

1. Find three consecutive integers whose sum is 57.

2. Find four consecutive <u>even</u> integers whose sum is 52.

3. Find three consecutive <u>odd</u> integers whose sum is 39.

4. Find three consecutive integers such that the sum of the smallest and largest is 100.

5. Find five consecutive even integers such that the largest is 3 times the smallest.

6. Find four consecutive odd integers such that the largest is 3 less than twice the smallest.

7. Find three consecutive integers such that the largest number is twice the middle number.

8. Find four consecutive even integers such that twice the smallest plus 4 times the largest equals 0.

9. Find three consecutive even integers such that the sum of the smallest and the largest is 20.

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10. Find five consecutive odd integers such that the largest is 4 less than 5 times the smallest.

11. The mean (average) of four consecutive integers is 8.5. Find the 4 numbers. Be sure you write and solve an equation.

12. Find three consecutive odd integers whose mean is 21.

13. Find five consecutive even integers whose mean is 0.

Solutions

1. 18, 19, 20   **2**. 10, 12, 14, 16   **3**. 11, 13, 15

4. 49, 50, 51   **5**. 4, 6, 8, 10, 12   **6**. 9, 11, 13, 15

7. 0, 1, 2   **8**. −4, −2, 0, 2   **9**. 8, 10, 12

10. 3, 5, 7, 9, 11   **11**. 7, 8, 9, 10   **12**. 19, 21, 23

13. −4, −2, 0, 2, 4

"**EDUCATION IS THE PASSPORT TO THE FUTURE, FOR TOMORROW BELONGS TO THOSE WHO PREPARE FOR IT TODAY.**"

Malcolm X (1925–1965)