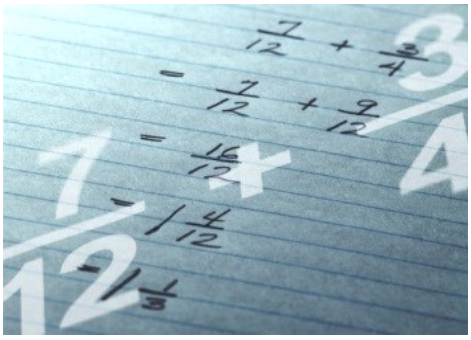
ADDING AND SUBTRACTING FRACTIONS

❑ *INTRODUCTION*



A previous chapter showed us that in order to add or subtract fractions, we must make sure the denominators are the same. So, even though the fractions in this chapter are a little more challenging than we've seen before, the same rules and techniques apply.

❑ *MORE ADDING AND SUBTRACTING*

EXAMPLE 1: **Add:** $\frac{4u-21}{u+7} + \frac{u^2}{u+7}$

Solution: We are adding fractions with the same denominator ($u + 7$), so all we have to do is add the numerators. The common denominator becomes the denominator of the sum:

$$\frac{4u-21+u^2}{u+7}$$

We've added the two fractions so that it's now a single fraction, but all fractional answers need to be reduced -- we'll rearrange the terms in the numerator so it's easier to factor:

$$\frac{u^2+4u-21}{u+7}$$

Factor the numerator and divide out the common factor:

$$\frac{(u+7)(u-3)}{u+7} = \frac{{}^{1}\cancel{(u+7)}(u-3)}{\cancel{u+7}_{1}} = \boxed{u-3}$$

EXAMPLE 2: **Subtract:** $\dfrac{\mathbf{12x-10}}{\mathbf{x^2-2x-8}} - \dfrac{\mathbf{11x-12}}{\mathbf{x^2-2x-8}}$

Solution: Remember that both addition and subtraction of fractions require a *common denominator*. The fractions in this example already have the same denominator, so there's nothing to worry about here.

The tricky part is subtracting the numerators. We must subtract the second numerator (all of it!) from the first numerator -- this is where parentheses come to the rescue:

$$\frac{12x-10}{x^2-2x-8} - \frac{11x-12}{x^2-2x-8}$$ (the original problem)

$$= \frac{(12x-10)-(11x-12)}{x^2-2x-8}$$ (one numerator minus all of the other)

$$= \frac{12x-10-11x+12}{x^2-2x-8}$$ (remove the parentheses, carefully)

$$= \frac{x+2}{x^2-2x-8}$$ (combine like terms)

$$= \frac{x+2}{(x+2)(x-4)}$$ (factor the denominator)

$$= \frac{\cancel{x+2}^{1}}{{}_{1}\cancel{(x+2)}(x-4)}$$ (divide out the common factor)

$$= \boxed{\frac{1}{x-4}}$$ (note that the numerator is 1)

EXAMPLE 3: **Subtract:** $\dfrac{10}{a^6} - \dfrac{7}{a^2}$

Solution: The denominators are different, so we can't subtract the fractions yet. To make the denominators the same, the second denominator (a^2) needs to be built up to match the first denominator (a^6). We ask, what should we multiply a^2 by to get a^6? Since a^2 is two factors of a, and a^6 is six factors of a, we would need four more factors of a to make them match. So we'll multiply the second fraction, top and bottom, by a^4.

$$\frac{10}{a^6} - \frac{7}{a^2}$$

$$= \frac{10}{a^6} - \frac{7 \times a^4}{a^2 \times a^4}$$

$$= \frac{10}{a^6} - \frac{7a^4}{a^6}$$

same denominator

$$= \boxed{\frac{10 - 7a^4}{a^6}}$$

EXAMPLE 4: **Add:** $\dfrac{2}{x^2y^3} + \dfrac{7}{y^7z}$

Solution: This problem also requires a common denominator. To make the denominators the same, we notice that the first denominator needs four more factors of y (so that there will be seven of them) and a factor of z; the second denominator needs an x^2 factor in it. Here's how we do it:

$$\frac{2}{x^2y^3} + \frac{7}{y^7z}$$ (the original problem)

$$= \frac{2 \cdot}{x^2y^3 \cdot} \quad \text{(build to an LCD)}$$

$$= \underbrace{\frac{2y^4z}{\quad} + \frac{7x^2}{\quad}}_{\text{same denominator}} \quad \text{(simplify tops and bottoms)}$$

Now the denominators are the same, so we add the numerators and put that result over the common denominator:

$$\boxed{\frac{2y^4z + 7x^2}{x^2y^7z}}$$

EXAMPLE 5: **Subtract:** $\dfrac{6k+4}{3} - \dfrac{2k+3}{2}$

Solution: The least common denominator (LCD) is 6, so we'll multiply the top and bottom of the first fraction by 2 and the top and bottom of the second fraction by 3:

$$\frac{6k+4}{3} - \frac{2k+3}{2} \quad \text{(the original problem)}$$

$$= \frac{\mathbf{2}(6k+4)}{\mathbf{2}(3)} - \frac{\mathbf{3}(2k+3)}{\mathbf{3}(2)} \quad \text{(create the LCD of 6)}$$

$$= \frac{12k+8}{6} - \frac{6k+9}{6} \quad \text{(simplify tops and bottoms)}$$

Notice the Parentheses!

$$= \frac{(12k+8)-(6k+9)}{6} \quad \text{(combine into a single fraction)}$$

$$= \frac{12k+8-6k-9}{6} \quad \text{(distribute)}$$

$$= \boxed{\frac{6k-1}{6}} \quad \text{(combine like terms)}$$

Homework

1. Add or subtract:

a. $\frac{d^2}{d-9} - \frac{d+72}{d-9}$

b. $\frac{3p+1}{p^2-6p-27} - \frac{2p-2}{p^2-6p-27}$

c. $\frac{n^2}{n-6} - \frac{7n-6}{n-6}$

d. $\frac{16x+63}{x+7} + \frac{x^2}{x+7}$

e. $\frac{-w-9}{w^2+8w-48} - \frac{-2w-5}{w^2+8w-48}$

f. $\frac{-6a-11}{a^2-10a-24} - \frac{-7a+1}{a^2-10a-24}$

2. Add or subtract:

a. $\frac{8}{5w^6} - \frac{3}{2w^4}$

b. $\frac{3}{b^6} + \frac{9}{5b^3}$

c. $\frac{2}{k^2} + \frac{7}{4k^5}$

d. $\frac{1}{9w^2} - \frac{4}{5w^4}$

e. $\frac{2}{5r^2} + \frac{5}{4r^4}$

f. $\frac{1}{z^3} - \frac{7}{6z^6}$

3. Add or subtract:

a. $\frac{2}{c^2m^2} + \frac{3}{m^2w^6}$

b. $\frac{4}{a^5c^5} - \frac{2}{c^6v^2}$

c. $\frac{5}{a^3b^2} + \frac{5}{b^5m^5}$

d. $\frac{2}{b^3h^2} - \frac{6}{h^3x^4}$

e. $\frac{5}{a^3c^2} + \frac{5}{c^4s^6}$

f. $\frac{6}{k^3t^4} - \frac{4}{t^5v^6}$

4. Add or subtract:

a. $\frac{2z-9}{8} + \frac{2z-9}{5}$

b. $\frac{8b+6}{6} - \frac{6b-8}{2}$

c. $\frac{-9m-9}{9} - \frac{-5m-9}{7}$

d. $\frac{4s-9}{3} + \frac{5s-3}{3}$

e. $\frac{-7u+9}{5} - \frac{4u-1}{6}$

f. $\frac{-2k-5}{4} + \frac{9k+1}{5}$

Solutions

1. a. $d + 8$ b. $\frac{1}{p-9}$ c. $n - 1$

d. $x + 9$ e. $\frac{1}{w+12}$ f. $\frac{1}{a+2}$

2. a. $\frac{16-15w^2}{10w^6}$ b. $\frac{15+9b^3}{5b^6}$ c. $\frac{8k^3+7}{4k^5}$

d. $\frac{5w^2-36}{45w^4}$ e. $\frac{8r^2+25}{20r^4}$ f. $\frac{6z^3-7}{6z^6}$

3. a. $\frac{2w^6+3c^2}{c^2m^2w^6}$ b. $\frac{4cv^2-2a^5}{a^5c^6v^2}$ c. $\frac{5b^3m^5+5a^3}{a^3b^5m^5}$

d. $\frac{2hx^4-6b^3}{b^3h^3x^4}$ e. $\frac{5c^2s^6+5a^3}{a^3c^4s^6}$ f. $\frac{6tv^6-4k^3}{k^3t^5v^6}$

4. a. $\frac{26z-117}{40}$ b. $\frac{-5b+15}{3}$ c. $\frac{-2m+2}{7}$

d. $3s - 4$ e. $\frac{-62u+59}{30}$ f. $\frac{26k-21}{20}$