
NUMBER WORD PROBLEMS

□ **TERMINOLOGY**

To **double** a number means to multiply it by **2**.

The double of 12 is $2(12) = 24$.

When n is doubled, it becomes $2n$.

To **square** a number means to multiply it by itself.

The square of 12 is $12^2 = 12 \times 12 = 144$.

When n is squared, it becomes n^2 .

To **triple** a number means to multiply it by **3**.

The result of tripling x is $3x$.

To **quadruple** a number means to multiply it by **4**.

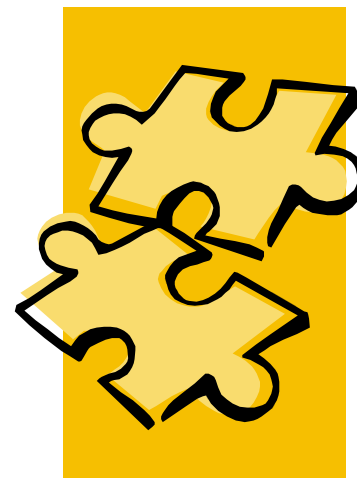
To **quintuple** a number means to multiply it by **5**.

10 is added to n : $n + 10$

N is subtracted from x : $x - N$

13 is subtracted from twice m : $2m - 13$

Phrases such as “the result is,” “the final result is,” “the answer is,” and “is the same as” translate to the **equal sign** in the equation. In fact, the word “is” alone usually (but not always) represents the equal sign.



Homework

1. Beginners frequently confuse *doubling* and *squaring*. Let's clear this up right now.
 - a. What is the double of 15?
 - b. What is the square of 15?
 - c. What's 9 squared?
 - d. What's 9 doubled?
 - e. 5 doubled gives ____.
 - f. 5 squared gives ____.
 - g. $(20)(2) =$
 - h. $20^2 =$
 - i. Doubling 25 gives ____.
 - j. Squaring 25 gives ____.
 - k. The square of y is ____.
 - l. The double of y is ____.

2. Now we add to the confusion. Consider the number 0. Notice that 0 doubled is $2(0) = \underline{0}$. Also, 0 squared is $0^2 = 0 \times 0 = \underline{0}$. In other words, 0 is a special number whose double is the same as its square. Try to find the only other number whose double is equal to its square.

□ I'M THINKING OF A NUMBER

EXAMPLE 1: I'm thinking of a number. If I double it and add 10, the result is 64. What is the number?

Solution: Let n represent the unknown number. Doubling it produces the expression $2n$. Adding 10 to that gives the expression $2n + 10$. The phrase "the result is" becomes an equal sign, and so the equation we need to solve is



$$\begin{aligned}
 2n + 10 &= 64 && \text{(English to Algebra)} \\
 \Rightarrow 2n + 10 - \mathbf{10} &= 64 - \mathbf{10} && \text{(subtract 10 from each side)} \\
 \Rightarrow 2n &= 54 && \text{(simplify)} \\
 \Rightarrow \frac{2n}{2} &= \frac{54}{2} && \text{(divide each side by 2)}
 \end{aligned}$$

$$\Rightarrow n = 27 \quad \text{(simplify)}$$

Since n stood for the number we were seeking, we can conclude that the number asked for in the question is

| |
|----|
| 27 |
|----|

EXAMPLE 2: **I'm thinking of a number. If 7 is subtracted from the number, and then that result is multiplied by 10, the final result is 150. What is the number?**

Solution: For variety, we'll select x to represent the unknown number. First 7 is subtracted from x ; this can be written $x - 7$ (not the other way around!). Next, that quantity is to be multiplied by 10. How shall we indicate that 10 must be multiplied by the quantity $x - 7$? We use parentheses: $10(x - 7)$. And, of course, the phrase "the final result is" produces our equal sign, yielding us the equation

$$\begin{aligned} 10(x - 7) &= 150 && \text{(English to Algebra)} \\ \Rightarrow 10x - 70 &= 150 && \text{(distribute)} \\ \Rightarrow 10x - 70 + \mathbf{70} &= 150 + \mathbf{70} && \text{(add 70 to each side)} \\ \Rightarrow 10x &= 220 && \text{(simplify)} \\ \Rightarrow \frac{10x}{\mathbf{10}} &= \frac{220}{\mathbf{10}} && \text{(divide each side by 10)} \\ \Rightarrow x &= 22 && \text{(simplify)} \end{aligned}$$

We conclude that my number was

| |
|----|
| 22 |
|----|

EXAMPLE 3: Solve each “I’m thinking of a number” problem:

- A. I’m thinking of a number. If I add 3 to the number, and then divide that sum by 2, the result is 7. What is the number?

Solution: Translating the words into algebra gives the equation

$$\begin{aligned} \frac{n+3}{2} &= 7 \\ \Rightarrow \frac{n+3}{2} [2] &= 7[2] && \text{(multiply each side by 2)} \\ \Rightarrow n+3 &= 14 && \text{(simplify)} \\ \Rightarrow \mathbf{n = 11} &&& \text{(subtract 3 from each side)} \end{aligned}$$

- B. I’m thinking of a number. If I divide the number by 9, and then subtract 5 from that quotient, the answer is 3. Find the number.

Solution: In this problem we divide first, and then subtract. Our equation to solve is therefore

$$\begin{aligned} \frac{x}{9} - 5 &= 3 \\ \Rightarrow \frac{x}{9} &= 8 && \text{(add 5 to each side)} \\ \Rightarrow \frac{x}{9} [9] &= 8[9] && \text{(multiply each side by 9)} \\ \Rightarrow \mathbf{x = 72} &&& \text{(simplify)} \end{aligned}$$

- C. I’m thinking of a number. If I subtract 5 from the number, and then divide that difference by 3.1, the result is -2.7 . What is the number?

Solution: Except for the ugliness of the numbers, this is just like part A above:

$$\begin{aligned}\frac{y-5}{3.1} &= -2.7 \\ \Rightarrow y-5 &= (-2.7)(3.1) && \text{(multiply each side by 3.1)} \\ \Rightarrow y-5 &= -8.37 && \text{(simplify)} \\ \Rightarrow y &= -3.37 && \text{(add 5 to each side)}\end{aligned}$$

Homework

Solve each of the following “I’m thinking of a number” problems by creating and solving an appropriate equation:

3.
 - a. If I add 7 to the number, the result is 53.
 - b. If I subtract 9 from the number, the result is 33.
 - c. If I double the number, the result is 23.
 - d. If I triple the number, the result is 72.
 - e. If I multiply the number by 7, the result is 861.
 - f. If I multiply the number by 8, the result is 100.
 - g. If I divide the number by 19, the result is 17.
 - h. If I divide the number by 3, the result is 287.

4.
 - a. If I double it and add 17, the result is 50.
 - b. If I triple it and subtract 9, the result is 0.
 - c. If I multiply it by 7 and then add 50, the result is -3 .
 - d. If I add 3 to the number, and then multiply that result by 9, the final result is 34.
 - e. If I subtract 5 from the number, and then multiply that result by 17, the final result is 20.

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- f. If 30 is added to the number, and then that result is multiplied by 3, the final result is 99.
 - g. If 17 is subtracted from the number, and then that result is multiplied by -5 , the result is 30.
- 5.
- a. If I add 7 to the number, and then divide that sum by 3, the result is 20.
 - b. If I subtract 9 from the number, and then divide that difference by 5, the result is -10 .
 - c. If I divide the number by 3, and then add 13 to the quotient, the result is 12.
 - d. If I divide the number by 12, and then subtract 50 from the quotient, the result is -23 .

□ MORE NUMBER PROBLEMS

The phrase “7 more than x ” represents the situation where 7 has been added to x , and therefore can be represented by the expression $x + 7$.

The phrase “9 less than w ” represents the situation where 9 has been subtracted from w , and thus can be written as the expression $w - 9$.

Here are some more translations from English to Algebra:

| | |
|-------------------------------|---|
| the sum of A and B | $A + B$ |
| the difference of x and y | $x - y$ |
| the product of a and b | ab |
| the quotient of m and n | $\frac{m}{n}$ (or m/n or $m \div n$) |
| a number p is divided by 7 | $\frac{p}{7}$ |
| 10 more than z | $z + 10$ |

| | |
|--|-------------|
| 8 less than a | $a - 8$ |
| 3 more than twice n | $2n + 3$ |
| 5 less than 11 times y | $11y - 5$ |
| the sum of 4 times a and 4 times b | $4a + 4b$ |
| 4 times the sum of a and b | $4(a + b)$ |
| the difference of twice q and 3 | $2q - 3$ |
| twice the difference of q and 3 | $2(q - 3)$ |
| a number n increased by 12 | $n + 12$ |
| a number w decreased by 7 | $w - 7$ |
| 100 less than the square of n | $n^2 - 100$ |
| 23 less than the double of u | $2u - 23$ |
| the square of the sum of x and y | $(x + y)^2$ |
| the difference of the squares of A and B | $A^2 - B^2$ |

Homework

6. Translate from English to Algebra:
- | | |
|------------------------------------|-------------------------------------|
| a. n doubled | b. x squared |
| c. a tripled | d. b quadrupled |
| e. t quintupled | f. 3 added to u |
| g. 9 subtracted from x . | h. twice Q |
| i. 7 added to 3 times n | j. 2 subtracted from 4 times z |
| k. 9 less than w | l. 4 more than x |
| m. 18 more than T | n. 27 less than P |
| o. 3 more than twice x | p. 3 less than 10 times y |
| q. 1 more than the triple of t . | r. 6 less than the quadruple of n |

- s. the quotient of x and 3 t. the product of a and b
 u. 3 times a added to 4 times a v. 9 times R
 w. 6 times the sum of x and y
 x. 9 times the difference of A and B
 y. 12 times the sum of x and 20
 z. 7 times the difference of n and 200
7. a. Translate “the square of the sum of u and w ” to Algebra.
 b. Translate “the sum of the squares of u and w ” to Algebra.
 c. Find a counterexample to the statement that the expressions in part a. and part b. are equal.

EXAMPLE 4: **Eight times a number is 6 less than twice the number. Find the number.**



Solution: Let x represent the number. We’ll translate the English sentence into algebra like this:

$$\begin{array}{ccc} \text{Eight times a number} & \text{is} & \text{6 less than twice the number} \\ \downarrow & & \downarrow \\ 8x & = & 2x - 6 \end{array}$$

In other words, the equation we must solve is

$$\begin{aligned} 8x &= 2x - 6 \\ \Rightarrow 8x - 2x &= 2x - 2x - 6 && \text{(subtract } 2x \text{ from each side)} \\ \Rightarrow 6x &= -6 && \text{(simplify)} \\ \Rightarrow \frac{6x}{6} &= \frac{-6}{6} && \text{(divide each side by 6)} \\ \Rightarrow x &= -1 && \text{(simplify)} \end{aligned}$$

Therefore, the number we’re searching for is -1

EXAMPLE 5: Find a number given that 3 times the difference of the number and 1 is 13 more than the number.

Solution: If we let n represent the unknown number, we can translate as follows:

“Three times difference of the number and 1” $\Rightarrow 3(n - 1)$

“is” $\Rightarrow =$

“13 more than the number” $\Rightarrow n + 13$

Putting it all together:

$$\begin{aligned} 3(n - 1) &= n + 13 && \text{(the equation we need to solve)} \\ \Rightarrow 3n - 3 &= n + 13 && \text{(distribute)} \\ \Rightarrow 2n - 3 &= 13 && \text{(subtract } n \text{ from each side)} \\ \Rightarrow 2n &= 16 && \text{(add 3 to each side)} \\ \Rightarrow n &= 8 && \text{(divide each side by 2)} \end{aligned}$$

The number we're seeking is 8

Homework

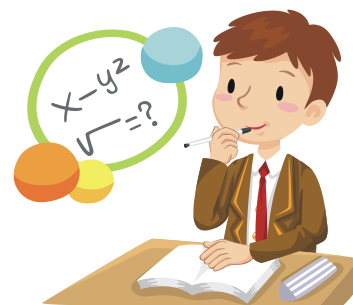
8. Seven times a number is 2 more than 9 times the number. What is the number?
9. Nine more than 2 times a number is equal to 9 more than 6 times the number. Find the number.
10. Six times the difference of a number and 3 is 13 less than the number. Find the number.
11. Find a number given that 5 times the sum of the number and 4 is 32 more than the number.

12. If 3 times a number is added to 4 times the number, the result is 700. Find the number.
13. Eight times a number is 3 more than 5 times the number. What is the number?
14. Nine less than 5 times a number is equal to 7 more than 8 times the number. Find the number.
15. Five times the difference of a number and 4 is 18 less than the number. Find the number.
16. Find a number given that 6 times the sum of the number and 7 is 8 less than the number.
17. Find a number given that 7 times the sum of the number and 9 is the same as 201 more than the number.
18. Find a number given that 10 times the difference of the number and 10 is the same as 64 less than the number.
19. Find a number given that 6 times the difference of the number and 7 is the same as 43 more than the number.
20. Find a number given that 5 times the sum of the number and 3 is the same as 141 less than the number.
21. Find a number given that 9 times the sum of the number and 8 is the same as 224 more than the number.
22. Find a number given that 9 times the difference of the number and 7 is the same as 263 less than the number.
23. 47 less than 8 times a number is equal to 26 less than the number. What is the number?
24. 5 less than 3 times a number is equal to 11 more than the number. What is the number?
25. 43 more than 12 times a number is equal to 122 less than the number. What is the number?

26. Find a number given that 12 times the sum of the number and 4 is the same as 499 more than the number.
27. 5 more than 6 times a number is equal to 50 more than the number. What is the number?
28. 47 more than 14 times a number is equal to 229 more than the number. What is the number?
29. Find a number given that 8 times the difference of the number and 1 is the same as 162 less than the number.

Review Problems

30. I'm thinking of a number. If 10 is subtracted from the number, and then that result is multiplied by 11, the final answer is -165 . What is the number?
31. I'm thinking of a number. If -2 is subtracted from the number, and then that result is multiplied by -11 , the final answer is -165 . What is the number?
32. I'm thinking of a number. If I divide the number by -5 , and then add -6 , the final result will be -17 . What is the number?
33. I'm thinking of a number. If 114 is subtracted from the number, and then that result is divided by -11 , the final answer is 10. What is the number?
34. I'm thinking of a number. If -46 is subtracted from the number, and then that result is divided by -8 , the final answer is -5 . What is the number?



35. Seventeen times a number is 2 less than 10 times the number. What is the number?
36. Nine less than 7 times a number is equal to 19 more than 4 times the number. Find the number.
37. Three times the difference of a number and 5 is 15 less than the number. Find the number.
38. Find a number given that 15 times the sum of the number and 4 is 32 less than the 6 times the number.

Solutions

1. a. 30 b. 225 c. 81 d. 18 e. 10 f. 25
 g. 40 h. 400 i. 50 j. 625 k. y^2 l. $2y$
2. I'd rather not give it away just yet.
3. a. $n + 7 = 53$; $n = 46$ b. $n - 9 = 33$; $n = 42$
 c. $2n = 23$; $n = \frac{23}{2}$ d. $3n = 72$; $n = 24$
 e. $7n = 861$; $n = 123$ f. $8n = 100$; $n = \frac{25}{2}$
 g. $\frac{n}{19} = 17$; $n = 323$ h. $\frac{n}{3} = 287$; $n = 861$
4. a. $2n + 17 = 50$; $n = \frac{33}{2}$ b. $3n - 9 = 0$; $n = 3$
 c. $7n + 50 = -3$; $n = -\frac{53}{7}$ d. $9(n + 3) = 34$; $n = \frac{7}{9}$
 e. $17(n - 5) = 20$; $n = \frac{105}{17}$ f. $3(n + 30) = 99$; $n = 3$
 g. $-5(n - 17) = 30$; $n = 11$

5. a. $\frac{n+7}{3} = 20; n = 53$ b. $\frac{n-9}{5} = -10; n = -41$
 c. $\frac{n}{3} + 13 = 12; n = -3$ d. $\frac{n}{12} - 50 = -23; n = 324$

6. -5 7. 13 8. -1 9. 4 10. -6

11. a. $2n$ b. x^2 c. $3a$ d. $4b$
 e. $5t$ f. $u + 3$ g. $x - 9$ h. $2Q$
 i. $3n + 7$ j. $4z - 2$ k. $w - 9$ l. $x + 4$
 m. $T + 18$ n. $P - 27$ o. $2x + 3$ p. $10y - 3$
 q. $3t + 1$ r. $4n - 6$ s. $\frac{x}{3}$ t. ab
 u. $4a + 3a$ v. $9R$ w. $6(x + y)$ x. $9(A - B)$
 y. $12(x + 20)$ z. $7(n - 200)$

12. a. $(u + w)^2$
 b. $u^2 + w^2$
 c. Pick a pair of numbers for u and w and show that you get a different answer for each expression.

13. $7x = 9x + 2; x = -1$

14. $2x + 9 = 6x + 9; x = 0$

15. $6(x - 3) = x - 13; x = 1$

16. $5(x + 4) = x + 32; x = 3$

17. 100 18. 1 19. $-\frac{16}{3}$ 20. $\frac{1}{2}$ 21. -10

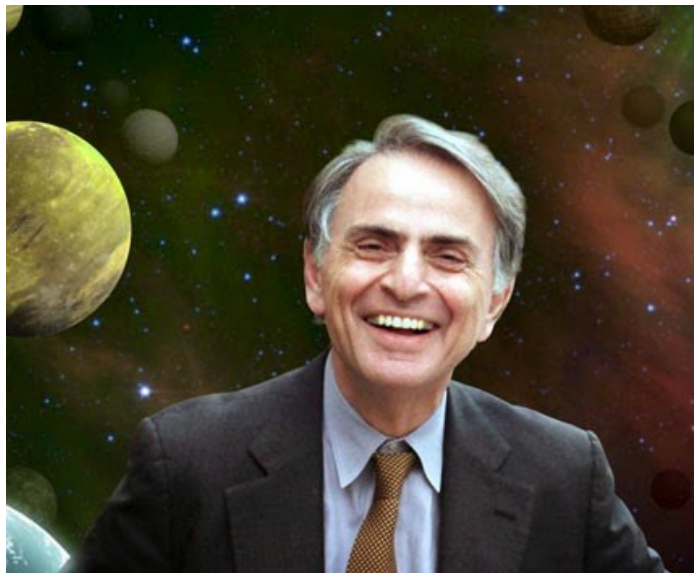
22. 23 23. 4 24. 17 25. -39 26. 19

27. -25 28. 3 29. 8 30. -15 31. 41

32. 9 33. 14 34. -22 35. $-\frac{2}{7}$ 36. $\frac{28}{3}$

37. 0 38. $-\frac{92}{9}$

“It is far better to grasp the Universe as it really is than to persist in delusion, however satisfying and reassuring.”



– Carl Sagan (1934 – 1996)