SOLVING QUADRATICS EQUATIONS BY FACTORING

❑ INTRODUCTION

Consider the quadratic equation

$$6x^2 + 7x = 0$$

a type of quadratic equation you migh already know how to solve. We first factored the left side of the equation, via the GCF:

$$x(6x + 7) = 0$$ NOTE: The factors are x and $6x + 7$.

Then we used the principle that if the product of two factors is 0, then either of the factors could be 0, so we then set each factor to 0, generating two linear equations to solve:

> If $ab = 0$ then $a = 0$ or $b = 0$.

$$x = 0 \quad \text{and} \quad 6x + 7 = 0$$

Solving these two equations gives us the solutions 0 and $-\frac{7}{6}$.

❑ SOLVING QUADRATIC EQUATIONS BY FACTORING

How do you think we should solve the quadratic equation

$$2w^2 - 31w + 84 = 0 \text{ ?}$$

We use the same logic as above -- factor the quadratic expression on the left side of the equation (noting that the right side is 0), set each factor to 0, and then solve each linear equation.

EXAMPLE 1: **Solve the quadratic equation $2w^2 - 31w + 84 = 0$.**

Solution:

	$2w^2 - 31w + 84 = 0$	(the original equation)
$\Rightarrow$	$(2w - 7)(w - 12) = 0$	(factor the quadratic)
$\Rightarrow$	$2w - 7 = 0$ or $w - 12 = 0$	(set each factor to 0)
$\Rightarrow$	$2w = 7$ or $w = 12$	(solve each equation)
$\Rightarrow$	$w = \frac{7}{2}$ or $w = 12$	

Thus, the solutions for w are

$$\boxed{\frac{7}{2}, 12}$$

If $ab = 0$,
then $a = 0$ or $b = 0$.

EXAMPLE 2: **Solve for x: $3x^2 - 7x - 40 = 0$**

Solution: This equation is in standard quadratic form, so it's all set to factor:

	$3x^2 - 7x - 40 = 0$	(the original equation)
$\Rightarrow$	$(3x + 8)(x - 5) = 0$	(factor the left side)
$\Rightarrow$	$3x + 8 = 0$ or $x - 5 = 0$	(set each factor to 0)
$\Rightarrow$	$x = \frac{-8}{3} = -\frac{8}{3}$ or $x = 5$	(solve each equation)

Thus, the final solutions to the quadratic equation are $\boxed{5, -\frac{8}{3}}$

It's time to practice our equation-checking. Letting $x = 5$ in the <u>original</u> equation $3x^2 - 7x - 40 = 0$ gives

$$3(\mathbf{5})^2 - 7(\mathbf{5}) - 40 = 3(25) - 7(5) - 40 = 75 - 35 - 40 = 0 \checkmark$$

Trying $x = -\frac{8}{3}$ in the original equation produces

$$3\left(-\frac{\mathbf{8}}{\mathbf{3}}\right)^2 - 7\left(-\frac{\mathbf{8}}{\mathbf{3}}\right) - 40 = 3\left(\frac{64}{9}\right) - 7\left(-\frac{8}{3}\right) - 40$$

$$= \frac{64}{3} + \frac{56}{3} - \frac{120}{3} = \frac{120}{3} - \frac{120}{3} = 0 \quad \checkmark$$

<u>EXAMPLE 3:</u> **Solve for y: $9y^2 - 16 = 0$**

<u>Solution:</u> It's a good-looking quadratic (even though the middle term, the y-term, is missing), so let's factor and set the factors to zero:

	$9y^2 - 16 = 0$	(the original equation)
$\Rightarrow$	$(3y + 4)(3y - 4) = 0$	(factor the left side)
$\Rightarrow$	$3y + 4 = 0$ or $3y - 4 = 0$	(set each factor to 0)
$\Rightarrow$	$3y = -4$ or $3y = 4$	(remove the 4's)
$\Rightarrow$	$y = -\frac{4}{3}$ or $y = \frac{4}{3}$	(divide both equations by 3)

Therefore, the solutions of the equation are

$$\boxed{\frac{4}{3}, -\frac{4}{3}}$$ which can also be written $\pm\frac{4}{3}$.

EXAMPLE 4: **Solve for u: $9u^2 = 42u - 49$**

Solution: This quadratic equation is not in standard form, so the first two steps will be to transform it into standard form:

	$9u^2 = 42u - 49$	(the original equation)
$\Rightarrow$	$9u^2 - 42u = -49$	(subtract $42u$)
$\Rightarrow$	$9u^2 - 42u + 49 = 0$	(add 49 -- standard form)
$\Rightarrow$	$(3u - 7)(3u - 7) = 0$	(factor)
$\Rightarrow$	$3u - 7 = 0$ or $3u - 7 = 0$	(set each factor to 0)
$\Rightarrow$	$u = \frac{7}{3}$ or $u = \frac{7}{3}$	(solve each equation)

We obtained two solutions, but they're equal to each other, so there's just one solution (or are there two solutions which are the same?):

$$\boxed{\frac{7}{3}}$$

Homework

1. Solve each quadratic equation by factoring:

a. $w^2 - 12w + 35 = 0$

b. $2x^2 - 13x + 15 = 0$

c. $n^2 - 25n + 150 = 0$

d. $6a^2 - 31a + 40 = 0$

e. $x^2 + 2x - 15 = 0$

f. $y^2 - 14y + 49 = 0$

g. $z^2 - 9 = 0$

h. $3h^2 - 17h + 10 = 0$

i. $16u^2 - 8u + 1 = 0$

j. $25w^2 - 4 = 0$

2. Solve each quadratic equation by factoring:

a. $2x^2 + 10x - 28 = 0$

b. $25y^2 = 4$

c. $2z^2 + 2 = -4z$

d. $4a^2 = 3 - 4a$

e. $6u^2 = 47u + 8$

f. $0 = 4t^2 + 8t + 3$

g. $x^2 + x - 42 = 0$

h. $2n^2 + n - 3 = 0$

i. $4x^2 = 1$

j. $a^2 + 12a + 36 = 0$

k. $9k^2 = 9k - 2$

l. $9n^2 - 30n + 25 = 0$

m. $6z^2 = 10z$

n. $49t^2 = 4$

o. $25h^2 = 30h - 9$

p. $6x^2 - 7x - 10 = 0$

q. $0 = 6y^2 + y - 2$

r. $4n^2 = 25n + 21$

s. $16q^2 - 24q + 9 = 0$

t. $64t^2 - 9 = 0$

u. $-n^2 + n + 56 = 0$ [Hint: multiply each side by -1]

v. $-2x^2 + x + 3 = 0$

w. $-14a^2 = 3 - 13a$

Review Problems

3. Solve by factoring: $25y^2 = 1$

4. Solve by factoring: $n^2 + 16n + 64 = 0$

5. Solve by factoring: $26y^2 = 5y$

6. Solve by factoring: $25t^2 + 30t + 9 = 0$

7. Solve by factoring: $24x^2 = 14x + 3$

8. Solve by factoring: $81h^2 = 25$

Solutions

1. a. 5, 7 b. $5, \frac{3}{2}$ c. 10, 15 d. $\frac{8}{3}, \frac{5}{2}$
e. 3, –5 f. 7 g. ± 3 h. $5, \frac{2}{3}$
i. $\frac{1}{4}$ j. $\pm\frac{2}{5}$

2. a. 2, –7 b. $\pm\frac{2}{5}$ c. –1 d. $\frac{1}{2}, -\frac{3}{2}$
e. $8, -\frac{1}{6}$ f. $-\frac{1}{2}, -\frac{3}{2}$ g. 6, –7 h. $1, -\frac{3}{2}$
i. $\pm\frac{1}{2}$ j. –6 k. $\frac{1}{3}, \frac{2}{3}$ l. $\frac{5}{3}$
m. $0, \frac{5}{3}$ n. $\pm\frac{2}{7}$ o. $\frac{3}{5}$ p. $2, -\frac{5}{6}$
q. $\frac{1}{2}, -\frac{2}{3}$ r. $7, -\frac{3}{4}$ s. $\frac{3}{4}$ t. $\pm\frac{3}{8}$
u. 8, –7 v. $-1, \frac{3}{2}$ w. $\frac{3}{7}, \frac{1}{2}$

3. $\pm\frac{1}{5}$ **4.** –8 **5.** $0, \frac{5}{26}$

6. $-\frac{3}{5}$ **7.** $\frac{3}{4}, -\frac{1}{6}$ **8.** $\pm\frac{5}{9}$

❑ To ∞ and Beyond

A. Solve for *x*: $x^3 + 3x^2 - 10x = 0$

Hint: There are 3 solutions.

B. Solve for *n*: $n^4 - 25n^2 + 144 = 0$

Hint: There are 4 solutions.

"There is no end to education. It is not that you read a book, pass an examination, and finish with education. The whole of life, from the moment you are born to the moment you die, is a process of learning."

Jiddu Krishnamurti

1895 – 1986