
LINEAR MODELING

□ INTRODUCTION

Sometimes it's not too hard to create a formula from words. For example the phrase *density is the quotient of mass and volume* translates to the formula $d = \frac{m}{V}$. But this method of formula

creation is useful only if we comprehend the words that need to be translated into Algebra.

More often, we're given a *situation* that needs translation. For example, if we buy 12 lbs of cashews at a unit price of \$14/lb, and we wish to determine the total cost, we have to understand that multiplication does the trick; no one's going to tell you that -- you have to reason it out yourself. So the total cost is

$$12 \text{ lbs} \times \$14/\text{lb} = \underline{\$168}$$

That's fine for one or a few calculations. But if you need to program a calculator, or perhaps a spreadsheet, or perhaps a programming



language, you need to teach the machine what to do, and that can only be done with *variables*. So in the cashew problem above, we could let Q stand for the quantity (# of pounds) of cashews, let P be the unit price (the price of 1 pound) of the cashews, and let C be the total cost. Again, if we understand that the key to the formula will

be multiplication, we can write a formula that can be entered into a computer (using $*$, the computer symbol for multiplication):

$$C = P * Q$$



□ JOINING THE CLUB

EXAMPLE 1: To join the Model Railroad Club, a member must pay an up-front fee of \$25 to join, and then pay \$10 per month for each month in the club.



- Find the total cost for someone to be a club member for 8 months.
- Find a formula for someone to be a club member for m months.
- Use your formula to calculate the total to be in the club for 36 months.

Solution: Let's begin with a simple table that shows the cost of membership for various months:

Months	0	1	2	3	4	5	6	7	8
Cost	\$25	\$35	\$45	\$55	\$65	\$75	\$85	\$95	\$105

- The cost, C , is \$25 plus 8 months at \$10 per month:

$$C = 25 + 10(8) = 25 + 80 = 105$$

Therefore, the cost of 8 months is **\$105**.

- Just change the 8 in part a. to the variable m :

$$C = 25 + 10m, \text{ or } C = 10m + 25$$

- Using the formula in part b., we get a cost of

$$C = 10m + 25 = 10(36) + 25 = 360 + 25 = \mathbf{\$385}$$

Homework

1.
 - a. It costs \$2 per mile to take a taxi. If m represents the total miles traveled, write a formula for the total cost, C .
 - b. Use your formula to calculate the total cost of a 20-mile trip.
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2. Sprint offers a monthly phone plan that has a base fee of \$20, plus \$0.25 for each minute of use. Write a formula for the monthly cost, C , if you used the phone for m minutes in a month.
 3.
 - a. It costs \$3 for the first mile of a taxi ride, and \$2 per mile for each additional mile. Calculate the total cost of a 21-mile taxi ride.
 - b. It costs \$3 for the first mile of a taxi ride, and \$2 per mile for each additional mile. If m represents the total miles traveled, write a formula for the total cost, C .
 - c. Use your new formula from part b. to calculate the total cost of a 15-mile trip.
 4. It costs \$7 for the first mile of a taxi ride, and \$4 per mile for each additional mile. If m represents the total miles traveled, write a formula for the total cost, C .
 5. The ultimate question on the taxi ride:

Let C = the total cost of the taxi ride

f = the cost of the first mile

a = the cost of each mile after the first

m = the total miles traveled

Write a formula for the total cost of a taxi ride.

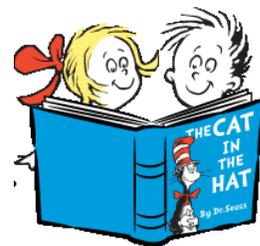
6. a. QRS, Incorporated sold 5000 widgets last month at a unit selling price of \$45/widget. Calculate the revenue obtained. Hint: Revenue is the total amount of money obtained.
- b. STU, Inc. sold w widgets last month at a unit selling price of $\$p$ /widget. Create a formula for the revenue, R , obtained.
7. a. WXY, Inc. produced 600 widgets last quarter, and it costs \$50 to produce each widget. (\$50/widget is called the *unit cost*). In addition, fixed costs (rent, utilities, salaries, etc.) totaled \$2,400 last quarter. Calculate the total cost of producing the 600 widgets last quarter.
- b. Let w = the number of widgets produced
 c = unit cost to produce one widget
 f = fixed costs (rent, utilities, salaries, etc.)
 E = total expense to produce the w widgets
- Create a formula for the total expense, E , of producing w widgets.
8. Ernie earns a base salary of \$2000/month and a commission of \$50 for each iPad he sells. Find Ernie's total salary during a month in which he sold 75 iPads. Construct a formula which would give Ernie's total salary, S , during a month if he sold i iPads.
9. Andrew wants to place a classified ad in the Chronicle to sell his old iPod Touch.
- a. The first 12 words cost \$7.50, and each additional word costs \$0.30. Find the total cost of an ad containing 20 words.
- b. The first 15 words cost \$9.75, and each additional word costs \$0.40. Construct a formula to calculate C , the total cost of an ad containing w words. [Assume that w is at least 15.]



10. A tole painter buys a piece of wood for $\$w$, paints it, and then sets the selling price at 4 times the cost of the wood. If the painter gets 75% of the selling price (the rest goes to the salesperson), find the tole painter's NET profit (money earned less expenses).

Review Problems

11. a. Suppose that the Foothill Middle School library currently has 7000 books, and is buying 150 more books per year. How many books will the library have 4 years from now?
- b. Write a formula that will give the number of books, B , that the library will have y years from now.
12. You must pay $\$75$ up front to join the Painting Club, and then pay $\$35$ per month for each month that you're a member of the club. Create a formula that will give the total cost, C , to be in the club for m months.
13. It costs $\$7.50$ for the first mile of a taxi ride, and $\$3.25$ per mile for each additional mile. If m represents the total miles traveled, write a formula for the total cost, C .
14. The Speedy Car Rental Co. charges $\$35/\text{day}$ and $\$0.15/\text{mile}$ to rent a car. Let d represent the number of days you rent the car, and let m represent the number of miles driven. Find a formula for C , the total rental cost.



Solutions

1. a. $C = 2m$ b. $C = 2m = 2(20) = \mathbf{\$40}$
2. $C = 20 + 0.25m$ [or, $C = 0.25m + 20$]
3. a. $3 + 2(20) = 3 + 40 = \mathbf{\$43}$
 b. $C = 3 + 2(m - 1)$, or $C = \mathbf{2(m - 1) + 3}$
 c. $C = 2(m - 1) + 3 = 2(15 - 1) + 3 = 2(14) + 3 = 28 + 3 = \mathbf{\$31}$
4. $C = 7 + 4(m - 1)$ or $C = 4(m - 1) + 7$
5. $C = f + a(m - 1)$ or $C = a(m - 1) + f$
6. a. $\$225,000$ b. $R = pw$
7. a. $600(50) + 2,400 = \mathbf{\$32,400}$ b. $E = cw + f$
8. $\$2000 + \$50(75) = \mathbf{\$5,750}$
 In general, $S = 2000 + 50(i)$, or $S = \mathbf{50i + 2000}$
9. a. $\$9.90$ b. $C = 9.75 + 0.40(w - 15)$
10. $0.75(4w) - w$
11. a. $7000 + 4(150) = \underline{7600}$ books
 b. $B = 150y + 7000$ or $B = 7000 + 150y$
12. The cost would be \$75 plus \$35 for each month in the club.
 That's $\$75 + \$35 \times$ the number of months in the club.
 That's $C = 75 + 35m$, which could be written $C = 35m + 75$
13. $C = 7.50 + 3.25(m - 1)$
14. $C = 35d + 0.15m$

□ TO ∞ AND BEYOND

Placing a Classified Ad

The first n words cost $\$I$, while each additional word costs $\$A$. Find the total cost, C , of an ad containing w words. You may assume that $w \geq n$.

*“Never doubt that a
small group of thoughtful,
committed citizens can
change the world;
indeed it is the only thing
that ever has.”*

Margaret Mead, Anthropologist