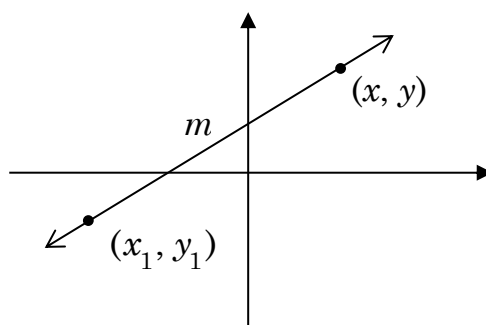

THE POINT-SLOPE FORM OF A LINE

The slope-intercept form of a line, $y = mx + b$, is perfect when you have the slope and the y -intercept. But the odds of this are slim. It's more likely that you'll be working with the slope and some point on the line other than the y -intercept.

THEOREM: The equation of the line with slope m and passing through the point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

PROOF: We begin by sketching the line and labeling the given point (x_1, y_1) , the slope m , and a generic point (x, y) :



On the one hand, the slope of the line is given by m . On the other hand, the slope of the line can be calculated using the two points

(x, y) and (x_1, y_1) : $\frac{y - y_1}{x - x_1}$. And, of course, these two slopes must

be the same:

$$\frac{y - y_1}{x - x_1} = m$$

$$\Rightarrow y - y_1 = m(x - x_1) \quad \text{Q.E.D.}$$

EXAMPLE 1: Find the equation of the line whose slope is -3 and which passes through the point $(8, -2)$.

Solution: This is precisely the data we need to use the point-slope form, $y - y_1 = m(x - x_1)$. We're given the slope, so $m = -3$. We're also given a point on the line, so $(x_1, y_1) = (8, -2)$. Plugging these values into the point-slope form gives us

$$y - (-2) = -3(x - 8), \text{ or}$$

$$\boxed{y + 2 = -3(x - 8)}$$

EXAMPLE 2: Find the equation of the line passing through the two points $(3, -5)$ and $(-2, -8)$.

Solution: The point-slope form, $y - y_1 = m(x - x_1)$, requires a point (we have two of them), and the slope, which we'll have to calculate ourselves.

$$m = \frac{\Delta y}{\Delta x} = \frac{-5 - (-8)}{3 - (-2)} = \frac{-5 + 8}{3 + 2} = \frac{3}{5}$$

Now, using the point $(3, -5)$ (either point would work), we get our equation

$$y - (-5) = \frac{3}{5}(x - 3), \text{ or } \boxed{y + 5 = \frac{3}{5}(x - 3)}$$

Homework

1. Use the point-slope formula to find the equation of the line with slope 7 and passing through the point $(6, -8)$.
2. Use the point-slope formula to find the equation of the line with slope 0 and passing through the point $(-17, 9)$.
3. Use the point-slope formula to find the equation of the line with slope $-\frac{4}{7}$ and passing through the point $(\frac{1}{2}, \pi)$.
4. Use the point-slope formula to find the equation of the line which passes through the points $(-2, 4)$ and $(5, -5)$.
5. Use the point-slope formula to find the equation of the line which passes through the points $(\pi, \sqrt{2})$ and $(-3, 1)$.

Solutions

1. Each output is identical to the input.
2. $y + 8 = 7(x - 6)$
3. $y - 9 = 0$
4. $y - \pi = -\frac{4}{7}(x - \frac{1}{2})$
5. $y - 4 = -\frac{9}{7}(x + 2)$

*An investment in
knowledge always pays
the best interest.*

Benjamin Franklin