PREPARING FOR COMPLETING THE SQUARE

❑ INTRODUCTION

I'm assuming that the only method you've learned for solving a quadratic equation is the **factoring method**: For example, to solve the quadratic equation

$$2x^2 + 3x - 20 = 0$$

you would proceed as follows:

$$2x^2 + 3x - 20 = 0$$

$$\Rightarrow \quad (2x - 5)(x + 4) = 0$$

$$\Rightarrow \quad 2x - 5 = 0 \quad \text{OR} \quad x + 4 = 0$$

$$\Rightarrow \quad x = \frac{5}{2} \quad \text{OR} \quad x = -4, \text{ and we're done.}$$

So factoring is a good enough method, you say? I think not: I submit to you the following quadratic equation

$$x^2 + 7x + 5 = 0$$

It's not a complicated equation – the numbers are small, and there aren't even any negative numbers to mess with. So go ahead; I dare you to factor that thing.

See my point? Trust me, that equation DOES have two solutions, but I think you believe me now when I say that that factoring just won't work. We need a new method for solving quadratic equations, and this chapter will give you some of the skills needed in a future chapter on completing the square.

❑ Factoring Perfect Square Trinomials

Recall the term "perfect square." The number 100 is a perfect square because 100 can be written as the square of 10: $100 = 10^2$. Also, the expression $(x + 5)^2$ is a perfect square, since it is the square of $x + 5$. One of the steps in solving a quadratic equation by *completing the square* is factoring a *perfect square trinomial*. Let's look at four examples.

<u>Example 1:</u>

$x^2 + 14x + 49$ is a perfect square trinomial because it's a trinomial that factors into the square of a binomial:

$$x^2 + 14x + 49 = (x + 7)(x + 7) = (x + 7)^2$$

<u>Example 2:</u>

$n^2 - 20n + 100$ is a perfect square trinomial:

$$n^2 - 20n + 100 = (n - 10)(n - 10) = (n - 10)^2$$

Now a couple of examples with fractions:

<u>Example 3:</u>

$$a^2 + 3a + \frac{9}{4} = \left(a + \frac{3}{2}\right)\left(a + \frac{3}{2}\right) = \left(a + \frac{3}{2}\right)^2$$

Check:

$$\left(a + \frac{3}{2}\right)^2 = \left(a + \frac{3}{2}\right)\left(a + \frac{3}{2}\right) = a^2 + \underbrace{\frac{3}{2}a + \frac{3}{2}a}_{\frac{3}{2}+\frac{3}{2}=\frac{6}{2}=3} + \frac{9}{4} = a^2 + 3a + \frac{9}{4} \ \checkmark$$

Example 4:

$$y^2 - \frac{2}{5}y + \frac{1}{25} = \left(y - \frac{1}{5}\right)\left(y - \frac{1}{5}\right) = \left(y - \frac{1}{5}\right)^2$$

Check:

$$\left(y - \frac{1}{5}\right)\left(y - \frac{1}{5}\right) = y^2 - \frac{1}{5}y - \frac{1}{5}y + \frac{1}{25} = y^2 - \frac{2}{5}y + \frac{1}{25} \checkmark$$

Homework

1. Factor each perfect square trinomial:

a. $x^2 + 10x + 25$

b. $y^2 - 18y + 81$

c. $a^2 + a + \frac{1}{4}$

d. $m^2 - \frac{4}{3}m + \frac{4}{9}$

e. $z^2 + \frac{2}{5}z + \frac{1}{25}$

f. $w^2 - \frac{5}{3}w + \frac{25}{36}$

g. $b^2 + \frac{9}{5}b + \frac{81}{100}$

h. $u^2 + \frac{3}{2}u + \frac{9}{16}$

i. $n^2 - \frac{4}{7}n + \frac{4}{49}$

j. $x^2 + \frac{10}{11}x + \frac{25}{121}$

❑ *Review of Solving Quadratics by Taking Square Roots*

Another skill required for completing the square is taking square roots to solve quadratic equations. You might want to review the Chapter entitled Solving Quadratics by Taking Square Roots. Here's an example from that chapter:

EXAMPLE 5: **Solve the quadratic equation: $(x + 7)^2 = 81$**

Solution: According to the Square Root Theorem, we can remove the squaring by taking the square root of both sides of the equation, remembering that the number 81 has two square roots:

$(x + 7)^2 = 81$ (the original equation)

$\Rightarrow \quad x + 7 = \pm\sqrt{81}$ (the Square Root Theorem)

$\Rightarrow \quad x + 7 = \pm 9$ ($\sqrt{81} = 9$)

$\Rightarrow \quad x = -7 \pm 9$ (subtract 7 from each side)

Using the plus sign yields $x = -7 + 9 = 2$.

Using the minus sign yields $x = -7 - 9 = -16$.

$$\boxed{x = 2 \text{ or } -16}$$

Review Problems

2. How many solutions does each quadratic equation have?

a. $(x + 3)(x + 4) = 0$ ______ b. $(x + 9)^2 = 0$ ______

c. $(x - 1)^2 = 10$ ______ d. $(x + 6)^2 = -9$ ______

3. Factor each trinomial:

a. $n^2 + 10n + 25$ b. $x^2 - 18x + 81$

c. $a^2 + 5a + \frac{25}{4}$ d. $x^2 - \frac{4}{3}x + \frac{4}{9}$

e. $y^2 + \frac{6}{7}y + \frac{9}{49}$ f. $t^2 - \frac{1}{5}t + \frac{1}{100}$

Solutions

1. a. $(x+5)^2$ b. $(y-9)^2$ c. $\left(a+\frac{1}{2}\right)^2$ d. $\left(m-\frac{2}{3}\right)^2$

e. $\left(z+\frac{1}{5}\right)^2$ f. $\left(w-\frac{5}{6}\right)^2$ g. $\left(b+\frac{9}{10}\right)^2$ h. $\left(u+\frac{3}{4}\right)^2$

i. $\left(n-\frac{2}{7}\right)^2$ j. $\left(x+\frac{5}{11}\right)^2$

2. a. 2 b. 1 c. 2 d. 0

3. a. $(n+5)^2$ b. $(x-9)^2$ c. $\left(a+\frac{5}{2}\right)^2$

d. $\left(x-\frac{2}{3}\right)^2$ e. $\left(y+\frac{3}{7}\right)^2$ f. $\left(t-\frac{1}{10}\right)^2$

Education is not

a preparation for life;

education is life itself.

John Dewey