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# DISTANCE ON THE LINE

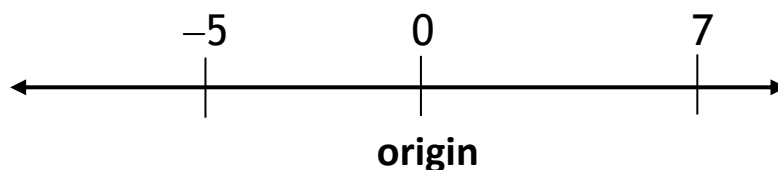
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## □ INTRODUCTION

Sometimes the “size” of a number is more important than whether it’s positive, zero, or negative. To measure the size of a number, we consider the *distance* between the number and zero (the origin) on the number line.

## □ EXAMPLES



What is the distance from the number 7 to 0? It’s 7.

How far from the origin is the number  $-5$ ? It’s 5 units away.

What is the distance from the origin to  $2\pi$ ? The distance is  $2\pi$ .

What is the distance between the number 0 and the origin? It’s 0.

## □ NOTES

1. We can calculate the distance from any number to 0 (the origin).
2. Distance must be greater than or equal to 0; that is, it’s never negative. If  $d$  represents distance, then  $d \geq 0$ .

# 2

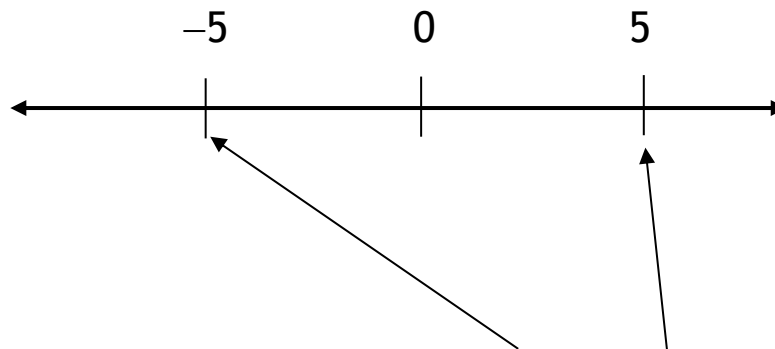
## □ EXAMPLE

I'm thinking of a number (or numbers). If the number is doubled and then 9 is added, the final result will be 5 units from the origin.

**Solution:** How about we let  $n$  represent the unknown number (or numbers). The problem states that

*“the number is doubled and then 9 is added”*

This maneuver transforms the  $n$  into the expression  $2n + 9$ , and it is this quantity which must be 5 units from the origin. Well, what does it really mean for a quantity to be 5 units from the origin? It means that the quantity must be either 5 or  $-5$  (since both 5 and  $-5$  are 5 units from the origin).



$2n + 9$  must be either here or here

Writing this idea with equations, we get two possibilities:

$$2n + 9 = -5 \quad \text{OR} \quad 2n + 9 = 5$$

$$\Rightarrow 2n = -14 \quad \text{OR} \quad 2n = -4$$

$$\Rightarrow n = -7 \quad \text{OR} \quad n = -2$$

Therefore, the numbers I was thinking of were

-7 and -2

## Homework

1. For each number, calculate its distance to 0 (the origin):
  - a. 18
  - b. -23
  - c. 0
  - d.  $\frac{2}{3}$
  - e.  $-2\frac{3}{5}$
  - f.  $\pi$
  - g.  $-\pi$
  - h.  $\sqrt{5}$
  - i.  $-\sqrt{17}$
  - f.  $\sqrt{-9}$
  
2. For each expression, calculate its distance to 0 (the origin):
  - a.  $20 - 12$
  - b.  $1 - 9$
  - c.  $3.7 - 3.7$
  - d.  $0 - 17$
  - e.  $\pi - \pi$
  - f.  $\sqrt{2} + \sqrt{2}$
  - g.  $\pi + \sqrt{2}$
  - h.  $0.5 - 2$
  
3. If  $d$  represents the distance from any point on the line to the origin, then
  - a.  $d > 0$
  - b.  $d \geq 0$
  - c.  $d < 0$
  - d.  $d \leq 0$
  
4. T/F: Given any number, we can always calculate its distance to the origin.
  
5. T/F: Given any number, its distance to the origin is always positive.
  
6. T/F: Given any number, its distance to the origin is never negative.
  
7. T/F: The distance between 0 and the origin does not exist.
  
8.
  - a. I'm 7 units from the origin on the number line. What number(s) am I?
  - b. The distance between me and 0 on the number line is 100. What am I?
  - c. I'm 0 units from the origin. What number(s) can I be?
  - d. The distance between me and the number 0 is 0. Who am I?

- e. The distance between me and the origin is  $-10$ . What number(s) can I be?
  - f. I'm  $-4$  units from 0 on the number line. Who am I?
9. I'm thinking of a number or two...
- a. If 1 is added to the number, you get a number whose distance to the origin is 9.
  - b. If 3 is subtracted from the number, the result is 10 units from 0 on the number line.
  - c. If the number is doubled, the resulting number is 12 units from the origin.
  - d. If the number is tripled, you get a number whose distance to the origin is 15.
  - e. If 7 is subtracted from the number, the resulting number is 0 units away from 0.
  - f. If 5 is added to the number, the distance between the final result and the origin is 0.
10. I'm thinking of a number or two...
- a. If 3 is added to the number, you get a number whose distance to the origin is 14.
  - b. If 5 is subtracted from the number, the result is 12 units from 0 on the number line.
  - c. If the number is doubled, the resulting number is 8 units from the origin.
  - d. If the number is tripled, you get a number whose distance to the origin is 9.
  - e. If 4 is subtracted from the number, the resulting number is 0 units from the origin.

11. I'm thinking of a number (or numbers). If the number is quadrupled and then 8 is subtracted, the final result will be 24 units from the origin.
12. I'm thinking of a number (or numbers). If the number is tripled and then 6 is added, the final result will be 9 units from the origin.
13. I'm thinking of a number (or numbers). If the number is doubled and then 9 is subtracted, the final result will be 10 units from the origin.
14. I'm thinking of a number (or numbers). If the number is quintupled and then  $\pi$  is added, the final result will be  $-7$  units from the origin.

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## Solutions

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1. a. 18      b. 23      c. 0      d.  $\frac{2}{3}$       e.  $2\frac{3}{5}$   
 f.  $\pi$       g.  $\pi$       h.  $\sqrt{5}$       i.  $\sqrt{17}$       f. Not a number
2. a. 8      b. 8      c. 0      d. 17  
 e. 0      f.  $2\sqrt{2}$       g.  $\pi + \sqrt{2}$       h. 1.5
3. b.      4. T      5. F      6. T      7. F
8. a. 7 or  $-7$       b. 100 or  $-100$       c. 0      d. 0  
 e. Impossible; no distance can be  $-10$   
 f. Impossible; no distance can be  $-4$
9. a.  $-10$  or 8      b.  $-7$  or 13      c. 6 or  $-6$

# 6

d. 5 or -5      e. 7      f. -5

**10.** a. 11 or -17      b. 17 or -7      c. 4 or -4  
d. 3 or -3      e. 4

**11.** a. 8 or -4      **12.** 1 or -5

**13.**  $\frac{19}{2}$  or  $-\frac{1}{2}$       **14.** Impossible

“Next in importance to freedom and justice is education, without which neither freedom nor justice can be permanently maintained.”

*President James A. Garfield (1831 - 1881)*