
SOLVING LINEAR EQUATIONS USING LIKE TERMS

□ INTRODUCTION

We now have the tool -- *combining like terms* -- that we need to solve equations like $7x - 2x - 9 = 12x + 1$. This chapter will start with solving equations, and then end with an application analyzing cell phone plans.

□ SOLVING EQUATIONS BY COMBINING LIKE TERMS

EXAMPLE 1: Solve each equation:

A. $7x + 3x = 20$

$$10x = 20 \quad \text{(combine like terms)}$$

$$\frac{10x}{10} = \frac{20}{10} \quad \text{(divide each side by 10)}$$

$$x = 2 \quad \text{(simplify)}$$

B. $4y - 2y = 15$

$$2y = 15 \quad \text{(combine like terms)}$$

$$\frac{2y}{2} = \frac{15}{2} \quad \text{(divide each side by 2)}$$

$$y = \frac{15}{2} \quad \text{(simplify)}$$

C. $23a = 3a - 80$

This is a little different from the equations we've seen so far in this course. Why? Because the variable sits on both sides of the equation -- it's hard to isolate something when pieces of it lie all over the place. Our plan is as follows: Get all the variables to one side of the equation. You have the option of which side, but I prefer the left side.

So, let's begin by writing the given equation:

$$23a = 3a - 80$$

I hope it's clear to you that if we're going to get all the a 's to the left side of the equation, we must remove all the a 's on the right side. So our first step is to subtract $3a$ from each side of the equation:

$$\begin{array}{r} 23a = 3a - 80 \\ - 3a \quad - 3a \end{array}$$

Now we combine the like terms on both sides of the equation:

$$20a = -80$$

The rest is old hat; divide each side by 20:

$$\frac{20a}{20} = \frac{-80}{20}$$

And simplify:

$$a = -4$$

CHECK:

$23a$	$3a - 80$
$23(-4)$	$3(-4) - 80$
<u>-92</u>	$-12 - 80$
	<u>-92</u>
	✓

D. $10n - 3 = 9 - 6n$

$$10n + \mathbf{6n} - 3 = 9 - 6n + \mathbf{6n} \quad (\text{add } 6n \text{ to each side})$$

$$16n - 3 = 9 \quad (\text{combine like terms})$$

$$16n - 3 + 3 = 9 + 3 \quad (\text{add } 3 \text{ to each side})$$

$$16n = 12 \quad (\text{simplify})$$

$$\frac{16n}{16} = \frac{12}{16} \quad (\text{divide each side by } 4)$$

$$n = \frac{3}{4} \quad (\text{simplify})$$

EXAMPLE 2: Solve for u : $3u - 7 + 8u - 1 = 9 - 2u + 12 - 6u$

Solution: This is one long equation! But we have all the skills required to solve it. The secret is to temporarily ignore the equation aspect of the problem, and start by simplifying each side of the equation separately -- by combining like terms. Then the equation will look similar to part D of the previous example, and we'll be in good shape.

$$3u - 7 + 8u - 1 = 9 - 2u + 12 - 6u \quad (\text{the original equation})$$

$$\Rightarrow 3u + 8u - 7 - 1 = -2u - 6u + 9 + 12 \quad (\text{rearrange the terms})$$

$$\Rightarrow 11u - 8 = -8u + 21 \quad (\text{combine like terms})$$

$$\Rightarrow 11u + 8u - 8 = -8u + 8u + 21 \quad (\text{add } 8u \text{ to each side})$$

$$\Rightarrow 19u - 8 = 21 \quad (\text{combine like terms})$$

$$\Rightarrow 19u - 8 + 8 = 21 + 8 \quad (\text{add } 8 \text{ to each side})$$

$$\Rightarrow 19u = 29 \quad (\text{do the arithmetic})$$

$$\Rightarrow \frac{19u}{19} = \frac{29}{19} \quad (\text{divide each side by } 19)$$

$$\Rightarrow \boxed{u = \frac{29}{19}} \quad (\text{simplify})$$

Homework

1. Solve each equation:

a. $2x + 1 = 10$

b. $-3n - 7 = -1$

c. $7z - 1 = -5$

d. $7x - 2x = 50$

e. $-6y + y = 12$

f. $30 = 7n - n$

g. $7a - 7 = 2a$

h. $8 - 2x = 10x$

i. $8m - 5 = m$

j. $4 - q = 10$

k. $20 - 7u = 7u$

l. $10 - 7x = 17x$

2. Solve each equation:

a. $4x - 7 = 2x + 1$

b. $20n - 20 = 18n - 1$

c. $12 - n = 14 - 2n$

d. $6v - 2v = 10 + 20$

e. $2h + 7 = -3h + 7$

f. $12p + 13 = 11p + 14$

g. $-7x - 8 = -9x - 8$

h. $9b + 7 = 8 + 12b$

i. $17a - 15 = -9a - 5$

j. $23c - 80 = -17c - 90$

3. Solve each equation:

a. $8x - 3x + 7 = 14x - 8$

b. $9e + 7 - 3e - 8 = 12 - 5e + 6 - 4e$

c. $3u - 7 + 9u - 1 = 19 - 2u + 13 - 5u$

d. $3x + 2x + 3 + 2 = 19x - 17x + 8 - 3$

e. $23a - 17 - 18 - 24a = 100 + a - 44a - 105$

$$f. -5h + 2 - 3h - 1 = -5h + 7 - 4h + 11$$

$$g. -10a + 8 + 5a + 8 = -2a + 4 + 4a + 12$$

$$h. -2j - 6 + 6j - 12 = 9j + 2 - 7j - 6$$

□ CELL PHONE APPLICATION

EXAMPLE 3: Sprint offers you a monthly phone plan that has a signup fee of \$20, plus \$0.25/min. Verizon's plan has no signup fee, but charges \$0.65/min.



- a. Assuming an average of 40 minutes per month, which is the better plan?
- b. Assuming an average of 70 minutes per month, which is the better plan?
- c. How many minutes of phone use would result in an equal charge from each company?

Solution: Let's write a formula for each company's cell phone plan. If we let m represent the number of minutes of use, we get the following two formulas:

$$S = 20 + 0.25m \quad (\text{for Sprint})$$

$$V = 0.65m \quad (\text{for Verizon})$$

- a. Calculate each plan's cost for 40 minutes of use:

$$S = 20 + 0.25m = 20 + 0.25(40) = 20 + 10 = 30$$

$$V = 0.65m = 0.65(40) = 26$$

For 40 minutes of use, Sprint charges \$30.00, while Verizon charges \$26.00. Thus, in this scenario, Verizon has the better plan.

- b. Now we'll assume 70 minutes of use:

$$S = 20 + 0.25m = 20 + 0.25(\mathbf{70}) = 20 + 17.5 = 37.5$$

$$V = 0.65m = 0.65(\mathbf{70}) = 44.5$$

For 70 minutes, Sprint charges \$37.50 and Verizon charges \$44.50. So this time it's Sprint that has the better plan.

- c. To determine the *break-even point* -- the number of minutes where each plan costs the same -- all we have to do is set Sprint's charges equal to Verizon's and solve the resulting equation:

$$S = V$$

Write each company's formula for their charges:

$$20 + 0.25m = 0.65m$$

To get all the variables on the left side, subtract $0.65m$ from each side of the equation:

$$20 + 0.25m - \mathbf{0.65m} = 0.65m - \mathbf{0.65m}$$

And simplify by combining like terms:

$$20 - 0.4m = 0$$

Subtract the constant from each side of the equation:

$$20 - \mathbf{20} - 0.4m = 0 - \mathbf{20}$$

And simplify:

$$-0.4m = -20$$

Last step -- divide each side of the equation by -0.4 :

$$\frac{-0.4m}{-\mathbf{0.4}} = \frac{-20}{-\mathbf{0.4}}$$

And we're done:

$$m = 50$$

We conclude that if we talk 50 minutes in a month, each plan will cost the same. From parts a. and b. of this problem, it should also be clear that Verizon is cheaper if you use less than 50 minutes of talk time each month, but that Sprint's plan is better if you usually exceed 50 minutes per month.

Homework

4. For each problem, I give you two cell phone plans. You need to determine the number of minutes that would result in the same charge for each plan:
- | | | | |
|----|---------|------------------|--------------------|
| a. | Plan #1 | Signup fee: \$40 | Per minute: \$0.10 |
| | Plan #2 | Signup fee: \$0 | Per minute: \$0.20 |
| b. | Plan #1 | Signup fee: \$25 | Per minute: \$0.20 |
| | Plan #2 | Signup fee: \$50 | Per minute: \$0.15 |
| c. | Plan #1 | Signup fee: \$15 | Per minute: \$0.25 |
| | Plan #2 | Signup fee: \$38 | Per minute: \$0.05 |



Review Problems

5. Solve each equation:

a. $w + 3 - 8w + 8 = 7w + 5 - 9w - 10$

b. $3b + 8 - 2b + 6 = -2b + 8 - 6b - 1$

c. $-10g + 6 + 5g + 2 = 10g + 9 - 3g + 6$

d. $-7t - 2 + 9t - 4 = 9t - 2 + 5t + 10$

6. Here are two cell phone plans:

Plan #1	Signup fee: \$19.00	Per minute: \$0.65
Plan #2	Signup fee: \$6.00	Per minute: \$0.90

How many minutes of talk time would result in the same charge for each plan?

Solutions

1. a. $x = \frac{9}{2}$ b. $n = -2$ c. $z = -\frac{4}{7}$ d. $x = 10$
 e. $y = -\frac{12}{5}$ f. $n = 5$ g. $a = \frac{7}{5}$ h. $x = \frac{2}{3}$
 i. $m = \frac{5}{7}$ j. $q = -6$ k. $u = \frac{10}{7}$ l. $x = \frac{5}{12}$
2. a. $x = 4$ b. $n = \frac{19}{2}$ c. $n = 2$ d. $v = \frac{15}{2}$
 e. $h = 0$ f. $p = 1$ g. $x = 0$ h. $b = -\frac{1}{3}$
 i. $a = \frac{5}{13}$ j. $c = -\frac{1}{4}$

3. a. $x = \frac{5}{3}$ b. $e = \frac{19}{15}$ c. $u = \frac{40}{19}$ d. $x = 0$

e. $a = \frac{5}{7}$ f. $h = 17$ g. $a = 0$ h. $j = 7$

4. a. 400 minutes b. 500 minutes c. 115 minutes

5. a. $w = \frac{16}{5}$ b. $b = -\frac{7}{9}$ c. $g = -\frac{7}{12}$ d. $t = -\frac{7}{6}$

6. 52 minutes

□ **TO ∞ AND BEYOND!**

Max wants to save up for a Kindle costing \$200. Max's dad said that if he gets a job, whatever he earns, dad will give him twice that amount toward the purchase of the Kindle. In addition, Aunt Edna has promised to contribute \$20 to the cause. By letting M represent the amount of money that Max earns at his job, write an equation which represents this situation. Solve the equation to see how much Max must earn himself in order to reach his \$200 goal.



“When I get a little money,
I buy books;
and if any is left,
I buy food and clothes.”

Erasmus, early 1500s

