
CONSTANTS – FINDING THE NUMBERS

□ FINDING A PAIR OF NUMBERS

To work the homework in this section, we need only the following four math terms:

The *sum* of two numbers is the result of adding them.

The *difference* of two numbers is the result of subtracting them.

The *product* of two numbers is the result of multiplying them.

The *quotient* of two numbers is the result of dividing them.

EXAMPLE 1: Find two numbers with a sum of 11 and a difference of 5.

Solution: What? How can we have an algebra problem with unknown values on the first page of an algebra book? Here's how: We don't care about algebra yet. Let's just use our knowledge of the words *sum* and *difference* and a little trial and error (guessing!).

Well, 10 and 1 have a sum of 11, so that's a possibility:

$$10 + 1 = 11 \quad \checkmark$$

$10 - 1 = 9$, not 5, like the problem requires.



We didn't get the right two numbers, so let's try 7 and 4:

$$7 + 4 = 11 \quad \checkmark$$

$$7 - 4 = 3, \text{ but the difference is supposed to be } 5.$$

Consider 8 and 3:

$$8 + 3 = 11 \quad \checkmark$$

$$8 - 3 = 5, \text{ as required by the problem!}$$

In short, the numbers 8 and 3 have a sum of 11 and a difference of 5. Both conditions of the problem have been met, and so our final answer is the pair of numbers



8 & 3

"Only education is capable of saving our societies from possible collapse, whether violent, or gradual."

– *Jean Piaget*

Homework

1. Find two numbers with a product of 20 and a sum of 12.
2. Find two numbers with a sum of 2 and a difference of 0.
3. Find two numbers with a product of 12 and a difference of 4.
4. Find two numbers with a product of 7 and a sum of 8.
5. Find two numbers with a difference of 9 and a sum of 11.
6. Find two numbers with a product of 42 and a sum of 13.

7. Find two numbers with a difference of 7 and a product of 18.
8. Find two numbers with a product of 16 and a difference of 6.
9. Find two numbers with a difference of 2 and a product of 48.
10. Find two numbers with a sum of 8 and a difference of 0.
11. Find two numbers with a difference of 14 and a quotient of 8.
12. Find two numbers with a product of 16 and a sum of 8.
13. Find two numbers with a sum of 19 and a product of 90.
14. Find two numbers with a difference of 4 and a sum of 10.
15. Find two numbers with a quotient of 2 and a product of 50.
16. Find two numbers with a sum of 11 and a difference of 7.
17. Find two numbers with a difference of 9 and a quotient of 2.
18. Find two numbers with a sum of 3 and a difference of 1.
19. Find two numbers with a product of 12 and a sum of 7.
20. Find two numbers with a quotient of 4 and a difference of 27.

I'M THINKING OF A NUMBER

I'm thinking of a number. If I add 10 to the number, the result is 45. What number was I thinking of?

Our goal in this chapter is to solve problems like the one above using just our common sense. We simply want to “reason” it out using just a little bit of logic. Can you find the number? The number I was thinking of was **35**. Next, we need some more terminology.

To ***double*** a number means to multiply it by **2**.

To ***triple*** a number means to multiply it by **3**.

To ***quadruple*** a number means to multiply it by **4**.

To ***quintuple*** a number means to multiply it by **5**.

There's no process that I intend to teach you at this point in the course, so on to the homework. One hint: If you see a phrase like "subtract 7 from 20," it means $20 - 7 = 13$.

Homework

21. Solve each of the following "*I'm thinking of a number*" problems using whatever reasoning you like:
- a. If I multiply the number by 2, the result is 24.
 - b. If I subtract 7 from the number, the result is 20.
 - c. If I divide the number by 5, the result is 10.
 - d. If I add 19 to the number, the result is 100.
 - e. If I multiply the number by 3, the result is 24.
 - f. If I subtract 17 from the number, the result is 20.
 - g. If I divide the number by 6, the result is 12.
 - h. If I add 29 to the number, the result is 50.
 - i. If I multiply the number by 20, the result is 300.
 - j. If I subtract 27 from the number, the result is 40.
 - k. If I divide the number by 9, the result is 8.
 - l. If I add 199 to the number, the result is 200.
 - m. If I double the number, the result is 34.
 - n. If I triple the number, the result is 72.
 - o. If I quadruple the number, the result is 300.
 - p. If I quintuple the number, the result is 95.



22. I'm thinking of a number . . .
- a. If I triple the number and then add 4, the result is 34.
 - b. If I multiply the number by 7, and then subtract 8, the result is 20.
 - c. If I add 3 to the number, and then double that result, the final result is 16.
 - d. If I subtract 2 from the number, and then triple that result, the final result is 99.
 - e. If I quintuple the number and then add 7, the result is 57.
 - f. If I quadruple the number, and then subtract 8, the result is 40.
 - g. If I add 13 to the number, and then double that result, the final result is 100.
 - h. If I subtract 20 from the number, and then triple that result, the final result is 51.
 - i. If I add 11 to the number, and then quadruple that result, the final result is 200.
 - j. If I subtract 22 from the number, and then quintuple that result, the final result is 75.

Review Problems

- 23. Find two numbers with a difference of 0 and a product of 9.
- 24. Find two numbers with a product of 16 and a quotient of 4.
- 25. Find two numbers with a sum of 7 and a product of 10.



26. Find two numbers with a quotient of 3 and a difference of 14.
27. Find two numbers with a quotient of 4 and a product of 4.
28. Find two numbers with a quotient of 5 and a sum of 36.
29. Find two numbers with a quotient of 4 and a sum of 10.
30. I'm thinking of a number. If 3 is added to the number, and then that result is doubled, the final answer is 14. What is the number?
31. I'm thinking of a number. If 4 is subtracted from the number, and then that result is multiplied by 6, the final answer is 18. What is the number?
32. I'm thinking of a number. If 2 is added to the number, and then that result is divided by 4, the final answer is 3. What is the number?
33. I'm thinking of a number. If I double the number, and then add 8, the final result will be 10. What is the number?
34. I'm thinking of a number. If I divide the number by 3, and then add 5, the final result will be 10. What is the number?
35. I'm thinking of a number. If 3 is subtracted from the number, and then that result is divided by 7, the final answer is 2. What is the number?
36. I'm thinking of a number. If I divide the number by 7, and then subtract 5, the final result will be 3. What is the number?
37. I'm thinking of a number. If the number is tripled, and then 9 is subtracted, the final result will be 15. What is the number?

***“The only real
mistake is the one
from which we
learn nothing.”***

– John Powell

Solutions

- | | | | | |
|--------------------------------------|------------------------|-------------------------|-------------------------|------------------------|
| 1. 2 & 10 | 2. 1 & 1 | 3. 2 & 6 | 4. 7 & 1 | |
| 5. 1 & 10 | 6. 6 & 7 | 7. 9 & 2 | 8. 8 & 2 | |
| 9. 8 & 6 | 10. 4 & 4 | 11. 16 & 2 | 12. 4 & 4 | |
| 13. 10 & 9 | 14. 3 & 7 | 15. 10 & 5 | 16. 2 & 9 | |
| 17. 18 & 9 | 18. 1 & 2 | 19. 4 & 3 | 20. 36 & 9 | |
| 21. a. 12
f. 37
k. 72
p. 19 | b. 27
g. 72
l. 1 | c. 50
h. 21
m. 17 | d. 81
i. 15
n. 24 | e. 8
j. 67
o. 75 |
| 22. a. 10
f. 12 | b. 4
g. 37 | c. 5
h. 37 | d. 35
i. 39 | e. 10
j. 37 |
| 23. 3 & 3 | 24. 8 & 2 | 25. 2 & 5 | 26. 21 & 7 | |
| 27. 4 & 1 | 28. 30 & 6 | 29. 8 & 2 | 30. 4 | |
| 31. 7 | 32. 10 | 33. 1 | 34. 15 | |
| 35. 17 | 36. 56 | 37. 8 | | |

□ *TO ∞ AND BEYOND*

- A. Find two numbers whose sum is 45 and whose product is 500.
- B. Find two numbers whose difference is $\frac{1}{4}$ and whose product is $\frac{1}{8}$.

“Live as if you were
to die tomorrow.
Learn as if you were
to live forever.”

Mahatma Gandhi

