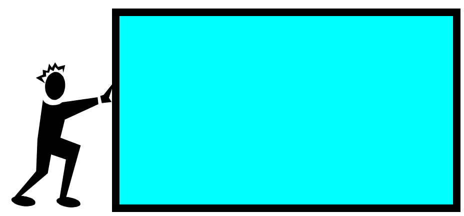
APPLICATIONS OF THE QUADRATIC FORMULA

❑ RECTANGLES



The examples in this chapter will utilize the Quadratic Formula in solving the equations which arise from the word problem. However, each problem can also be solved by the Factoring Method, so you might want to practice your factoring skills and solve some of the Homework problems by factoring.

EXAMPLE 1: **The length of a rectangle is 6 less than 4 times its width. If the area is 18, find the dimensions of the rectangle.**

Solution: Letting w represent the width of the rectangle, the length l must be $l = 4w - 6$. We'll start our solution with the formula for the area of a rectangle, $A = lw$, substitute the given area of 18 for A, and substitute the expression $4w - 6$ for l.

	$A = lw$	(Area = length × width)
$\Rightarrow$	$18 = (4w - 6)w$	(substitute 18 for A and $4w - 6$ for l)
$\Rightarrow$	$18 = w(4w - 6)$	(turn the factors around)
$\Rightarrow$	$18 = 4w^2 - 6w$	(distribute . . . look, it's quadratic!)
$\Rightarrow$	$0 = 4w^2 - 6w - 18$	(bring all terms to same side)
$\Rightarrow$	$4w^2 - 6w - 18 = 0$	(turn it around for standard form)

We can see that this equation is quadratic, and it's in standard form, so we can confidently state that

$$a = 4,\ b = -6, \text{ and } c = -18$$

We begin by writing the Quadratic Formula, and then we plug in the values for a, b and c:

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{(the Quadratic Formula)}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(-18)}}{2(4)} \qquad \text{(insert values for } a, b, \text{ and } c\text{)}$$

$$= \frac{6 \pm \sqrt{36 - (-288)}}{8} \qquad \text{(square and multiply)}$$

$$= \frac{6 \pm \sqrt{36 + 288}}{8} \qquad \text{(subtract the negative)}$$

$$= \frac{6 \pm \sqrt{324}}{8} \qquad \text{(add)}$$

$$= \frac{6 \pm 18}{8} \qquad \text{(take square root)}$$

$$= \frac{6+18}{8} \qquad = \frac{6-18}{8} \qquad \text{(split the plus/minus sign)}$$

$$= \frac{24}{8} \qquad = \frac{-12}{8} \qquad \text{(simplify numerators)}$$

$$= \mathbf{3} \qquad = -\frac{\mathbf{3}}{\mathbf{2}} \qquad \text{(simplify fractions)}$$

Our quadratic equation has two solutions, one positive and one negative. In spite of this fact -- that the equation indeed has two solutions -- we must remember that for this particular word problem, these solutions are supposed to be for the width, w, of the rectangle. Since a width can never be negative (or even zero), we'll discard the negative solution and stick with the positive solution, $w = 3$. Thus, we know that the width of the rectangle is 3. The length is calculated from the formula $l = 4w - 6$, and so

$$l = 4(\mathbf{3}) - 6 = 12 - 6 = \mathbf{6}.$$

The dimensions of the rectangle are therefore

$$\boxed{6 \times 3}$$

Let's make sure these are the dimensions. First, is the length equal to 6 less than 4 times the width? $4w - 6 = 4(\mathbf{3}) - 6 = 6$, which is the length. Second, is the area of the rectangle 18? Of course: $6 \times 3 = 18$.

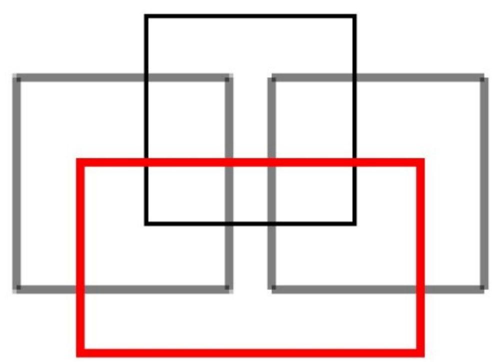
Homework

1. The length of a rectangle is 3 more than twice its width. If the area is 65, find the dimensions of the rectangle.

2. The length of a rectangle is 2 less than 10 times its width. If the area is 36, find the dimensions of the rectangle.

3. The length of a rectangle is 1 more than its width. If the area is 156, find the dimensions of the rectangle.

4. The length of a rectangle is 11 less than 3 times its width. If the area is 190, find the dimensions of the rectangle.

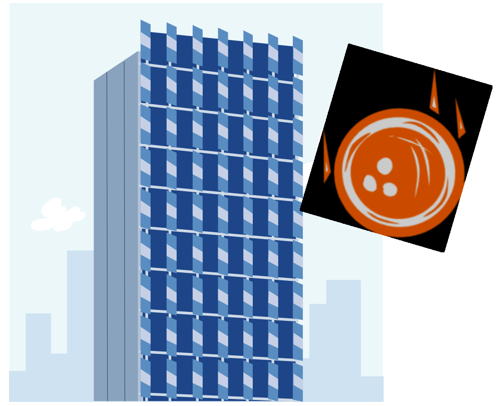


5. The length of a rectangle is 12 less than twice its width. If the area is 144, find the dimensions of the rectangle.

6. The length of a rectangle is 9 more than 10 times its width. If the area is 19, find the dimensions of the rectangle.

❑ FALLING OBJECTS (AND MATH TEACHERS)

EXAMPLE 2: **A bowling ball is thrown upward from the ledge of a building and falls to the ground under the effects of just gravity (we neglect air resistance). Let h represent its height (in meters) above the ground after t seconds have gone by. Suppose the formula for h is given by:**



$$h = -2t^2 + 10t + 100$$

a. Prove that when the ball is thrown (that is, when $t = 0$), it is 100 m above the ground.

b. Calculate the height of the ball 4 seconds after the ball is thrown.

c. How many seconds after the bowling ball is thrown does it crash into the ground?

Solution: In the formula

$$h = -2t^2 + 10t + 100$$

t is the time and h is the height above the ground. So if we put a particular value of t into the formula, we should be able to calculate the height of the bowling ball at that moment in time. Conversely, we can put in a value of h, and then calculate the moment in time, t, when the ball was at that given height.

a. When the ball is first thrown, the value of t is 0 (like starting a stopwatch). So let's plug $t = 0$ into the formula and see what we get for h, the height:

$$h = -2t^2 + 10t + 100 \qquad \text{(the height formula)}$$
$$\Rightarrow \quad h = -2(\mathbf{0})^2 + 10(\mathbf{0}) + 100 \qquad \text{(set } t = 0\text{)}$$
$$\Rightarrow \quad h = -2(0) + 10(0) + 100 \qquad \text{(exponents first)}$$
$$\Rightarrow \quad h = 0 + 0 + 100 \qquad \text{(then multiplication)}$$
$$\Rightarrow \quad h = 100 \qquad \text{(and then addition)}$$

And so we've proved that the height when $t = 0$ is **100 m**.

b. This is the same idea as the first question -- we'll just set $t = 4$ and determine the value of h:

$$h = -2t^2 + 10t + 100 \qquad \text{(the height formula)}$$
$$\Rightarrow \quad h = -2(\mathbf{4})^2 + 10(\mathbf{4}) + 100 \qquad \text{(set } t = 4\text{)}$$
$$\Rightarrow \quad h = -2(16) + 10(4) + 100 \qquad \text{(Order of Operations)}$$
$$\Rightarrow \quad h = -32 + 40 + 100$$
$$\Rightarrow \quad h = 8 + 100$$
$$\Rightarrow \quad h = 108$$

We conclude that the bowling ball is **108 m** above the ground after 4 seconds have transpired.

c. Now for the good part . . . I mean the hard part. In this problem the unknown is t, the time. But the height is not given, so what are we supposed to do? We ask ourselves, What is the height of the bowling ball the moment it hits the ground? It's 0 meters. Thus, we place $h = 0$ into the formula to calculate the value (or values) of t. Are you ready?

$$h = -2t^2 + 10t + 100 \qquad \text{(the height formula)}$$

$$\mathbf{0} = -2t^2 + 10t + 100 \qquad \text{(hits the ground when } h = 0\text{)}$$

Notice that we have a quadratic equation. First, turn the whole thing around so that it's in standard form:

$$-2t^2 + 10t + 100 = 0$$

Let's multiply each side of the equation by -1 so that the leading coefficient (the value of a) is positive (not a necessary step, but a nice one):

$$2t^2 - 10t - 100 = 0$$

We should now be able to deduce that $a = 2$, $b = -10$, and $c = -100$. Placing these values into the Quadratic Formula, we can solve for t:

$$t = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(2)(-100)}}{2(2)}$$

$$t = \frac{10 \pm \sqrt{100 + 800}}{4} = \frac{10 \pm \sqrt{900}}{4} = \frac{10 \pm 30}{4}$$

Using the plus sign:

$$t = \frac{10 + 30}{4} = \frac{40}{4} = 10$$

Using the minus sign:

$$t = \frac{10 - 30}{4} = \frac{-20}{4} = -5$$

Like most quadratic equations, we've got two solutions at hand. But do they both work? Well, certainly they both work in the equation we solved, but let's look at the big picture. The value $t = 0$ represented the beginning of the experiment, the moment the bowling ball was thrown from the ledge of the building. Therefore, the solution $t = -5$ makes no sense; that moment in time occurred 5 seconds <u>before</u> the ball was thrown -- that's absurd.

So we conclude that the ball hit the ground **10 seconds** after it was thrown.

Homework

7. A math teacher is thrown upward from the ledge of a building. Let h represent his height (in meters) above the ground after t seconds have gone by. Suppose the formula for h is given by:

$$h = -3t^2 - 12t + 288$$

a. Prove that when the teacher is tossed, he is 288 m above the ground.

b. Calculate the height of the teacher 5 seconds after he is thrown.

c. How many seconds after the math teacher is thrown does he hit the ground?

8. An object is falling and its height (in meters), after t seconds have gone by, is given by:

$$h = -4t^2 + 121$$

a. At what height was the object released?

b. How many seconds until the object hits the ground?

9. A soccer ball is thrown from the top of a mountain and its height (in meters), after t seconds have gone by, is given by

$$h = -t^2 + 11t + 26$$

a. When the soccer ball is tossed, how far above the ground is it?

b. What is the ball's height 5 seconds after it's thrown?

c. How long will it take for the ball to hit the ground?

Solutions

1. 13×5 **2.** 18×2 **3.** 13×12

4. 19×10 **5.** 12×12 **6.** 19×1

7. a. Hint: let $t = 0$
b. 153 m
c. 8 sec

8. a. 121 m
b. 5.5 sec

9. a. 26 m
b. 56 m
c. 13 sec

"Knowledge is the eye of desire and can become the pilot of the soul."

WILL DURANT