DISTANCE IN THE PLANE

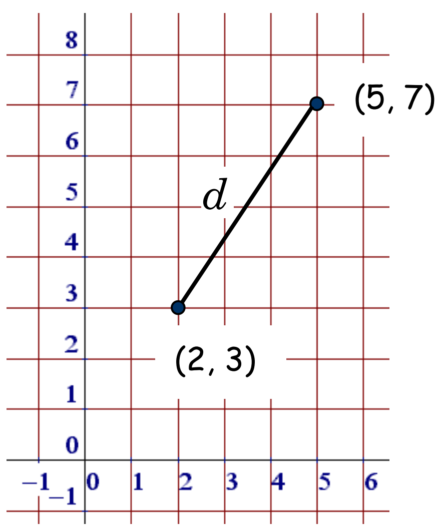
❑ INTRODUCTION

Assuming that you know how to plot points in the plane and remember the Pythagorean Theorem, we can tackle the question: How do we find the ***distance*** between two points in the plane? If the Earth were flat, it would be like asking how far apart two cities are if we know the latitude and longitude of each city.

❑ DISTANCE USING A TRIANGLE

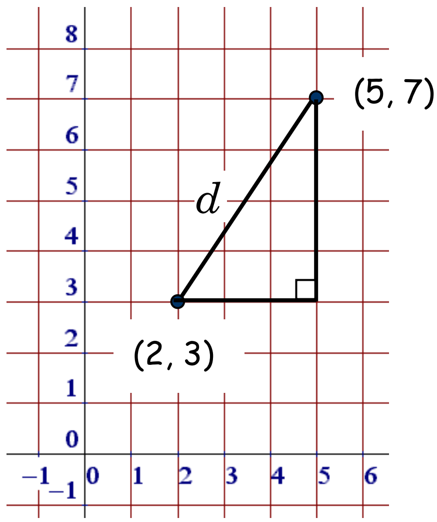
<u>EXAMPLE 1:</u> **Find the distance between the points (2, 3) and (5, 7) in the plane.**

<u>Solution:</u> Let's draw a picture and see what we can see. We'll plot the two given points and connect them with a straight line segment. The distance between the two points, which we'll call d, is simply the length of that line segment.



How far is it between the two points (2, 3) and (5, 7)? Equivalently, what is the length of the line segment connecting the two points?

Now what do we do? Well, here comes the interesting part. If we're creative enough, we might see that the segment connecting the two points can be thought of as the *hypotenuse* of a right triangle -- as long as we sketch in a pair of legs to create such a triangle. Let's do that:



We've created a right triangle whose legs have lengths 3 and 4, and whose hypotenuse has a length equal to the distance between the two given points.

Sure enough, we've constructed a right triangle where d is the length of the hypotenuse. If we can determine the lengths of the legs, then we can use the Pythagorean Theorem to find the length of the hypotenuse. By counting squares along the base of the triangle, we see that one leg is 3. Similarly, the other leg (the height) is 4. Since the square of the hypotenuse is equal to the sum of the squares of the legs, we can write the equation

$$
\begin{aligned}
& d^2 = 3^2 + 4^2 && \text{(Pythagorean Theorem: hyp}^2 = \text{leg}^2 + \text{leg}^2\text{)} \\
\Rightarrow\ & d^2 = 9 + 16 && \text{(square the legs)} \\
\Rightarrow\ & d^2 = 25 && \text{(add)} \\
\Rightarrow\ & d = 5 && \text{(since } \sqrt{25} = 5\text{)}
\end{aligned}
$$

You may notice that $d = -5$ also satisfies the equation $d^2 = 25$, since $(-5)^2 = 25$. But does a negative value of d make sense? No, because distance can never be negative; so we conclude that

The distance between the two points is **5**

Homework

1. By plotting the two given points in the plane and using the Pythagorean Theorem, find the **distance** between the points:

   a. (1, 1) and (4, 5)

   b. (2, –3) and (6, –6)

   c. (–3, 5) and (2, –7)

   d. (–4, –5) and (1, 7)

   e. (–5, 0) and (1, 8)

   f. the origin and (6, 8)

   g. the origin and (–5, –12)

   h. (2, 5) and (2, –1)

   i. (–3, 4) and (2, 4)

   j. (π, 99) and (π, 99)

❑ *DISTANCE USING A FORMULA*

Solutions

1. a. 5 b. 5 c. 13 d. 13 e. 10
f. 10 g. 13 h. 6 i. 5 j. 0

"Give a man a fish

and you feed him for a day.

Teach a man to fish

and you feed him for a lifetime."

Chinese Proverb