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# EXPONENTIAL EQUATIONS

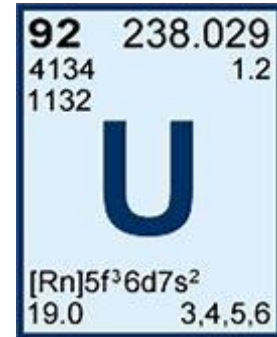
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## □ INTRODUCTION

Watching your investments grow, tracking populations, the decaying of a radioactive substance – these are the kinds of problems which lead to exponential equations.



Uranium, atomic number 92, decays exponentially – the less that remains, the less that decays.

## □ EXAMPLES

An *exponential equation* is an equation with the variable in the exponent, not something we're used to seeing. Let's start with two basic examples.

**#1:** Consider the exponential equation

$$5^x = 25$$

Five raised to what power equals 25? Well, 5 to the 2nd power is 25, so  $x = 2$ . Not so tough.

**#2:** How about the equation

$$3^{2n-1} = 81?$$

You might go about solving this equation by asking yourself “3 to what power is 81?” Since 3 to the 4th power is 81, it follows that the exponent,  $2n - 1$ , must be equal to 4.

Writing this last phrase as an equation, we can find the value of  $n$ :

$$2n - 1 = 4 \Rightarrow 2n = 5 \Rightarrow n = \frac{5}{2}. \text{ Done!}$$

**Check:** Placing  $n = \frac{5}{2}$  into the equation  $3^{2n-1} = 81$  gives

$$3^{2\left(\frac{5}{2}\right)-1} \stackrel{?}{=} 81$$

$$3^{\cancel{2}\left(\frac{5}{\cancel{2}}\right)-1} \stackrel{?}{=} 81$$

$$3^{5-1} \stackrel{?}{=} 81$$

$$3^4 \stackrel{?}{=} 81$$

$$81 = 81 \quad \checkmark$$

**EXAMPLE 1:** Solve for  $x$ :  $3^{7x} = 3^{14}$

**Solution:** Each side of the equation is an exponential expression. Notice that the bases are the same (the 3's), so the only way the two sides of the equation can be equal is if the exponents are equal. In other words,  $7x$  must equal 14:

$$7x = 14,$$

from which we determine that

$x = 2$
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**EXAMPLE 2:** Solve for  $y$ :  $5^{4y} = 25$

**Solution:** We're not as lucky here as we were in Example 1 -- the bases are not the same. But maybe we can make them the same. Suppose we think of 25 as  $5^2$ . Then each side of the equation will have the same base, and we can set the exponents equal to each other to find the value of  $y$ . Let's try all of this:

$$5^{4y} = 25 \quad (\text{the original equation})$$

$$\begin{aligned} \Rightarrow 5^{4y} &= 5^2 && \text{(rewrite 25 with a base of 5)} \\ \Rightarrow 4y &= 2 && \text{(set the exponents equal to each other,} \\ &&& \text{since the bases are the same)} \\ \Rightarrow \boxed{y = \frac{1}{2}} \end{aligned}$$

**EXAMPLE 3:** Solve for  $z$ :  $27^{-4z} = \frac{1}{9}$

**Solution:** This one's terrible! The bases aren't the same, and it contains a fraction. But look at the 27 in the equation -- it's equal to  $3^3$ . And check out the 9 in the denominator -- it can be written as  $3^2$ . So maybe we can write everything in terms of the base 3:

$$\begin{aligned} 27^{-4z} &= \frac{1}{9} && \text{(the original equation)} \\ \Rightarrow (3^3)^{-4z} &= \frac{1}{3^2} && \text{(express 27 and 9 as powers of 3)} \\ \Rightarrow 3^{-12z} &= 3^{-2} && \text{(exponent rules)} \\ \Rightarrow -12z &= -2 && \text{(bases are the same - set exponents equal)} \\ \Rightarrow \boxed{z = \frac{1}{6}} &&& \text{(solve for } z\text{)} \end{aligned}$$

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## Homework

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Solve each exponential equation:

1.  $16^{6p} = 8$

2.  $4^{10q} = 16$

3.  $125^{3t} = \frac{1}{5}$

4.  $16^{6w} = 16$

5.  $4^{-2h} = 64$

6.  $25^{6b} = \frac{1}{25}$

7.  $25^{8w} = 125$

8.  $625^{-4x} = \frac{1}{625}$

9.  $125^{-10a} = \frac{1}{25}$

10.  $16^{3v} = 2$

11.  $4^{9r} = \frac{1}{2}$

12.  $9^{-4p} = 729$

13.  $625^{-3y} = \frac{1}{25}$

14.  $81^{9x} = \frac{1}{729}$

15.  $25^{-2k} = \frac{1}{25}$

16.  $16^{-5x} = 512$

17.  $27^{8b} = \frac{1}{729}$

18.  $81^{6a} = 3$

19.  $81^{-5r} = 81$

20.  $625^{6b} = \frac{1}{25}$

21.  $27^{-5v} = 3$

22.  $81^{-4k} = \frac{1}{9}$

23.  $125^{5p} = \frac{1}{25}$

24.  $125^{-10d} = \frac{1}{5}$

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## Solutions

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1.  $p = \frac{1}{8}$

2.  $q = \frac{1}{5}$

3.  $t = -\frac{1}{9}$

4.  $w = \frac{1}{6}$

5.  $h = -\frac{3}{2}$

6.  $b = -\frac{1}{6}$

7.  $w = \frac{3}{16}$

8.  $x = \frac{1}{4}$

9.  $a = \frac{1}{15}$

10.  $v = \frac{1}{12}$

11.  $r = -\frac{1}{18}$

12.  $p = -\frac{3}{4}$

13.  $y = \frac{1}{6}$

14.  $x = -\frac{1}{6}$

15.  $k = \frac{1}{2}$

16.  $x = -\frac{9}{20}$

17.  $b = -\frac{1}{4}$

18.  $a = \frac{1}{24}$

19.  $r = -\frac{1}{5}$

20.  $b = -\frac{1}{12}$

21.  $v = -\frac{1}{15}$

22.  $k = \frac{1}{8}$

23.  $p = -\frac{2}{15}$

24.  $d = \frac{1}{30}$

*Education* is not merely a means for earning a living or an instrument for the acquisition of wealth. It is an initiation into life of spirit, a training of the human soul in the pursuit of truth, and the practice of virtue."

– Vijaya Lakshmi Pandit