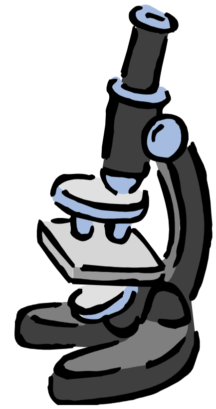
FINDING THE EQUATION OF A LINE

❑ *INTRODUCTION*

Given an equation like $y = 7x + 3$, we've learned that its graph is a line with a slope of 7 and a y-intercept of (0, 3), and we certainly could find other points on the line. In this chapter we turn the tables -- given some information about the line, we try to seek the equation of the line. For you CSI fans, it's like using blood and hair samples to determine the identity of the killer.



❑ *FINDING THE LINE EQUATION GIVEN THE SLOPE AND THE y-INTERCEPT*

EXAMPLE 1: **Find the equation of the line which has a slope of –3 and whose *y*-intercept is the point (0, 9).**

Solution: This is the easiest possible problem asking us to find the equation of a line. Here's why: The slope-intercept form of a line we've learned is

$$y = mx + b$$

where m is the slope and $(0, b)$ is the y-intercept. To "fill in the blanks" of this equation, we need m and b. Now look at what's given to us in the problem: the slope and the y-intercept. That is, we're told that $m = -3$ and $b = 9$. We're done!

$$\boxed{y = -3x + 9}$$

❑ FINDING THE LINE EQUATION GIVEN THE SLOPE AND A POINT

EXAMPLE 1: **Find the equation of the line which has a slope of 7 and which passes through the point (–5, 3).**

Solution: The line equation we are using is $y = mx + b$, where m is the slope and $(0, b)$ is the y-intercept. In this example, we are given the slope of 7. That's good.

And so the line equation $y = mx + b$

becomes $y = 7x + b$ (putting the 7 in for slope)

But did the problem give us the y-intercept? No; instead, we are given a point on the line, but I know that it's not the y-intercept. (How can I be so sure?) So now our goal is to find the value of b.

Consider that the problem tells us that the point (–5, 3) is on the line. Therefore, the point <u>must</u> work in the equation we just wrote, $y = 7x + b$. So we plug –5 in for x and plug 3 in for y:

$y = 7x + b$ (our line with the slope plugged in)

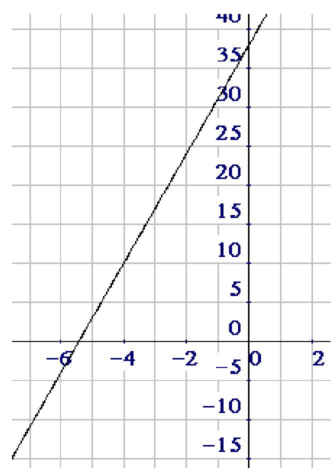
$\Rightarrow$ $\mathbf{3} = 7(\mathbf{-5}) + b$ (since (–5, 3) lies on the line)

$\Rightarrow$ $3 = -35 + b$ (multiply)

$\Rightarrow$ $b = 38$ (solve for b)

Now we put it all together. The value of m is 7 (given in the problem) and the value of b is 38 (we just calculated it). We get our final answer:

$$\boxed{y = 7x + 38}$$



Homework

1. Find the equation of the line with the given slope and passing through the given point:

a. $m = -3;$ (8, –16)

b. $m = 4;$ (–1, –11)

c. $m = 2;$ (0, –10)

d. $m = -3;$ (3, –2)

e. $m = 9;$ (2, 18)

f. $m = -1;$ (–3, 11)

g. $m = 1;$ (5, 5)

h. $m = 7;$ (0, 10)

i. $m = -8;$ (–2, 29)

j. $m = 1;$ (10, –89)

❑ *FINDING THE LINE EQUATION GIVEN TWO POINTS*

EXAMPLE 2: **Find the equation of the line passing through the points (–1, 3) and (8, –15).**

Solution: This problem will really test our deductive skills. Let's begin with the slope-intercept form of a line:

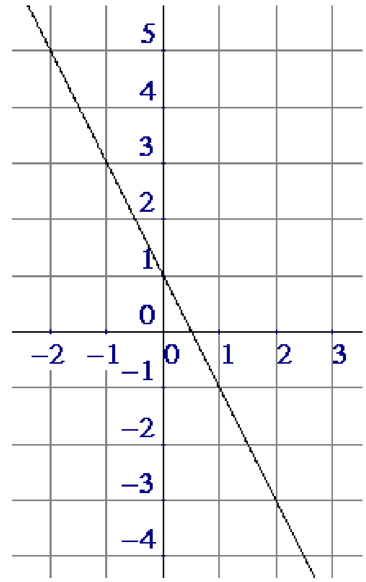
$$y = mx + b$$ [we need the slope and the *y*-intercept]

Did the problem tell us what the slope is? No. Did the problem give us the *y*-intercept? No. This is not good -- how can we possibly solve this problem? Well, even though the slope was not handed to us on a silver platter, we can use the two given points on the line to calculate the slope, using our $m = \frac{\Delta y}{\Delta x}$ formula. So, using the given points (–1, 3) and (8, –15), we find the slope:

$$m = \frac{\Delta y}{\Delta x} = \frac{3-(-15)}{-1-8} = \frac{3+15}{-1-8} = \frac{18}{-9} = -2$$

We can now write our line as

$$y = -2x + b$$



How do we find b? The same way we did in the previous example: Plug the coordinates of one of the original points in for x and y -- either point will do the job, since each of them lies on the line. Let's use the point (–1, 3).

$$
\begin{aligned}
& & y &= -2x + b \\
&\Rightarrow & \mathbf{3} &= -2(\mathbf{-1}) + b \\
&\Rightarrow & 3 &= 2 + b \\
&\Rightarrow & b &= 1
\end{aligned}
$$

Putting the values of m and b into the formula $y = mx + b$ gives us the line which passes through the two points:

$$\boxed{y = -2x + 1}$$

✓ **<u>Check:</u>** Let's check our final answer. If $y = -2x + 1$ is really the line passing through the two given points, then obviously each point should lie on the line. That is, each point should satisfy the equation of the line.

$y = -2x + 1$ Is this the right equation?

(–1, 3): $\mathbf{3} = -2(\mathbf{-1}) + 1 \Rightarrow 3 = 2 + 1 \Rightarrow 3 = 3$ ✓

(8, –15): $\mathbf{-15} = -2(\mathbf{8}) + 1 \Rightarrow -15 = -16 + 1 \Rightarrow -15 = -15$ ✓

Homework

2. Find the equation of the line passing through the two given points. Also be sure you know how to check your answer:

a. (3, 13) and (–1, 5)

b. (1, –9) and (–5, 39)

c. (0, –1) and (2, 9)

d. (1, –11) and (6, –1)

e. (1, –8) and (–2, 31)

f. (0, 0) and (–5, 35)

g. (7, 7) and (–3, –3)

h. (0, 17) and (–17, 0)

i. (–5, –4) and (2, –11)

j. (1, –3) and (–4, –38)

k. (2, –30) and (–1, –3)

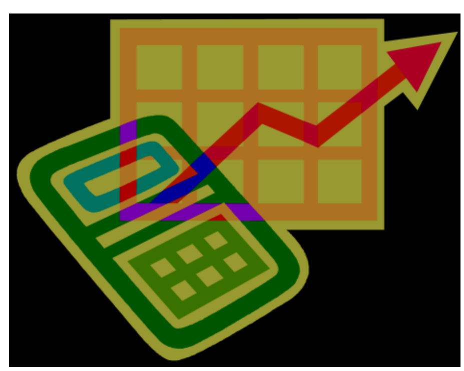
l. (–4, –23) and (–1, 7)

m. (25, –43) and (–10, 27)

n. (1, –13) and (–13, –97)

Review Problems

3. Find the slope and y-intercept of the line $-28x + 4y = 16$ by converting the line to $y = mx + b$ form.



4. Find the equation of the line which has slope 5 and which passes through the point (6, 30).

5. Find the equation of the line which passes through the points (2, 15) and (–2, –9).

6. Find the slope and y-intercept of the line $-45x - 5y = -15$ by converting the line to $y = mx + b$ form.

7. Find the equation of the line which has slope –4 and which passes through the point (–5, 22).

8. Find the equation of the line which passes through the points (–2, –6) and (7, 3).

Solutions

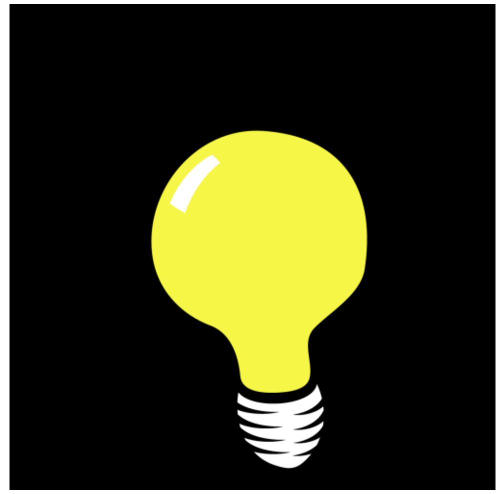
1.
a. $y = -3x + 8$ b. $y = 4x - 7$ c. $y = 2x - 10$
d. $y = -3x + 7$ e. $y = 9x$ f. $y = -x + 8$
g. $y = x$ h. $y = 7x + 10$ i. $y = -8x + 13$
j. $y = x - 99$

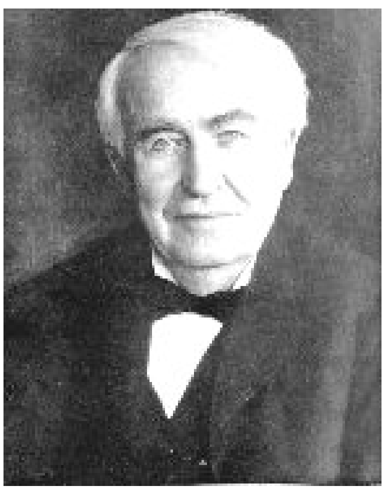
2.
a. $y = 2x + 7$ b. $y = -8x - 1$ c. $y = 5x - 1$
d. $y = 2x - 13$ e. $y = -13x + 5$ f. $y = -7x$
g. $y = x$ h. $y = x + 17$ i. $y = -x - 9$
j. $y = 7x - 10$ k. $y = -9x - 12$ l. $y = 10x + 17$
m. $y = -2x + 7$ n. $y = 6x - 19$

3. $m = 7$; y-int: (0, 4) **4.** $y = 5x$ **5.** $y = 6x + 3$

6. $m = -9$; y-int: (0, 3) **7.** $y = -4x + 2$ **8.** $y = x - 4$

“I haven't failed, I've found 1,000 ways that don't work.”





Thomas Edison