MULTIPLYING BINOMIALS

❑ THE DOUBLE DISTRIBUTIVE PROPERTY

What we need now is a way to multiply two binomials together, a skill absolutely necessary for success in this class. For example, how do we simplify the product $(3x + 1)(2x - 11)$? The **double distributive property** says, in a nutshell,

Multiply each term in the first binomial
by each term in the second binomial.

The following example demonstrates the general concept of the double distributive property.

EXAMPLE 1: **Use the double distributive property to multiply out the product $(a + b)(c + d)$.**

Solution: We carry out the multiplication in four steps:

i) Multiply the a by the c: ac
ii) Multiply the a by the d: ad
iii) Multiply the b by the c: bc
iv) Multiply the b by the d: bd

> **$(a + b)(c + d)$**
>
> a and b are dogs; c and d are cats. Each dog wants to chase each cat. That makes four possible pairings.

Notice that each term in the first set of parentheses is multiplied by each term in the second set of parentheses. In other words, we first distribute the a and then distribute the b -- hence the phrase *double distribute*.

Now add the four terms together:

$$ac + ad + bc + bd$$

EXAMPLE 2: **Multiply out (simplify): $(x + 7)(x + 5)$**

Solution: We apply the four steps to double distribute:

i)	Multiply the first x by the second x:	x^2
ii)	Multiply the first x by the 5:	$5x$
iii)	Multiply the 7 by the second x:	$7x$
iv)	Multiply the 7 by the 5:	35

Add the four terms together: $x^2 + 5x + 7x + 35$, and then combine like terms

$$\boxed{x^2 + 12x + 35}$$

EXAMPLE 3: **Simplify the given expression:**

A. $(2n + 1)(n - 8)$

$= 2n^2 - 16n + n - 8$ (double distribute)

$= \mathbf{2n^2 - 15n - 8}$ (combine like terms)

B. $(7a - 3)(4a - 5)$

$= 28a^2 - 35a - 12a + 15$ (double distribute)

$= \mathbf{28a^2 - 47a + 15}$ (combine like terms)

C. $(6k - 7)(6k + 7)$

$= 36k^2 + 42k - 42k - 49$ (double distribute)

$= \mathbf{36k^2 - 49}$ (combine like terms)

D. $(11y + 10)(11y - 10)$

$= 11y^2 - 110y + 110y - 100$

$= \mathbf{121y^2 - 100}$

E. $(2x + 9)^2$ The square of a quantity is the product of the quantity with itself:

$= (2x + 9)(2x + 9)$ (since $N^2 = N \cdot N$)

$= 4x^2 + 18x + 18x + 81$ (double distribute)

$= \mathbf{4x^2 + 36x + 81}$ (combine like terms)

Homework

1. Simplify each expression by double distributing and combining like terms:

a. $(x + y)(w + z)$ b. $(c + d)(a - b)$ c. $(x + 2)(y + 3)$

d. $(x + 3)(x + 4)$ e. $(n - 4)(n - 1)$ f. $(a + 3)(a - 7)$

g. $(y + 9)(y - 9)$ h. $(u - 3)(u + 3)$ i. $(t - 20)(t - 19)$

j. $(z + 3)(z + 3)$ k. $(v - 4)(v - 4)$ l. $(N + 1)(N - 1)$

2. Simplify:

a. $(3a + 7)(a - 9)$ b. $(2n - 3)(n + 4)$ c. $(3n - 8)(n - 1)$

d. $(5x + 7)(5x + 6)$ e. $(7w + 2)(7w - 2)$ f. $(x + 12)(x - 12)$

g. $(2y + 1)(2y + 1)$ h. $(7x + 3)(6x - 7)$ i. $(q + 7)(3q - 7)$

j. $(3n + 1)(3n + 1)$ k. $(3x - 7)(6x + 5)$ l. $(u - 7)(u - 7)$

3. Simplify:

a. $(y + 4)^2$ b. $(z - 9)^2$ c. $(3x + 5)^2$

d. $(2a - 1)^2$ e. $(n + 12)^2$ f. $(6t - 7)^2$

g. $(q - 15)^2$ h. $(5b + 3)^2$ i. $(7u - 1)^2$

j. $(2x + 1)^2$ k. $(3h - 12)^2$ l. $(5y - 5)^2$

4. Simplify:

a. $(a + b)(c - d)$	b. $(2x - 3)(2x + 3)$	c. $(3n - 1)^2$
d. $(3t + 1)(2t - 3)$	e. $(2x + 4)(3x - 6)$	f. $(n + 1)(n - 1)$
g. $(7a - 10)(6a - 10)$	h. $(10c + 7)^2$	i. $(L + 4)^2$
j. $(7x - 3)(3x + 7)$	k. $(13n - 7)(13n + 7)$	l. $(12d - 20)^2$

5. Prove that $(a + b)^2 \neq a^2 + b^2$ in two ways:
   i) Plug in numbers to provide a counterexample.
   ii) Simplify $(a + b)^2$ the correct way.

6. Find a counterexample to the proposition that $(x + y)^3 = x^3 + y^3$.

❑ *OTHER VIEWS OF THE DOUBLE DISTRIBUTIVE PROPERTY*

Using the Double Distributive Property for Simple Arithmetic

Let's multiply 34 by 75. Obviously, the double distributive property is overkill for an arithmetic problem like this, but this should convince you that the property is indeed valid.

	(34)(75)	
=	(30 + 4)(70 + 5)	(expand each number)
=	30(70) + 30(5) + 4(70) + 4(5)	(double distribute)
=	2100 + 150 + 280 + 20	(multiply before add)
=	**2550**, which is, of course, exactly what 34 times 75 is.	

A Geometric Picture of the Double Distributive Property

	c	d
a	ac	ad
b	bc	bd

Looking at the entire shaded rectangle, its dimensions are $a + b$ by $c + d$. Since the area of a rectangle is the product of its length and its width, the area of the entire rectangle is $(a + b)(c + d)$.

On the other hand, the areas of the four smaller rectangles have been labeled using the "length × width" formula. And it's clear that the area of the entire rectangle is equal to the sum of the areas of the four smaller rectangles.

Therefore,

$$\underbrace{(a+b)(c+d)}_{\text{the large rectangle}} = \underbrace{ac+ad+bc+bd}_{\text{the 4 small rectangles}}$$

Yet Another View of Double Distributing

Let's multiply $a + b$ times $c + d$ the way it might be done in elementary school:

$$\begin{array}{r} a + b \\ \times \quad c + d \\ \hline ad \quad bd \\ ac \quad bc \quad\quad \\ \hline \boldsymbol{ac + ad + bc + bd} \end{array}$$

Multiply *d* by *b*, and then multiply *d* by *a*. Now go down a row and shift over. Multiply *c* by *b* and then multiply *c* by *a*. Draw a line and add.

Review Problems

7. Simplify:

a. $(x + 9)(x + 8)$
b. $(y - 1)(y - 8)$
c. $(2z + 5)(2z - 5)$
d. $(N + 10)(N - 10)$
e. $(x - 9)^2$
f. $(a + 5)^2$
g. $(t + 9)(t - 5)$
h. $(a - 22)(a + 1)$
i. $(a - 11)(a + 2)$
j. $(2x + 1)(x - 5)$
k. $(3x + 8)(2x - 5)$
l. $(6x + 5)(x - 3)$
m. $(6a + 17)(a - 1)$
n. $(R + 12)(R - 12)$
o. $(5n - 3)^2$
p. $(1 - a)(2 - a)$
q. $(7w + 5)^2$
r. $(3a - 1)(3a - 2)$
s. $(9a - 1)(a - 2)$
t. $(x + 18)(x - 2)$
u. $(x + 36)(x + 1)$
v. $(5c - 1)(6c - 1)$
w. $(8a + 1)(2a - 1)$
x. $(6q + 5)^2$
y. $(3 + n)(3 - n)$
z. $(16n - 9)(2n - 3)$

8. Prove that $(u + w)^4 \neq u^4 + w^4$.

Solutions

1.
a. $xw + xz + wy + yz$
b. $ac - bc + ad - bd$
c. $xy + 3x + 2y + 6$
d. $x^2 + 7x + 12$
e. $n^2 - 5n + 4$
f. $a^2 - 4a - 21$
g. $y^2 - 81$
h. $u^2 - 9$
i. $t^2 - 39t + 380$
j. $z^2 + 6z + 9$
k. $v^2 - 8v + 16$
l. $N^2 - 1$

2.
a. $3a^2 - 20a - 63$
b. $2n^2 + 5n - 12$
c. $3n^2 - 11n + 8$
d. $25x^2 + 65x + 42$
e. $49w^2 - 4$
f. $x^2 - 144$

g. $4y^2 + 4y + 1$ h. $42x^2 - 31x - 21$ i. $3q^2 + 14q - 49$

j. $9n^2 + 6n + 1$ k. $18x^2 - 27x - 35$ l. $u^2 - 14u + 49$

3. a. $y^2 + 8y + 16$ b. $z^2 - 18z + 81$ c. $9x^2 + 30x + 25$

d. $4a^2 - 4a + 1$ e. $n^2 + 24n + 144$ f. $36t^2 - 84t + 49$

g. $q^2 - 30q + 225$ h. $25b^2 + 30b + 9$ i. $49u^2 - 14u + 1$

j. $4x^2 + 4x + 1$ k. $9h^2 - 72h + 144$ l. $25y^2 - 50y + 25$

4. a. $ac - ad + bc - bd$ b. $4x^2 - 9$ c. $9n^2 - 6n + 1$

d. $6t^2 - 7t - 3$ e. $6x^2 - 24$ f. $n^2 - 1$

g. $42a^2 - 130a + 100$ h. $100c^2 + 140c + 49$ i. $L^2 + 8L + 16$

j. $21x^2 + 40x - 21$ k. $169n^2 - 49$ l. $144d^2 - 480d + 400$

5. i) By letting $a = 3$ and $b = 4$, for instance, we get:

$(a + b)^2 = (\mathbf{3} + \mathbf{4})^2 = 7^2 = 49$, whereas

$a^2 + b^2 = \mathbf{3}^2 + \mathbf{4}^2 = 9 + 16 = 25$.

Clearly, $(a + b)^2 \neq a^2 + b^2$ (except in rare circumstances)

ii) $(a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$

6. Choosing, for example, $x = 1$ and $y = 2$, we would get the following results:

$$(x + y)^3 = (\mathbf{1} + \mathbf{2})^3 = 3^3 = 27;$$

on the other hand, $x^3 + y^3 = \mathbf{1}^3 + \mathbf{2}^3 = 1 + 8 = 9$.

7. a. $x^2 + 17x + 72$ b. $y^2 - 9y + 8$ c. $4z^2 - 25$

d. $N^2 - 100$ e. $x^2 - 18x + 81$ f. $a^2 + 10a + 25$

g. $t^2 + 4t - 45$ h. $a^2 - 21a - 22$ i. $a^2 - 9a - 22$

j. $2x^2 - 9x - 5$ k. $6x^2 + x - 40$ l. $6x^2 - 13x - 15$

m. $6a^2 + 11a - 17$ n. $R^2 - 144$ o. $25n^2 - 30n + 9$
p. $a^2 - 3a + 2$ q. $49w^2 + 70w + 25$ r. $9a^2 - 9a + 2$
s. $9a^2 - 19a + 2$ t. $x^2 + 16x - 36$ u. $x^2 + 37x + 36$
v. $30c^2 - 11c + 1$ w. $16a^2 - 6a - 1$ x. $36q^2 + 60q + 25$
y. $9 - n^2$, or $-n^2 + 9$ z. $32n^2 - 66n + 27$

8. Choosing, for example, $u = 2$ and $w = 3$,

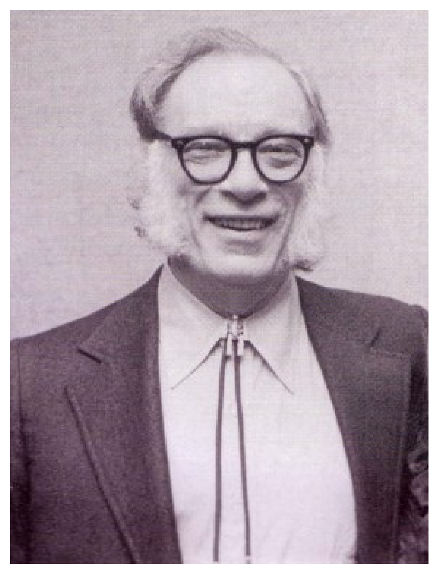
$$(2 + 3)^4 = 5^4 = 625, \text{ whereas } 2^4 + 3^4 = 16 + 81 = 97.$$

❑ *To ∞ and Beyond*

A. Simplify: $(x + 2)(x^2 - 3x - 10)$

B. Simplify: $(2x + 5)^3$

"If knowledge can create problems, it is not through ignorance that we can solve them."



– Isaac Asimov (1920 - 1992)